Non-Linear Aeroelastic Modelling and Prediction

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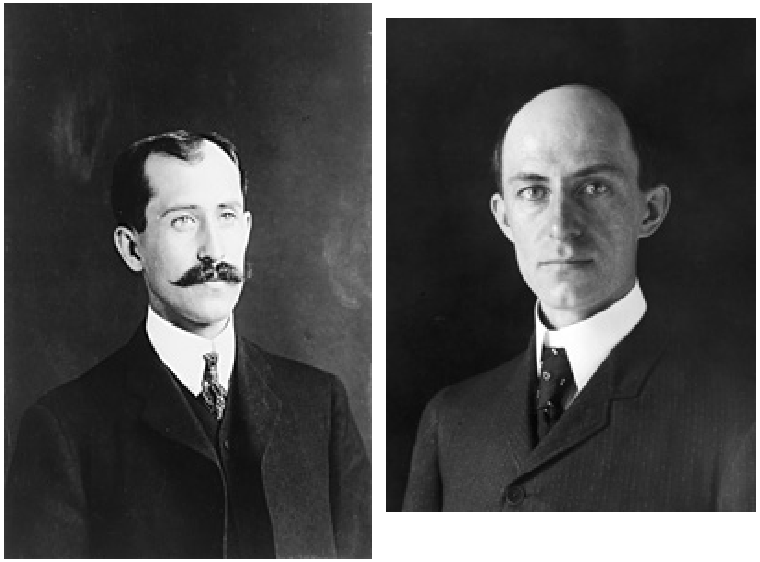
Outline

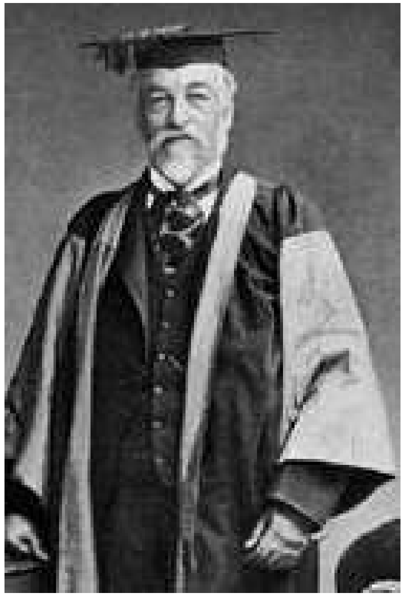
- Overview of Aeroelasticity
- Outline of several aeroelastic phenomena
- Flutter
  - Prediction of stability bounds
- Effect of non-linearities
  - Structural
  - Aerodynamic
  - Control system
- Limit Cycle Oscillations
  - Determination of LCO characteristics
- Future Directions

104 Years Ago in the USA

- Wright brothers were perfecting their “Flyer” at Kitty Hawk.

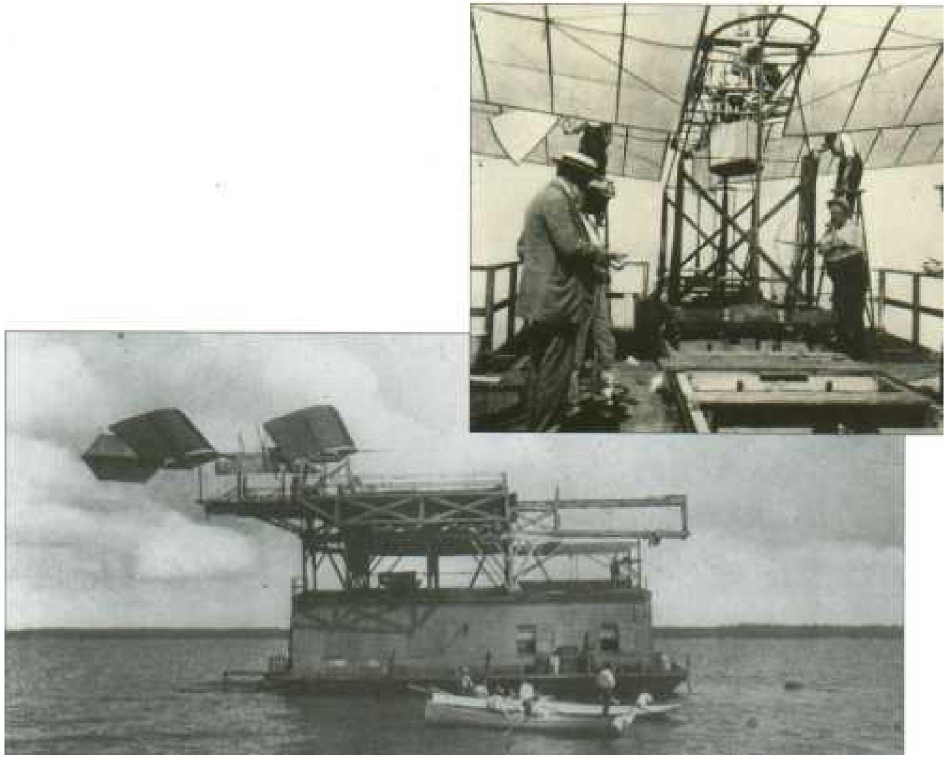
- Samuel Langley, backed by the Smithsonian Institute, attempted to fly his “Aerodrome” off a houseboat on the Potomac River.

