Supervisors Project Submission Form

Supervisor: Charles Walkden

1	Project Title	Ergodic Theory		
2	Category	Pure		
3	Level	4		
4	Semesters (length of project)	1 or 2		
5	Description	Dynamical systems (popularly. if inaccurately, called 'chaos theory') is about studying how a mathematical system evolves under 'time'. A (discrete time) dynamical system consists of a phase space X and a map T : X \to X. One thinks of applying the map T as moving forward one step in time. For x \in X, the sequence (also called the orbit of x) x, T(x), T(T(x)), T(T(T(x))),... shows how the point x evolves. Thus dynamical systems is the study of iterating a map T : X \to X. In dynamical systems often one wants to understand the orbit of every point x \in X. Ergodic theory takes a more qualitative view: instead of understanding the orbit of every point, instead we'll try to understand the long-term behaviour of typical points. To make 'typical' precise we need to use ideas from measure theory. There are many connections between ergodic theory and number theory. For example, a number x \in [0,1] is simply normal (in base 10) if, when written as a decimal, each of the digits 0,1,...,9 appear with equal frequency in the decimal expansion of x. (Thus .125125... (recurring) is not simply normal in base 10 as digit 3 never appears, but .01234567890123456789 0... (recurring) is simply normal in base 10 as each digit occurs with frequency 1/10th.) Ergodic theory can be used to show that almost every number is simply normal in base 10. In fact, we can use ergodic theory to prove that almost every number is simultaneously normal in every base, despite there being no known examples of such numbers (\pi is a good candidate, but nobody can prove this!). Ergodic theory also connects with other areas of mathematics. The project would begin by giving a brief summary of the main ideas in ergodic theory up to and including Birkhoff's Ergodic Theorem. You would then go on to study applications of ergodic theory to (for example) number theory, or to hyperbolic geometry, or to fractal geometry, ... This project contains a lot of hardcore analysis is only suitable for people who like epsilon-delta analysis. You will need to have taken Metric Spaces, but you do not need to have taken a course on Measure Theory.		
6	References			
7	Prerequisite courses			
8	Additional notes			