Stochastic Finite Element Modelling of Elementary Random Media

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We present a numerical framework for elastostatics of random media. Firstly, after a mathematical investigation of the popular white noise model in an engineering context, the smooth spatial stochastic dependence between material properties is identified as a fundamental feature of practical random media. Based on the recognition of the probabilistic essence of practical random media and driven by engineering simulation requirements, a comprehensive random medium model, namely elementary random media (ERM), is consequently defined and its macro-scale properties including stationarity, smoothness and principles for material measurements are systematically explored. Moreover, an explicit representation scheme, namely the Fourier-Karhunen-Loève (F-K-L) discretization scheme, is developed for the general elastic tensor of ERM by combining the spectral representation theory of wide-sense stationary stochastic fields and the standard dimensionality reduction technology of principal component analysis. Then, based on the concept of ERM and the F-K-L representation for its random elastic tensor, the stochastic partial differential equations regarding elastostatics of random media are formulated and further discretized, in a similar fashion as for the standard finite element method, to obtain a stochastic system of linear algebraic equations. For the solution of the resulting stochastic linear algebraic system, a novel solution strategy, termed the joint diagonalization solution strategy, is developed.

Original contributions include the theoretical analysis of practical random medium modelling, establishment of the ERM model and its F-K-L discretization scheme, and development of the novel numerical solver for the stochastic linear algebraic system. In particular, for computational challenges arising from the proposed framework, two novel numerical algorithms are developed: (a) a quadrature algorithm for multidimensional highly oscillatory functions, which reduces the computational cost of the F-K-L representation by up to several orders of magnitude; and (b) a Jacobi-like joint diagonalization solution method for relatively small mesh structures, which can effectively solve the associated stochastic linear algebraic system with a large number of random variables.

References:

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