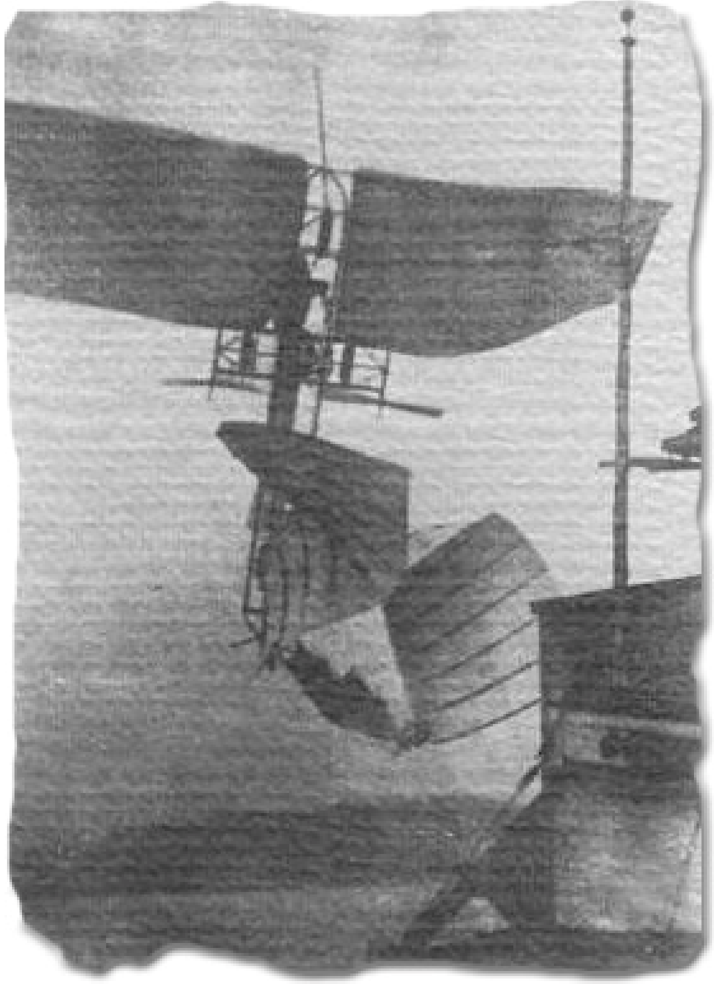




Langley’s Tests

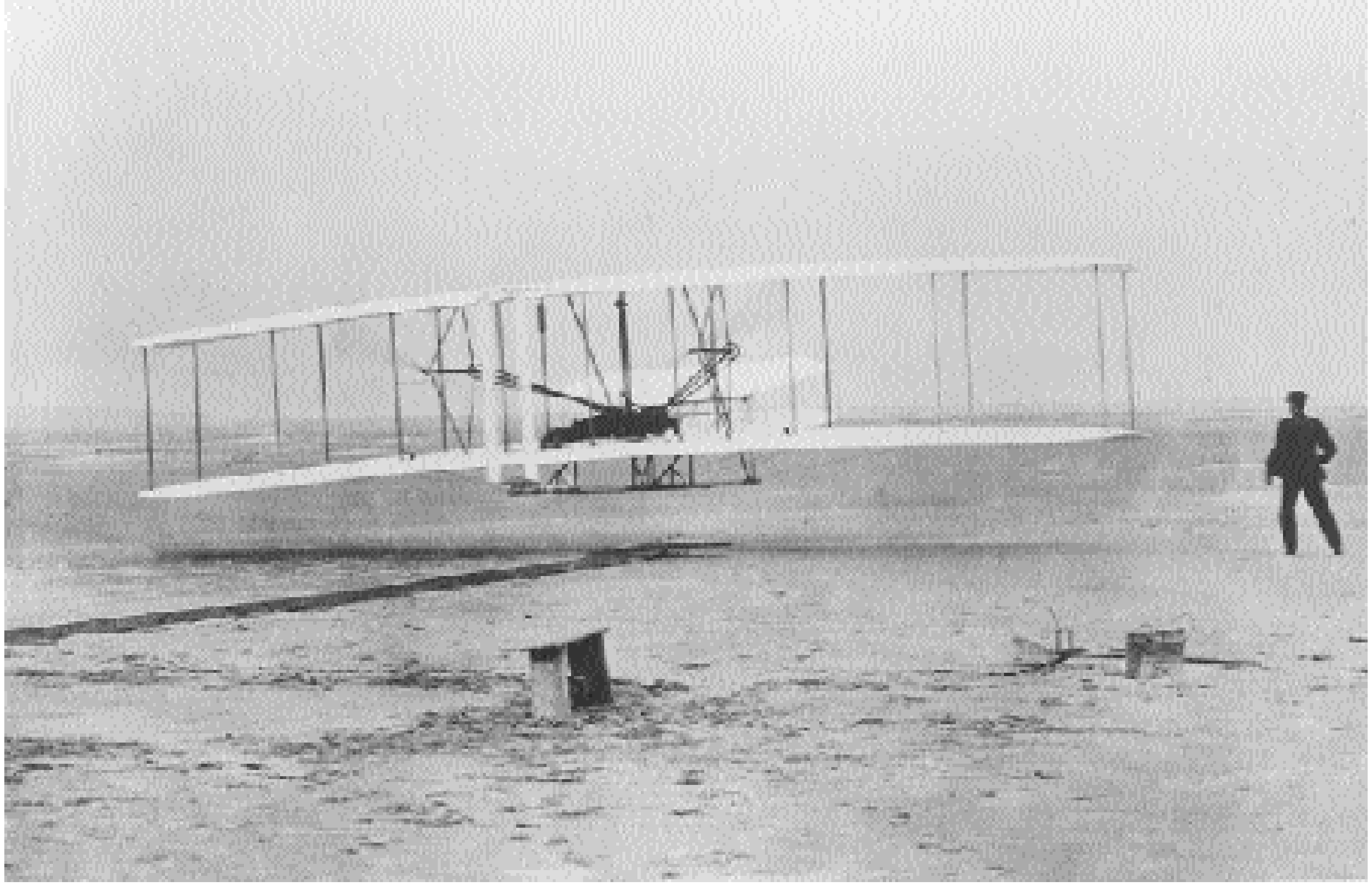
- Structural failure





Wright Brothers

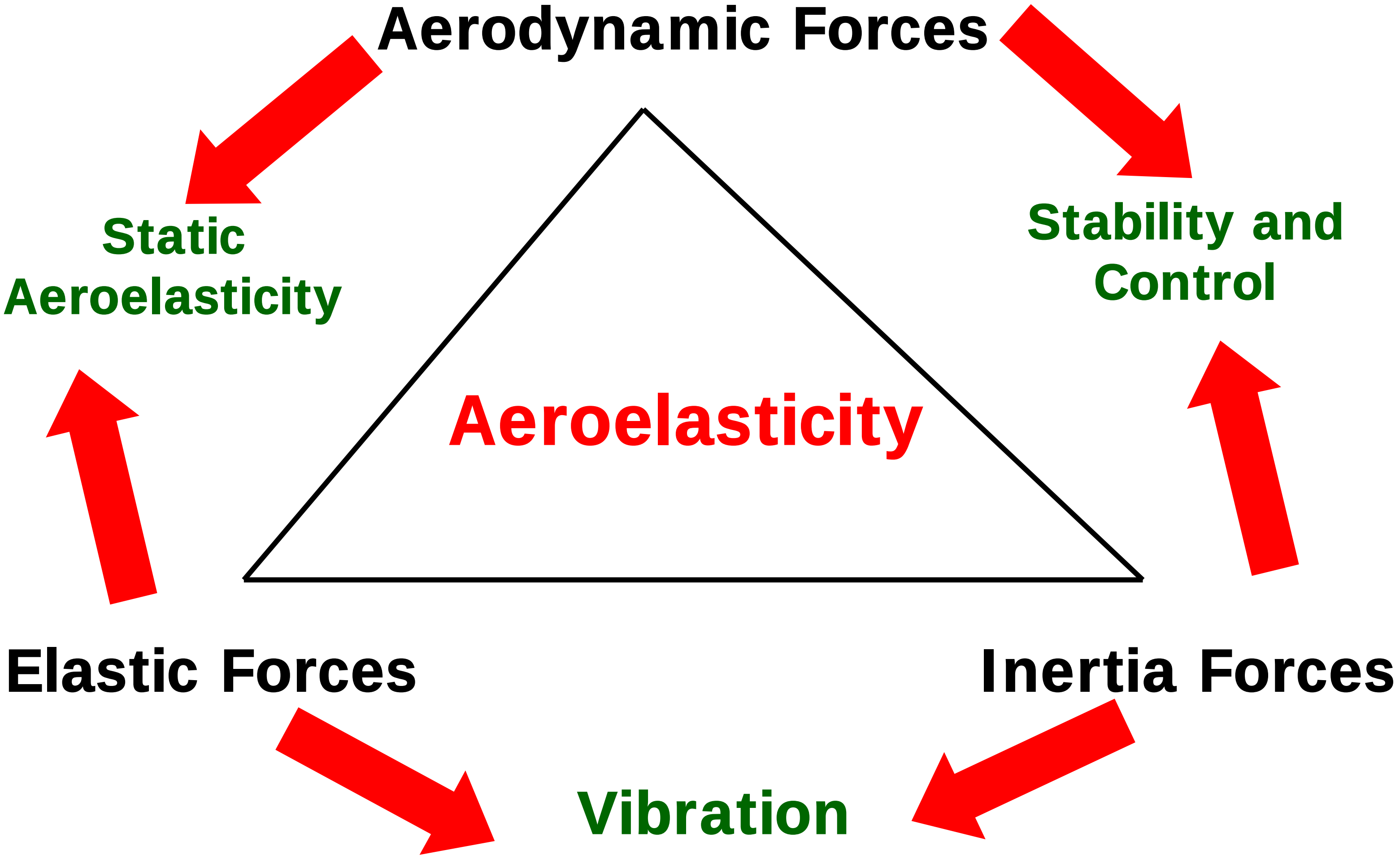
- Success
- Wing warping for roll control



Langley - First Known Aeroelastic Failure

- Wings were not stiff enough
- “Divergence” – torsional loads overcome structural restoring forces
- “Aerodrome” rebuilt some years later by Curtis with stiffer wings – it flew
- Interaction of flexible structure and aerodynamic forces need to be considered
- Science of Aeroelasticity

Collar’s Aeroelastic Triangle



Aerodynamic Forces

Static Aeroelasticity

Stability and Control

Aeroelasticity

Elastic Forces

Inertia Forces

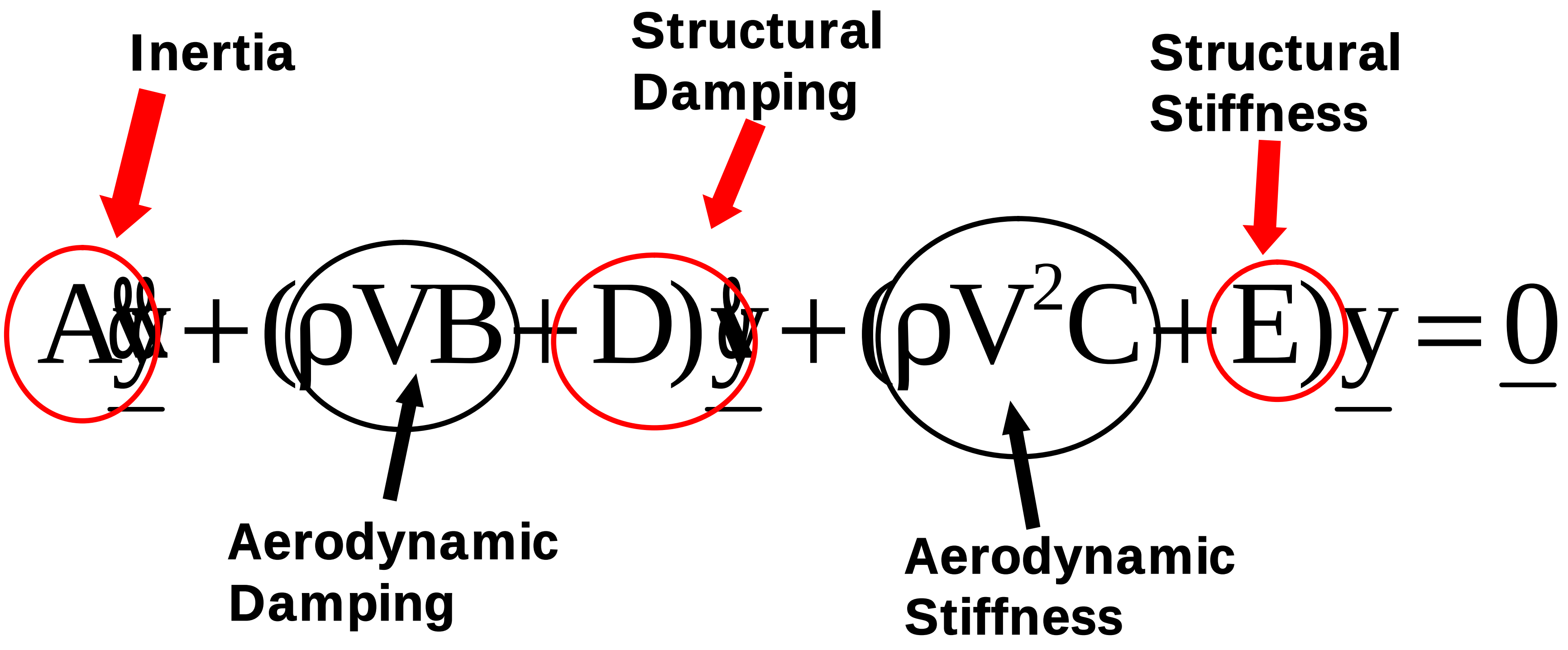
Vibration

Aeroelastic Phenomena

- Mostly undesirable
- Often catastrophic
  - Flutter / Divergence
- Response
  - Gusts / Manoeuvres / Control surface inputs
  - Buffet
- Linear and non-linear response
- Key criteria for aircraft design and certification
  - Many (1000s) of cases need to be considered
- Still unable to accurately predict some types of behaviour

F-18 HARV
Smoke Test
late 1980's
Dryden
Flight Research Center

Aeroelastic Equations



Inertia

Structural Damping

Structural Stiffness

$$A\ddot{\underline{y}} + (\rho V B + D)\dot{\underline{y}} + (\rho V^2 C + E)\underline{y} = \underline{0}$$

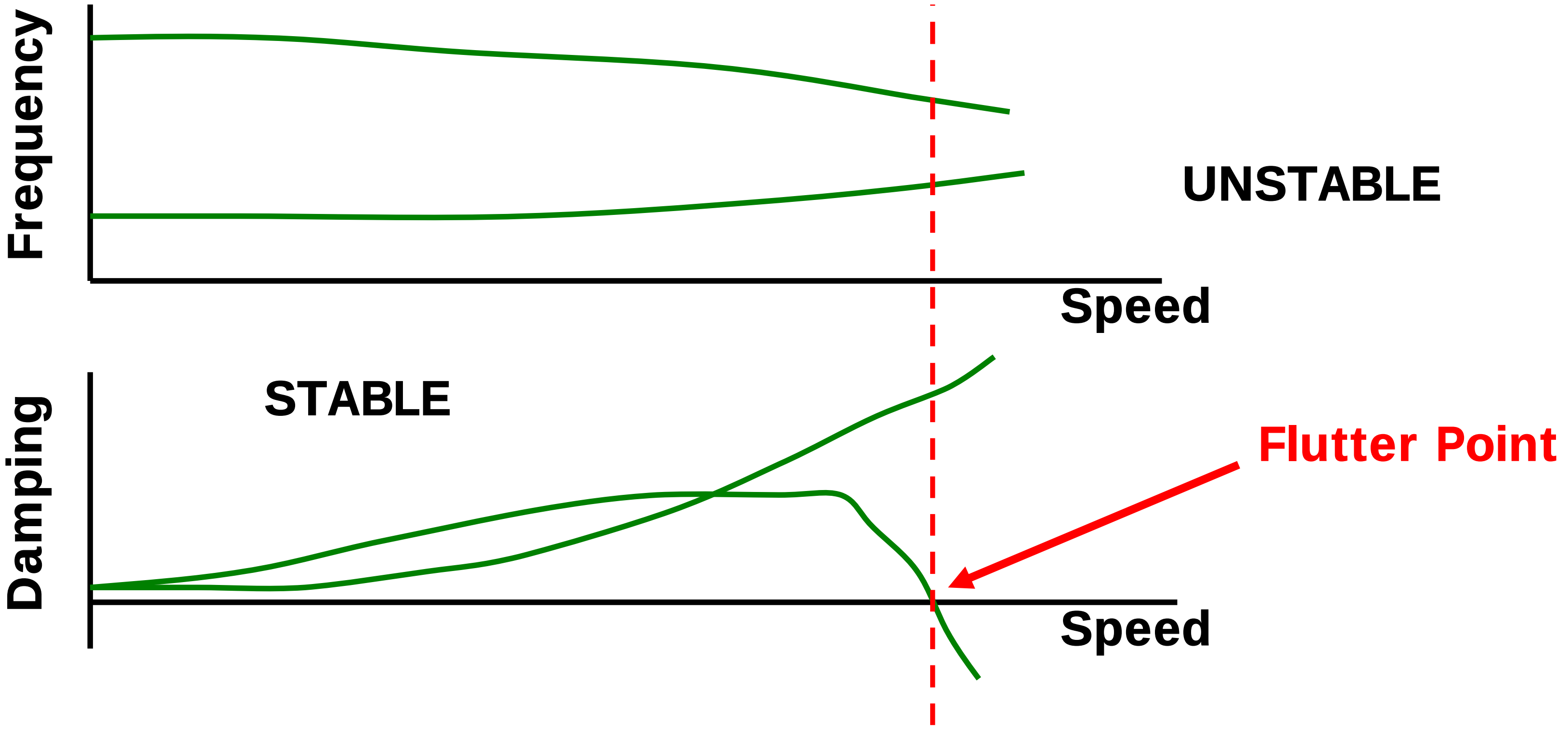
Aerodynamic Damping

Aerodynamic Stiffness

- •2nd order differential equation cf. $M\ddot{\underline{x}} + C\dot{\underline{x}} + K\underline{x} = \underline{0}$
- •Stiffness and damping change with speed and height
- •Matrices of order 80 x 80
- •Right hand side for gusts / control inputs

Flutter

- Violent unstable vibration often resulting in structural failure
- Two modes interact with each other



Aeroelasticity at its Worst

AEROSTRUCTURES TEST WING

Dryden
Flight Research Center
April 24, 2001

Prediction of Aeroelastic Stability

- Aeroelastic equations

$$\mathbf{A}\ddot{\underline{\mathbf{q}}} + (\rho V\mathbf{B} + \mathbf{D})\dot{\underline{\mathbf{q}}} + (\rho V^2\mathbf{C} + \mathbf{E})\underline{\mathbf{q}} = \underline{\mathbf{0}}$$

- First order form

$$\begin{Bmatrix} \dot{\underline{\mathbf{q}}} \\ \ddot{\underline{\mathbf{q}}} \end{Bmatrix} - \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{A}^{-1}\left(\rho V^2\mathbf{C} + \mathbf{E}\right) & -\mathbf{A}^{-1}\left(\rho V\mathbf{B} + \mathbf{D}\right) \end{bmatrix} \begin{Bmatrix} \underline{\mathbf{q}} \\ \dot{\underline{\mathbf{q}}} \end{Bmatrix} = \underline{\mathbf{0}}$$

- Eigenvalue problem

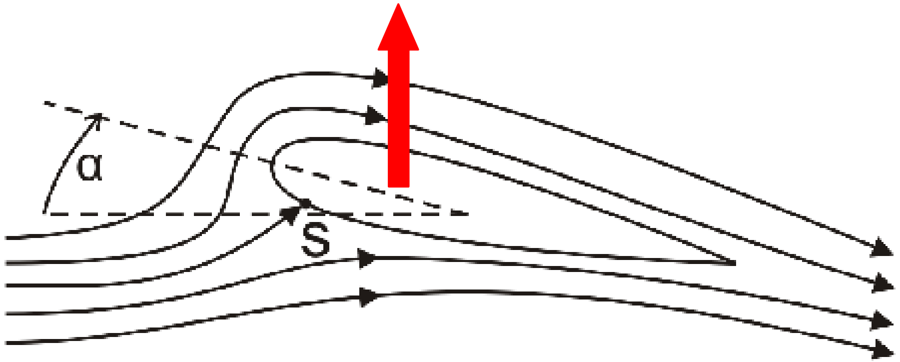
$$\dot{\underline{\mathbf{x}}} - \mathbf{Q}\underline{\mathbf{x}} = \underline{\mathbf{0}} \qquad \underline{\mathbf{x}} = \underline{\mathbf{x}}_{\mathbf{0}} e^{\lambda t} \qquad \Longrightarrow \qquad \left(\mathbf{Q} - \mathbf{I}\lambda\right)\underline{\mathbf{x}}_{\mathbf{0}} = \underline{\mathbf{0}}$$

- Eigenvalues of Q

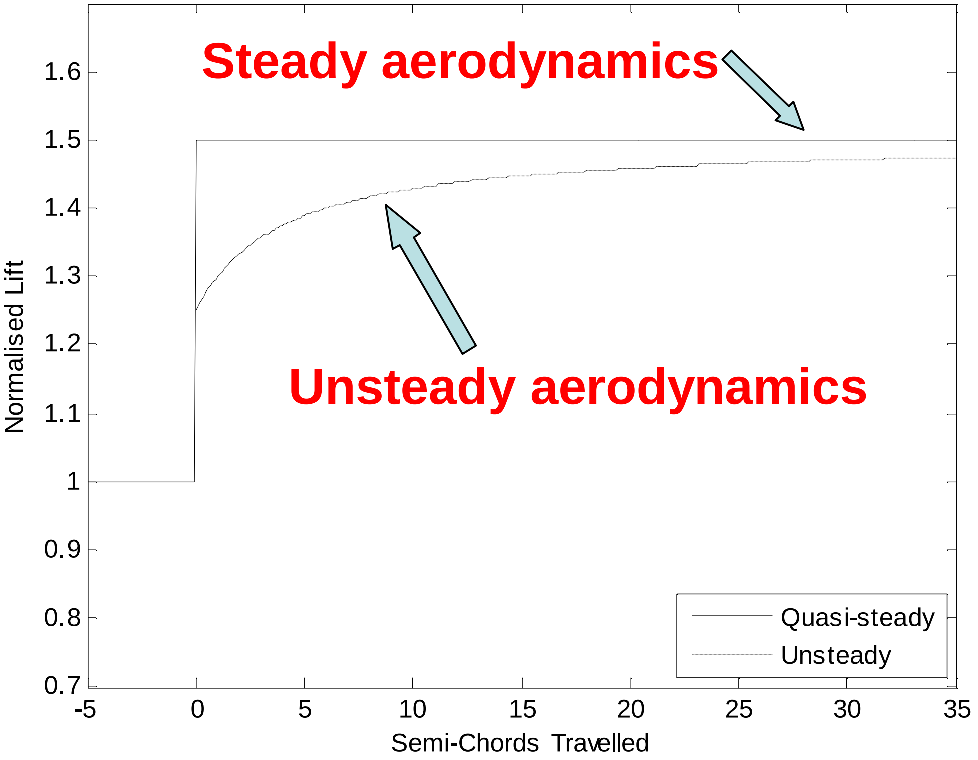
$$\lambda_j = -\zeta_j\omega_j \pm i\omega_j\sqrt{1-\zeta_j^2} \qquad j = 1, 2 \ldots N$$

Unsteady Aerodynamics

- Steady lift proportional to angle of incidence

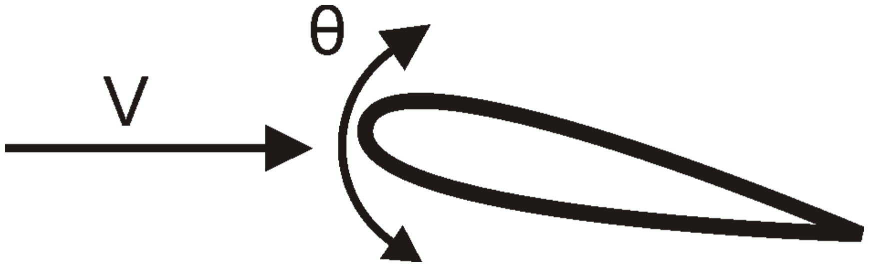


- Consider sudden change in incidence



Effect of Harmonic Motion

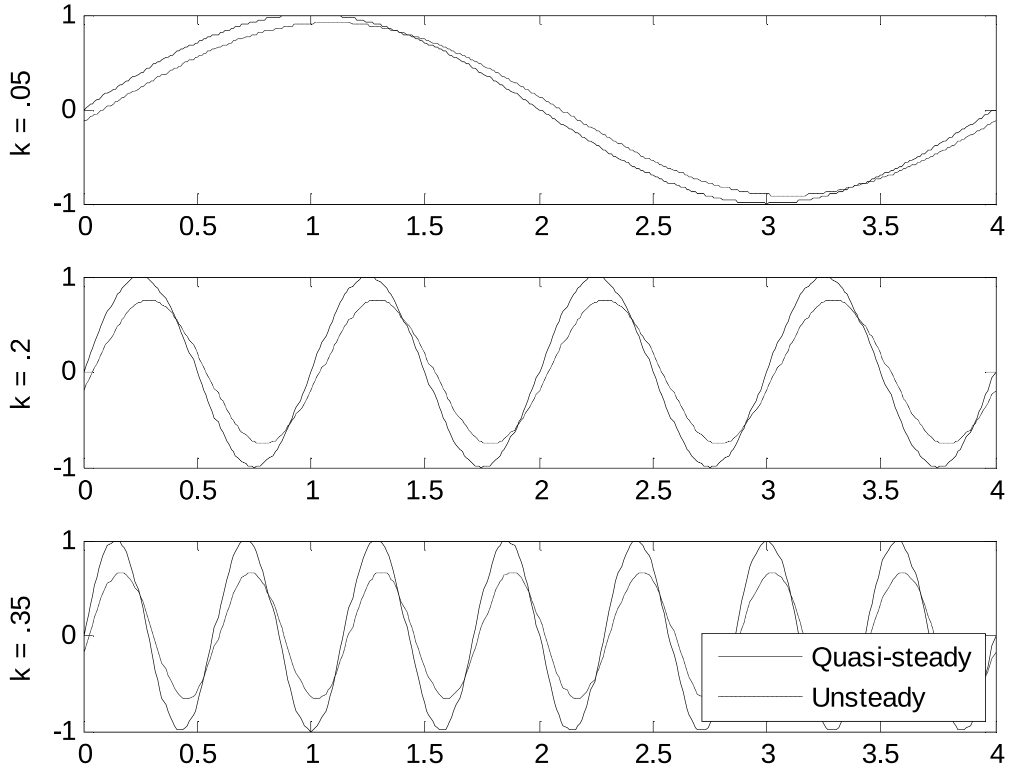
- Oscillatory motion of aerofoil



- Lift depends upon the reduced frequency

$$k = \frac{\omega b}{V} = \frac{\omega c}{2V}$$

- B and C are reduced frequency dependent



Frequency Matching

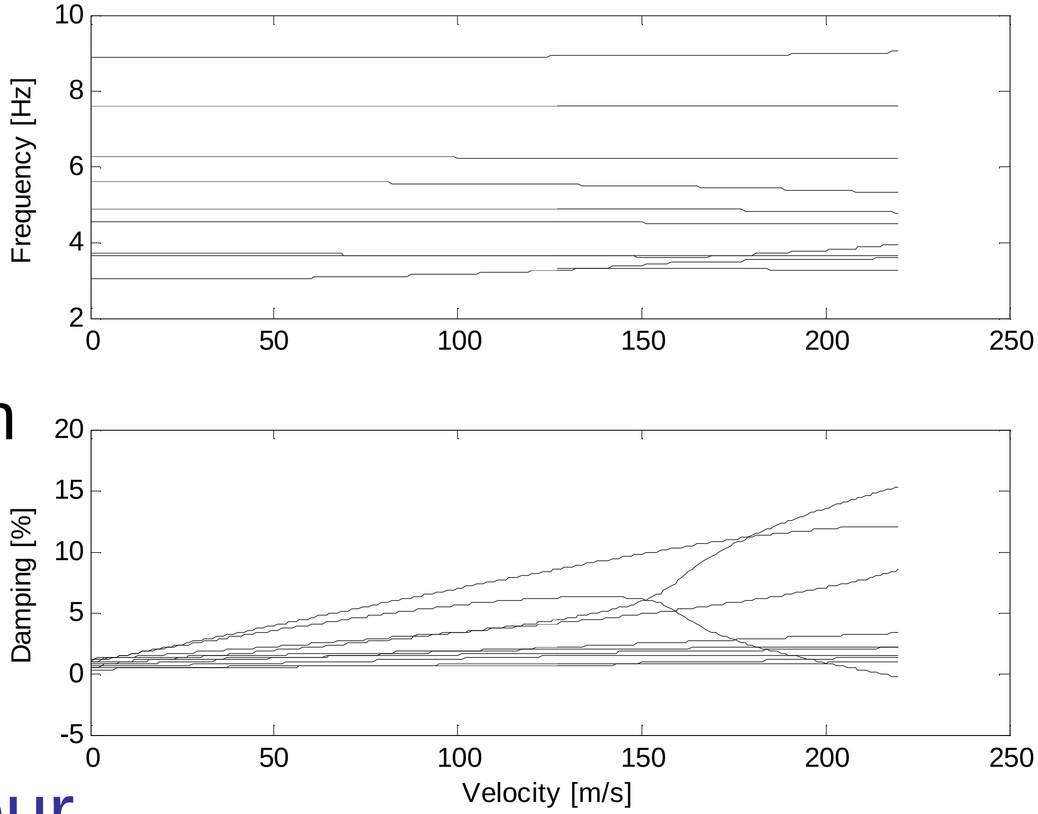
- Aeroelastic equations

$$\mathbf{A}\ddot{\underline{\mathbf{q}}} + (\rho V\mathbf{B} + \mathbf{D})\dot{\underline{\mathbf{q}}} + (\rho V^2\mathbf{C} + \mathbf{E})\underline{\mathbf{q}} = \underline{\mathbf{0}}$$

- If A,B,C,D,E are known
  - Typically using “panel methods”
  - Find ω and ζ from eigen problem
- But, need to know ω and ζ
  - To find B and C
- “Chicken and Egg” situation
  - Frequency matching problem
  - Consider individual harmonics

PK Method

- At each speed and frequency
  - Guess frequency
  - Calculate B and C
  - Solve eigenproblem
  - Repeat process with new frequency
- Exact at flutter condition
- Sub-critical behaviour not exact



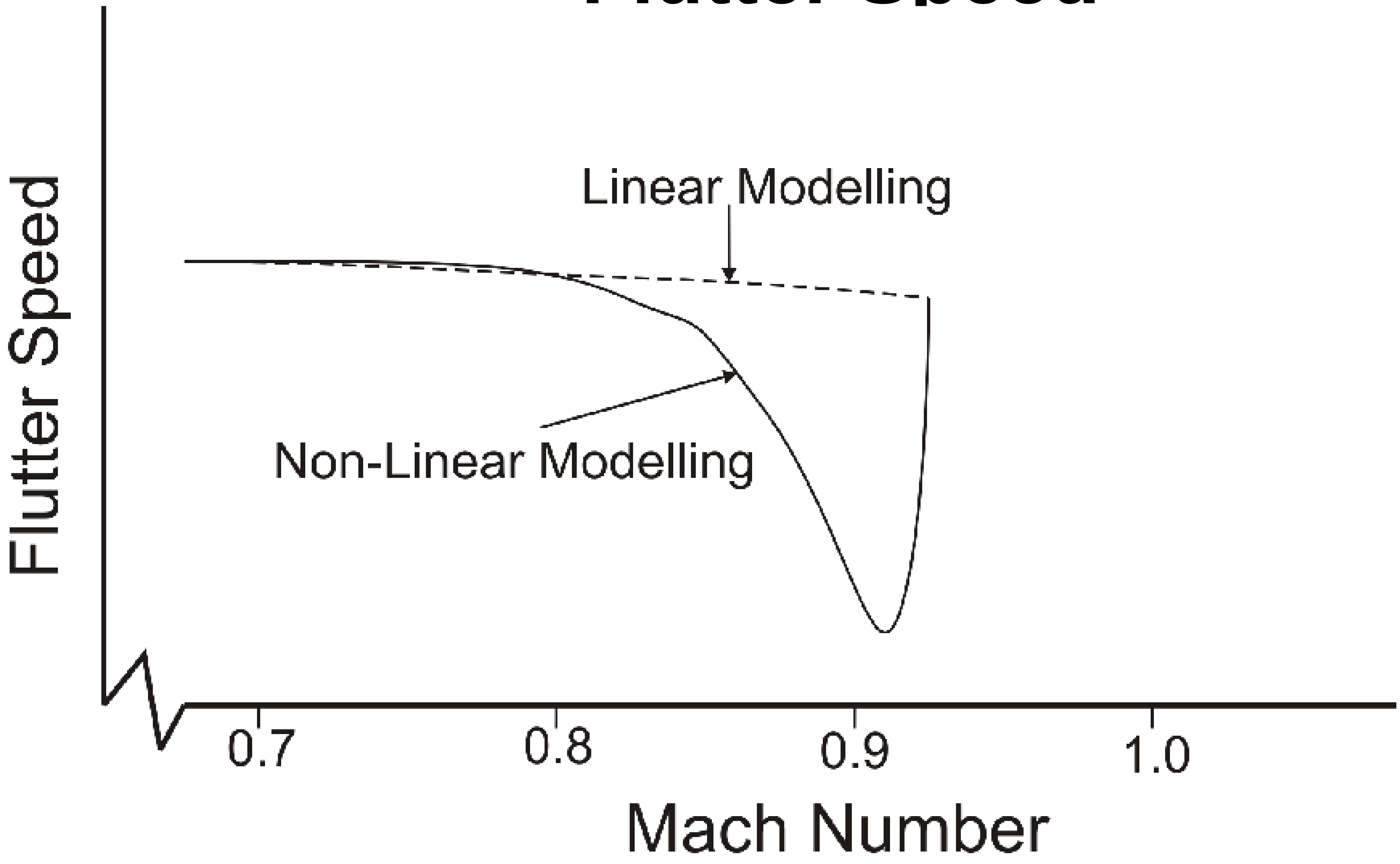
Control Surface Flutter

- Interaction of control surface and wing
- F-117 Stealth Fighter
    - Freeplay of control surfaces

Effect of Non-Linearities

- Non-linear phenomena change aeroelastic behaviour
  - Structural
    - Cubic stiffening - joints
    - Freeplay – control surfaces
  - Aerodynamic
    - Transonic behaviour
    - Moving shocks
    - Stall flutter
  - Control
    - Control surface rate and deflection limits
    - Control circuit time delays

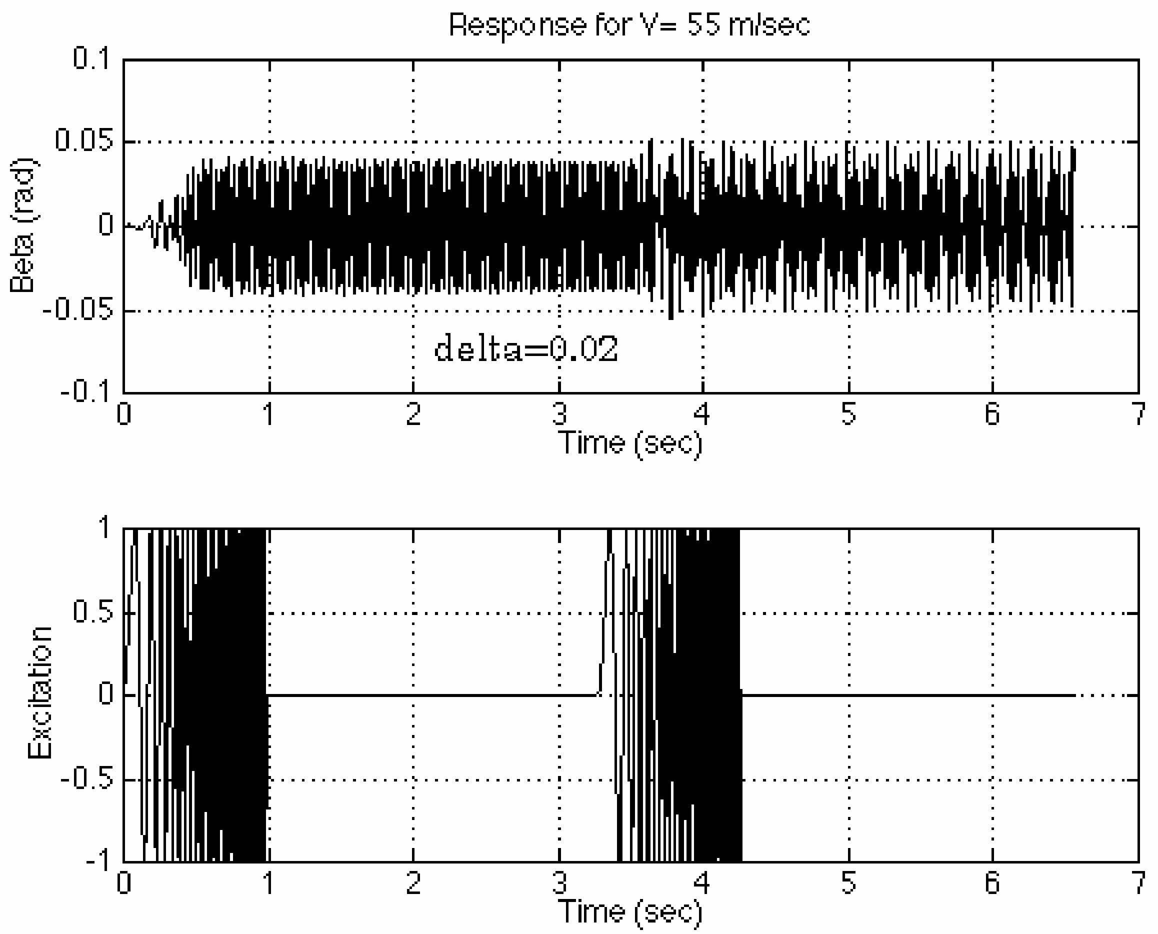
Effect of Transonic Flow on Flutter Speed



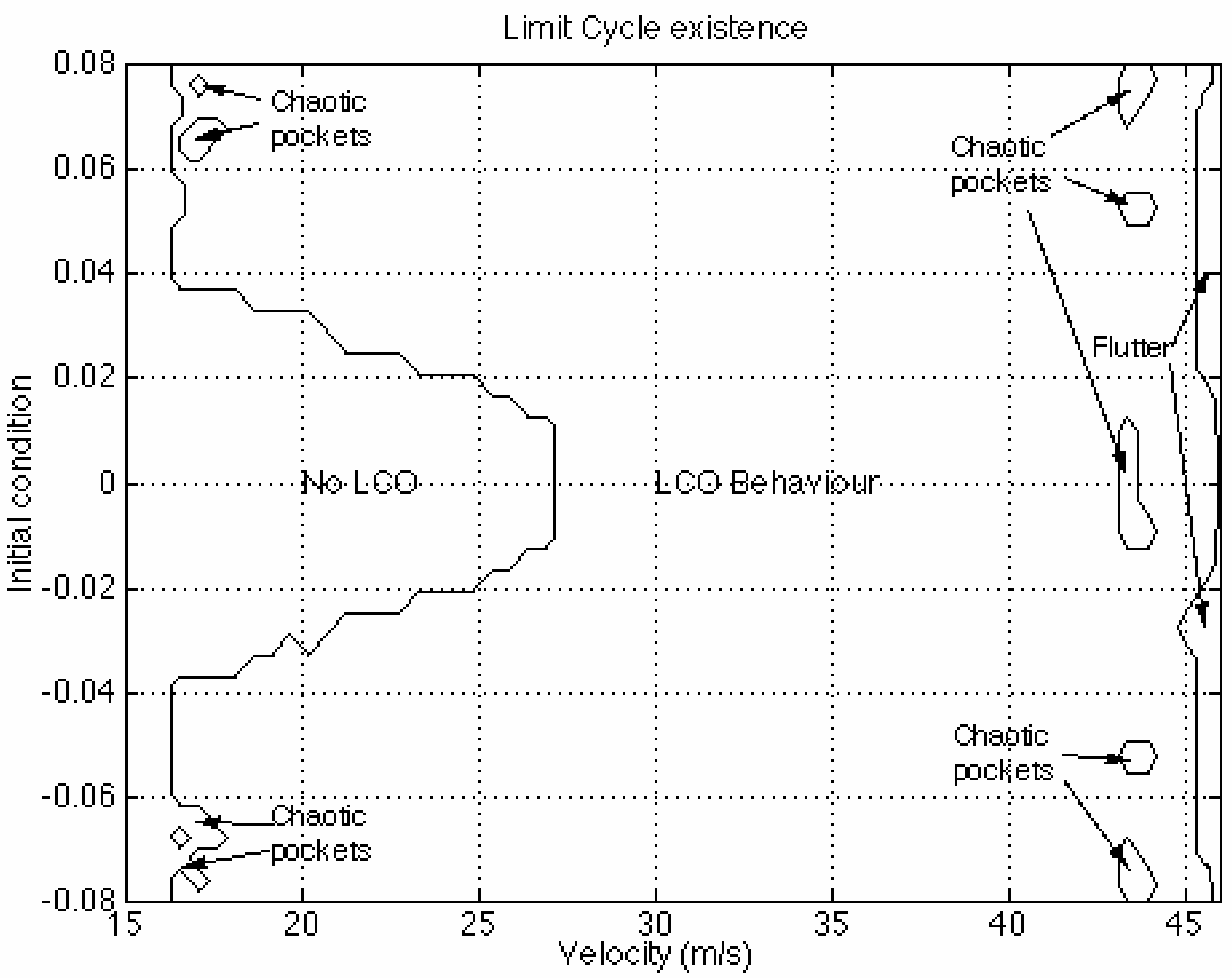
Need high fidelity aerodynamics to model accurately

Limit Cycle Oscillations

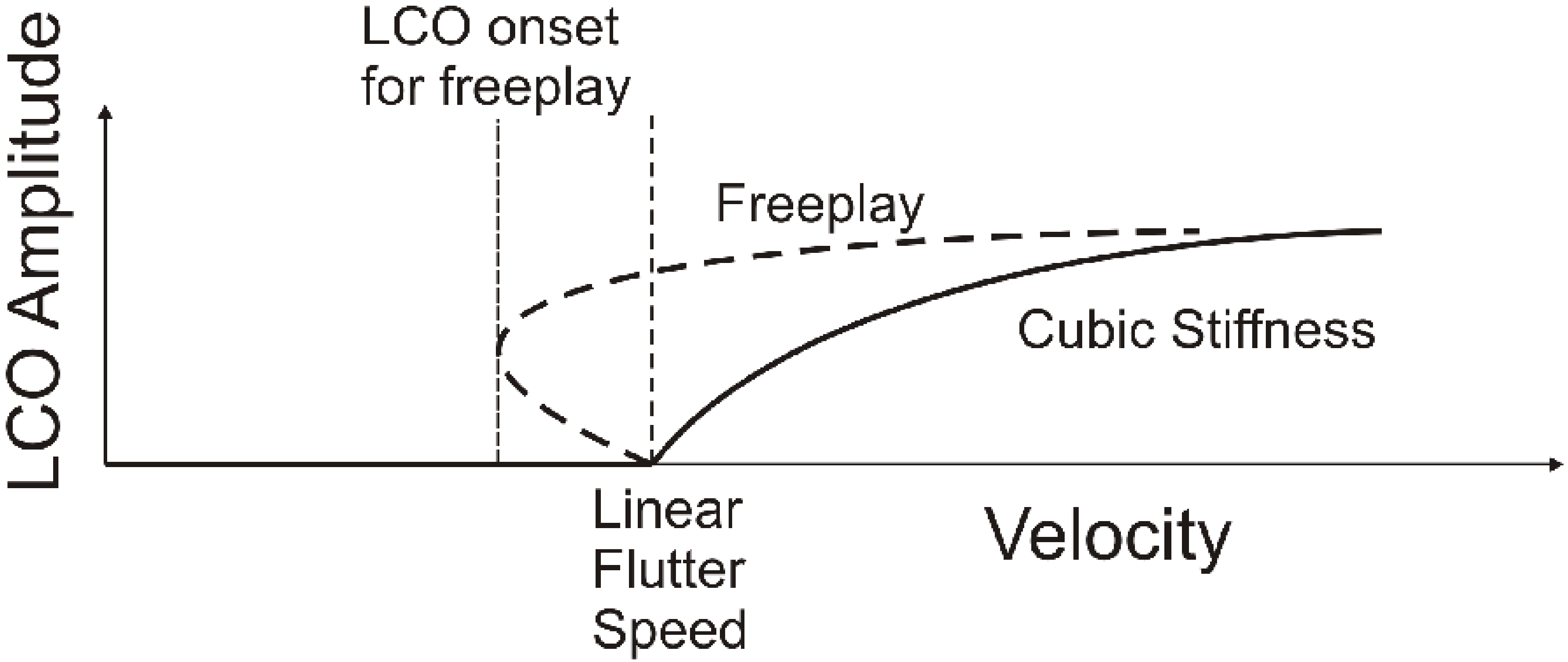
- Non-linearities cause a bounded flutter to occur
- Not disastrous
    - – Fatigue problem
    - – Other problems
        - weapon aiming
        - pilot control



Typical Parameter Space Behaviour



LCO Amplitude Behaviour

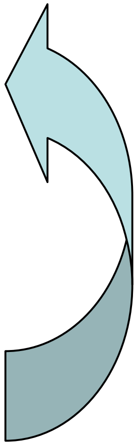


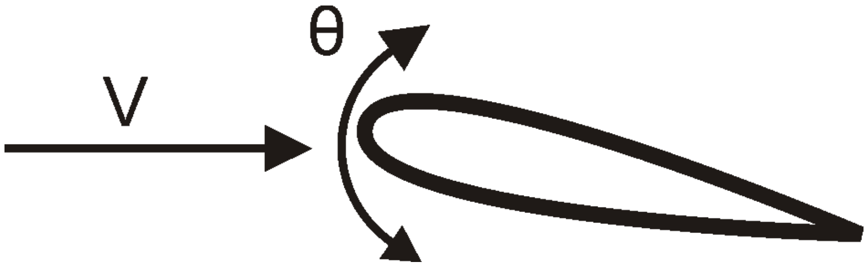
Limit Cycle Oscillations

- Interaction of control surface and wing
- Example here is limited amplitude
  - Limit Cycle Oscillation (LCO)

Stall Flutter

- If angle of incidence gets too high
  - Flow separates
  - Lift is lost
  - Incidence reduces
  - Flow reattaches
  - Incidence increases
- LCO results
- Occurs at wing tips





Military Aircraft with Stores

- Cannot predict LCO using current methods
- Problems if unpredicted vibration occurs in flight test
- Lots of (expensive) testing needed


USAF Photo

www.f-16.net

Prediction of Stability Boundaries and Characteristics

- Possible to compute coupled FE/CFD models but very expensive in transonic region
- Many design cases need to be considered
- Aim to use non-linear dynamics methods to determine regions of interest
- Direct interesting areas where the FE/CFD analysis should be used
- Interested in
  - Stability boundaries
  - Amplitudes
  - Frequencies

Non-Linear Aeroelastic Prediction

- Continuous Non-linearities (Normal Form)
  - – structural and aerodynamic non-linearities
  - – determination of LCO frequencies and amplitudes
- Discontinuous Non-linearities (Normal Form / Harmonic Balance methods / Cell Mapping / numerical continuation)
  - – structural and control system non-linearities

Aeroelastic Equations of Motion

$$\mathbf{A}\ddot{\underline{\mathbf{q}}} + (\rho V\mathbf{B} + \mathbf{D})\dot{\underline{\mathbf{q}}} + (\rho V^2\mathbf{C} + \mathbf{E})\underline{\mathbf{q}} + \underline{\mathrm{F}}(\underline{\mathbf{q}}, \dot{\underline{\mathbf{q}}}) = \underline{\mathbf{0}}$$

$$\begin{Bmatrix} \dot{\underline{\mathbf{q}}} \\ \ddot{\underline{\mathbf{q}}} \end{Bmatrix} - \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{A}^{-1}\left(\rho V^2\mathbf{C} + \mathbf{E}\right) & -\mathbf{A}^{-1}\left(\rho V\mathbf{B} + \mathbf{D}\right) \end{bmatrix} \begin{Bmatrix} \underline{\mathbf{q}} \\ \dot{\underline{\mathbf{q}}} \end{Bmatrix} + \begin{Bmatrix} \underline{\mathbf{0}} \\ \underline{\mathrm{F}} \end{Bmatrix} = \begin{Bmatrix} \underline{\mathbf{0}} \\ \underline{\mathbf{0}} \end{Bmatrix}$$

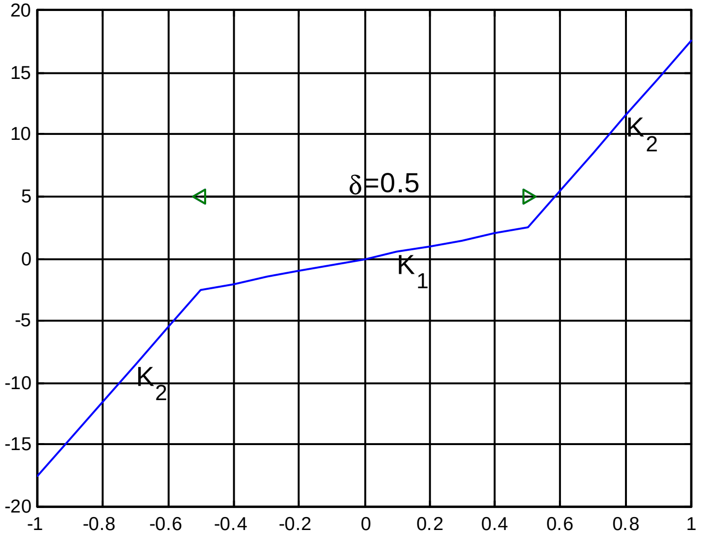
- $\underline{\mathrm{F}}$ – nonlinearity
- Several methods used
  - – Normal Form
  - – Higher Order Harmonic Balance
  - – Numerical Continuation

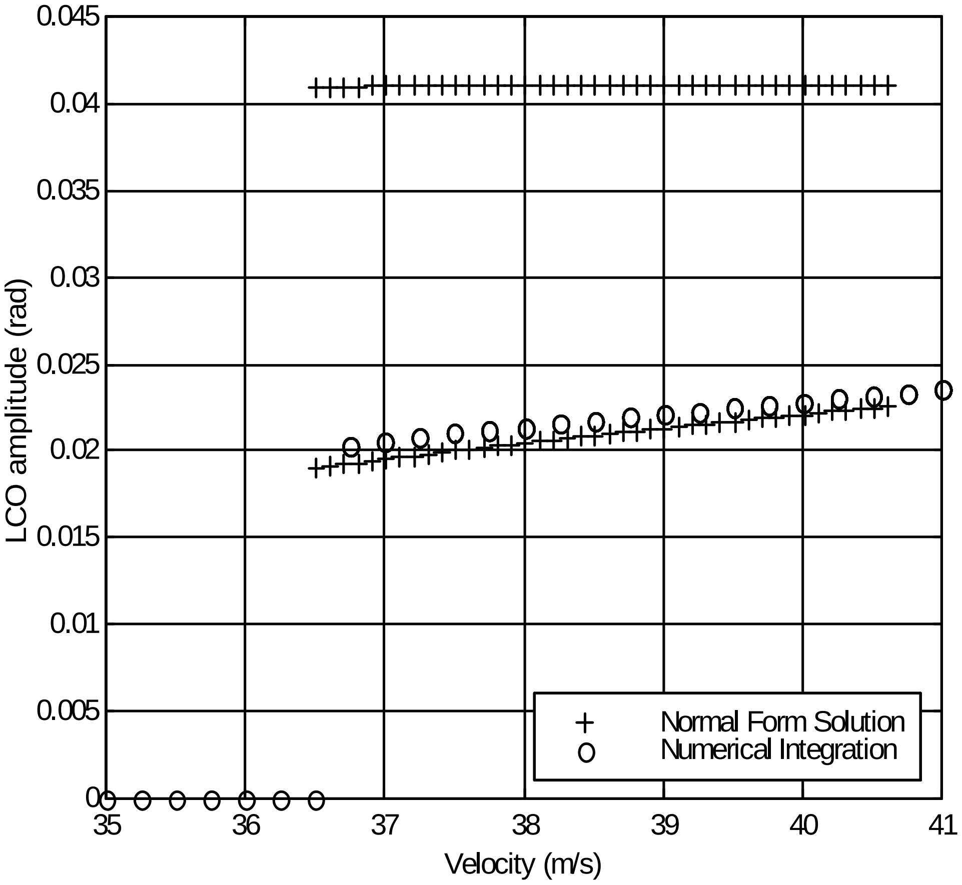
Normal Form Theory

- Technique to obtain the local post-critical behaviour on a 1DOF undergoing Hopf Bifurcation
- Requires Centre manifold theory to simplify system from MDOF to 1DOF
- Applied to structural discontinuous non-linearities
  - Curve-fit the non-linearity
  - Define the equation of motion
  - Define the linear flutter condition
  - Reduce the model (Centre Manifold)
  - Apply Normal Form Theory
  - Determine amplitude and frequency of LCO

NFT Solution

- 18 DOF aeroelastic model
- Bi-linear non-linearity with various ratio of inner / outer stiffness





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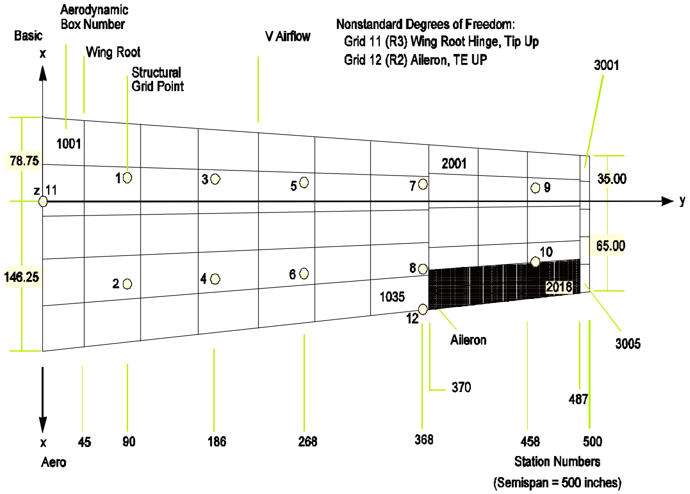
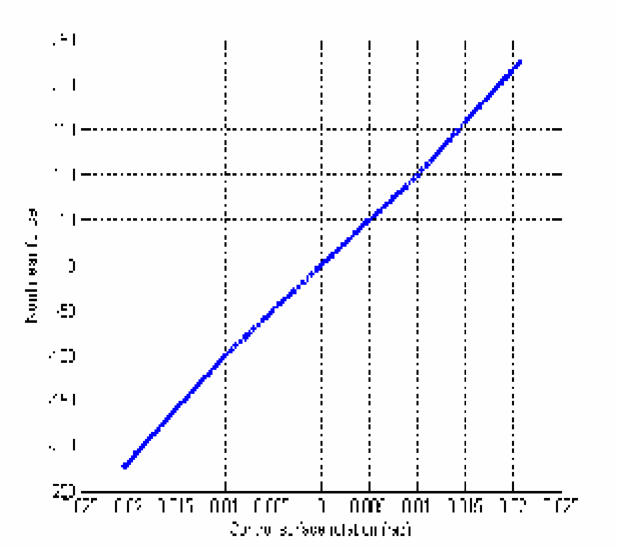
Higher Order Harmonic Balance

- Various types of bifurcation can be treated using Harmonic Balance methods
  - Hopf (sub-critical and super-critical)
  - Period Doubling
  - Folds
- Harmonic Balance
  - Approximates LCO with single sinusoidal component
  - Implementation
    - Describing function
    - Equivalent linearisation
  - Accuracy reduced if significant higher order components exists
- Higher Order Harmonic Balance
  - Similar to HB but higher order terms included
  - Approximates LCO with multiple sinusoids

BAH Aeroelastic Model

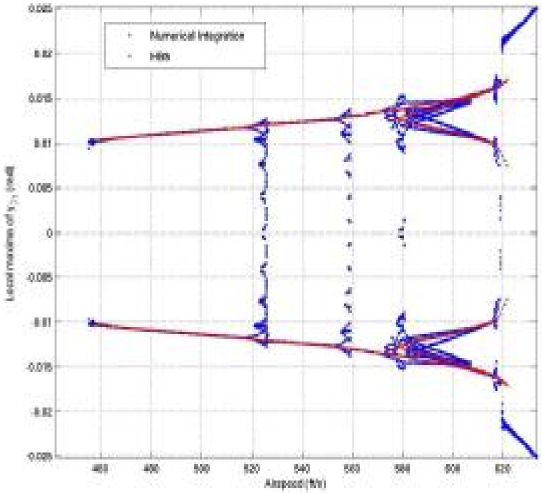
- Bisplinghoff, Ashley and Halfman (BAH) wing
- 12 Finite Element nodes, 72 degrees of freedom
- 9 modes
- Unsteady aerodynamics with 4 aerodynamic lags
- A total of 54 states
- Piecewise linear nonlinearity in control surface rotation degree of freedom

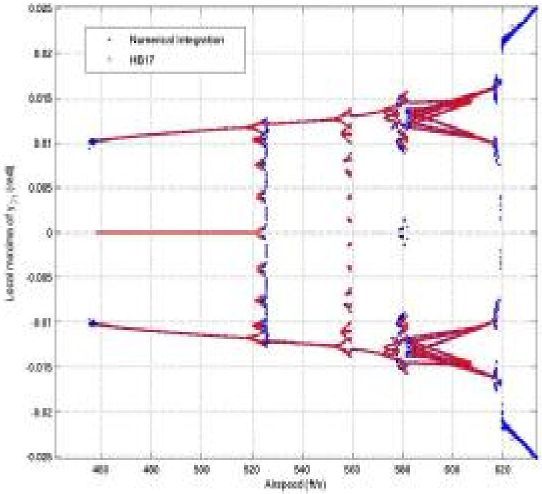




Bifurcation plots from HOHB

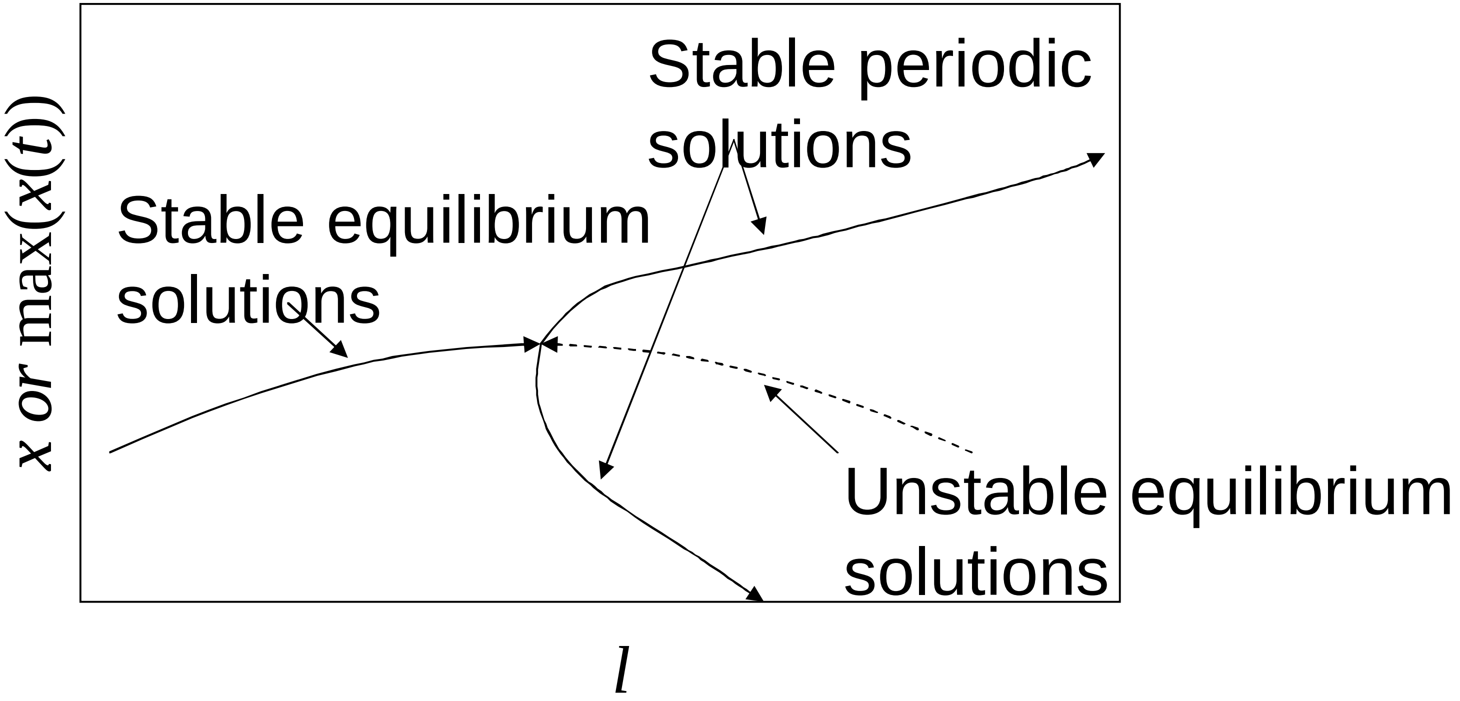
- Bifurcation plots from HB 5 and HB 17 for lower LCO branch
- 17th order HB results are more accurate but slower





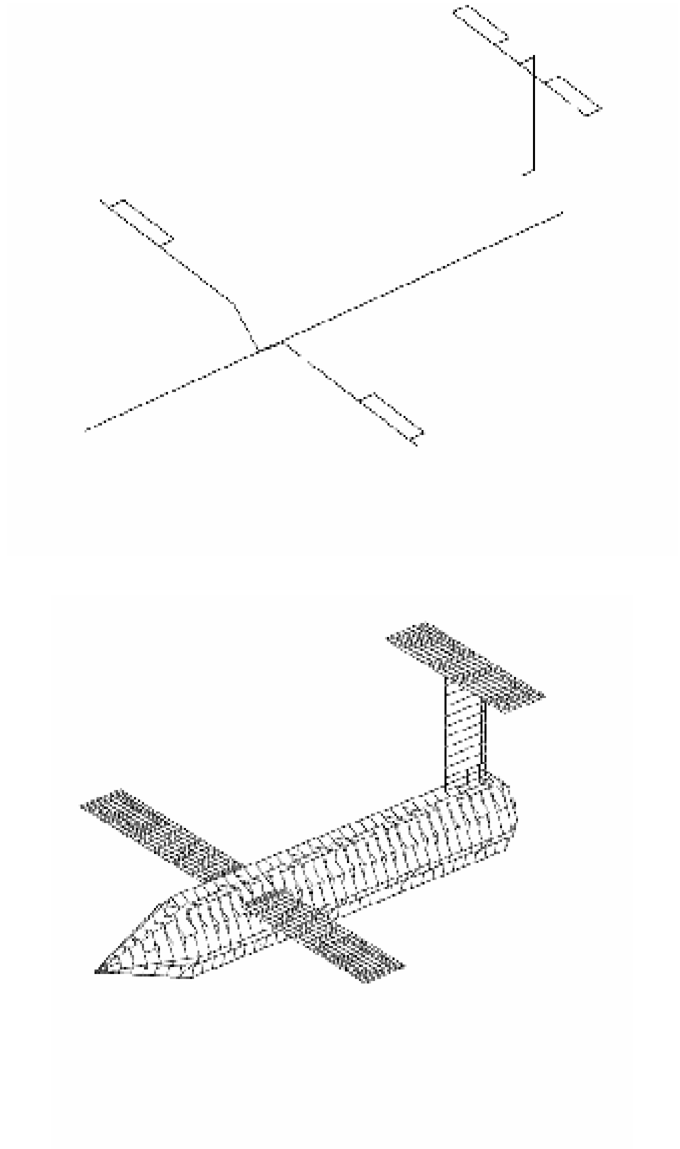
Numerical Continuation

- Numerical Continuation is a method designed to solve nonlinear algebraic equations that depend on one or more parameters
- Having achieved a solution, can then change the parameters slightly and track the new solution



NC applied to full aircraft

- 9 structural modes
- 4 aerodynamic lag roots
- DOF-2-DOF cubic / freeplay non-linearity applied to control surface



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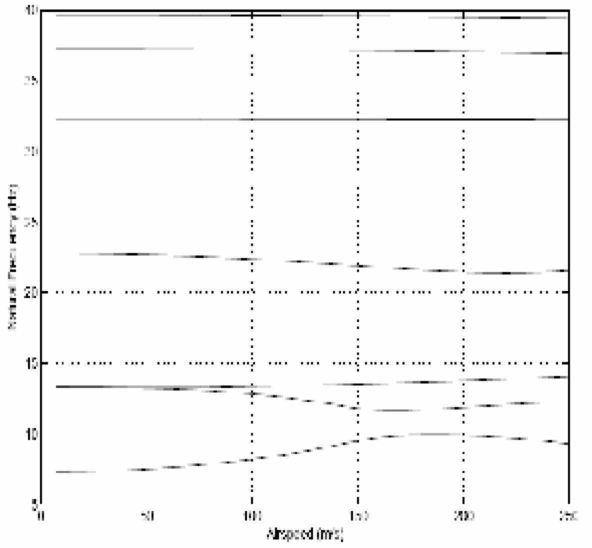
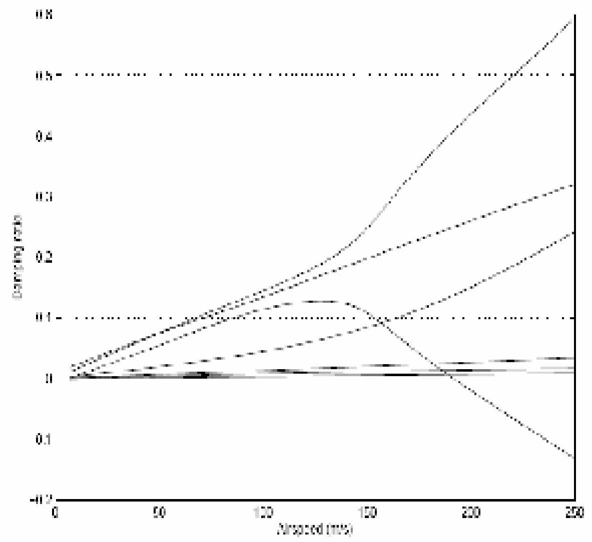
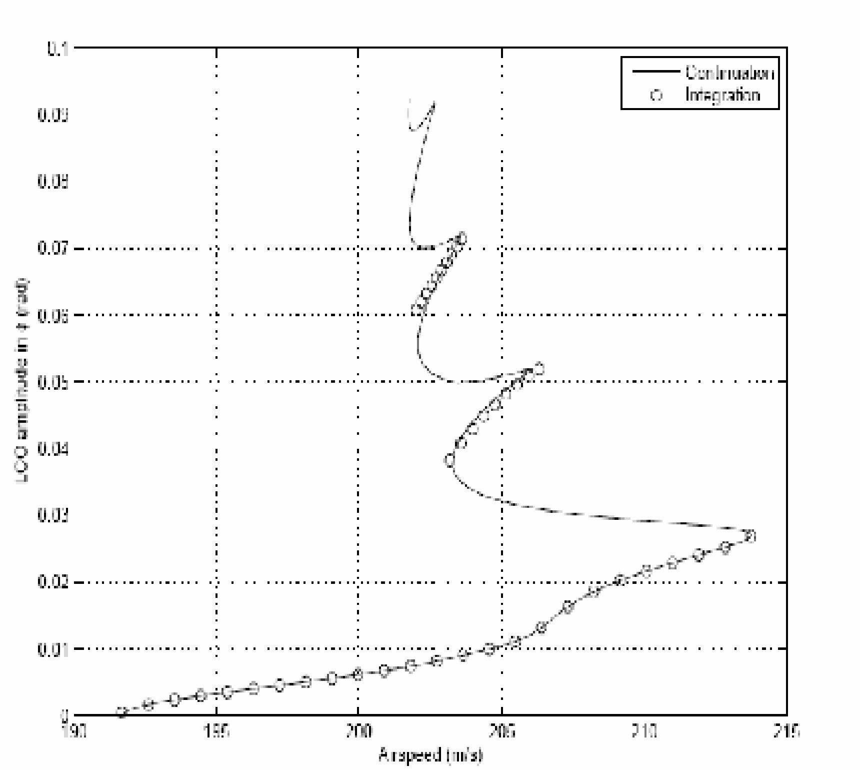
Frequency-damping plot

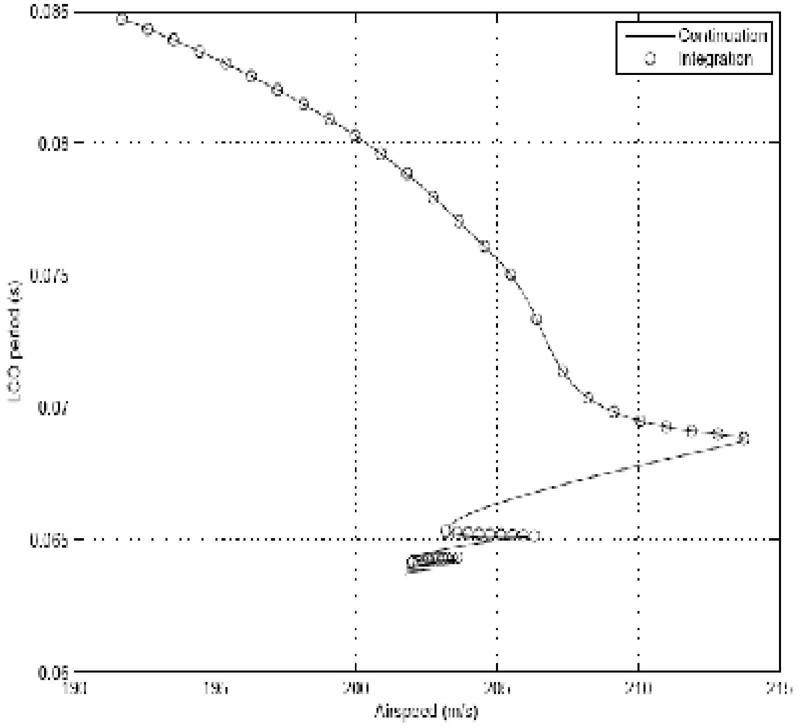
- M,C,K, AIC extracted from Nastran
- Roger approximation applied to transform AIC matrices to time domain



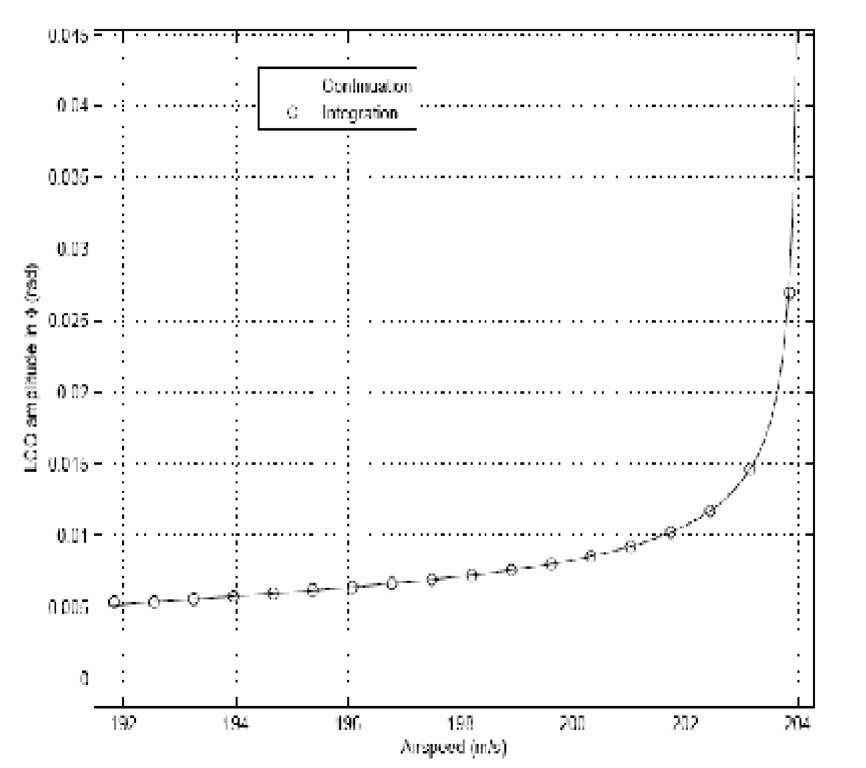


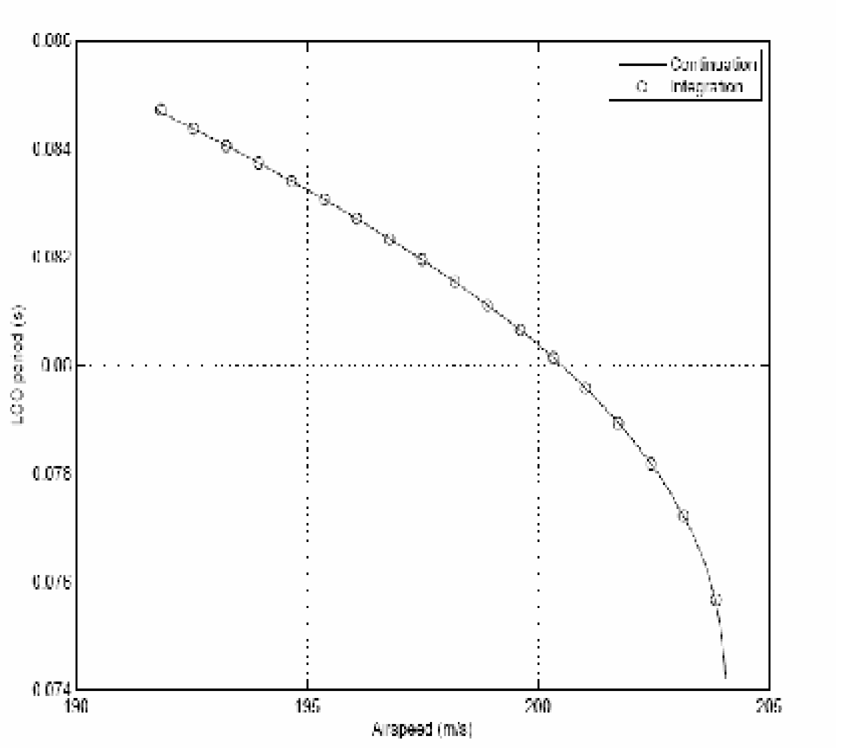
Cubic non-linearity





Freeplay non-linearity





Conclusions

- A number of different approaches have been described for modeling and prediction of linear and non-linear aeroelastic behaviour
- Prediction of non-linear aeroelastic phenomena presents a number of challenges for analysis, testing and FE/CFD modelling
- Further work involves the extension to larger order models and transonic aerodynamics