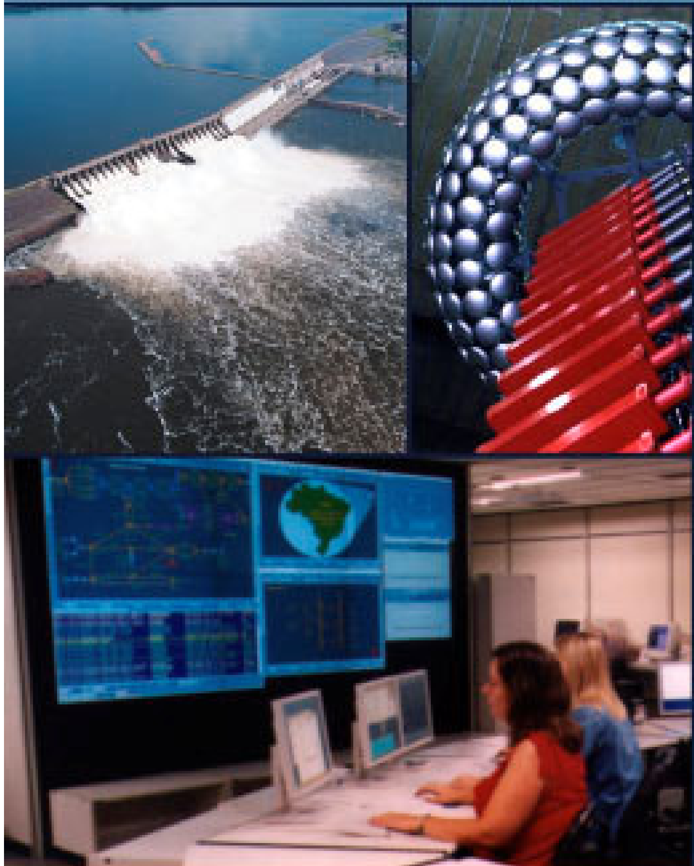
CEPEL

Centro de Pesquisas de Energia Elétrica

Grupo Eletrobrás

DSE



Descriptor and nonlinear eigenvalue problems in the analysis of large electrical power systems

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Workshop on

Nonlinear Eigenvalue Problems

March 2007

Presentation Outline

- Electrical Network Modeling
  - ✓ Descriptor Systems
  - ✓ Non-linear Eigenvalue formulation - Y(s) modeling
- Y(s) Dominant Pole Algorithm (YDPA)
- Y(s) Multiple Dominant Pole Algorithm (YMDPA)
- Results
  - ✓ Small test system
  - ✓ Medium-scale test system

Electrical Network Modeling

Network Modeling Approaches

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- The dynamic behavior of a linear system can be generically written in the s-domain as:

$$\mathbf{Y}(s)\,\mathbf{x} = \mathbf{b}\,u \qquad\qquad y = \mathbf{c}^t\,\mathbf{x} + d\,u$$

where $\mathbf{x}$ is the vector containing the relevant system variables, u and y are the system input and output respectively.

- Depending on the technique adopted to model the system dynamics the matrix $\mathbf{Y}(s)$ can, for instance, be equal to:

$$\mathbf{Y}(s) = \begin{cases} (s\,\mathbf{I} - \mathbf{A}) & \text{State-space} \\ (s\,\mathbf{T} - \mathbf{A}) & \text{Descriptor System (DAE)} \\ \mathbf{Y}_{\mathbf{bus}}(s) & \text{Nodal Admittance} \end{cases}$$

where $\mathbf{I}$, $\mathbf{T}$ and $\mathbf{A}$ are constant matrices.

Network Modeling Approaches

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- State space and DAE models yield matrices that are linear functions of *s.* The efficient SADPA and SAMDP algorithms may be applied to obtain the set of dominant poles for SISO and MIMO transfer functions.
- The elements of Ybus(s) can be non-linear functions of *s*, allowing the modeling of long transmission lines (distributed and frequency dependent parameters)
- The relevant system poles of Y(s) and associated residues can be computed using the Multiple Dominant Pole Algorithm.
- This algorithm requires the determination of $\mathbf{Y}(s)$ and its derivative with respect to *s*. This derivative is built applying the same rules used to build $\mathbf{Y}(s)$ but using the admittance derivatives of the system elements.

Electrical Network Modeling

Descriptor System Equations

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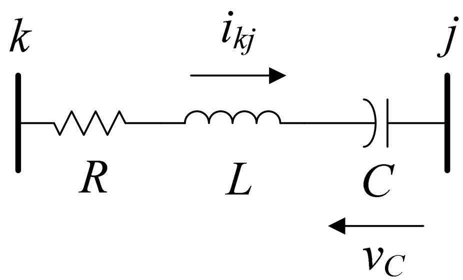
•RLC Series Branch



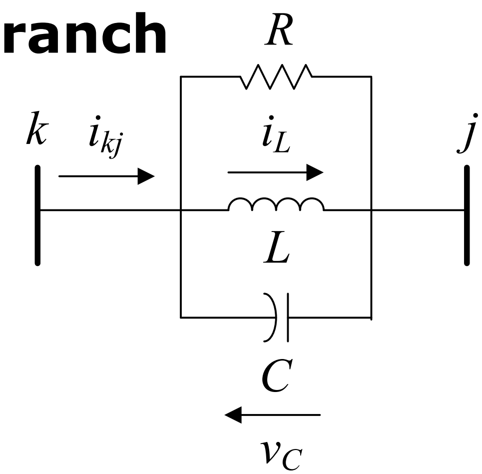
$$v_k - v_j = R\, i_{kj} + L \frac{di_{kj}}{dt} + v_C$$

$$C \frac{dv_C}{dt} = i_{kj}$$

Inexistent C

$$v_k - v_j = R\, i_{kj} + L \frac{di_{kj}}{dt}$$

•RLC Parallel Branch



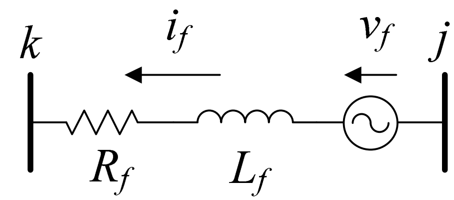
$$\frac{v_C}{R} + i_L + C \frac{dv_C}{dt} = i_{kj}$$

$$L \frac{di_L}{dt} = v_C \qquad v_C = v_k - v_j$$

Inexistent L

$$\frac{v_C}{R} + C \frac{dv_C}{dt} = i_{kj}$$

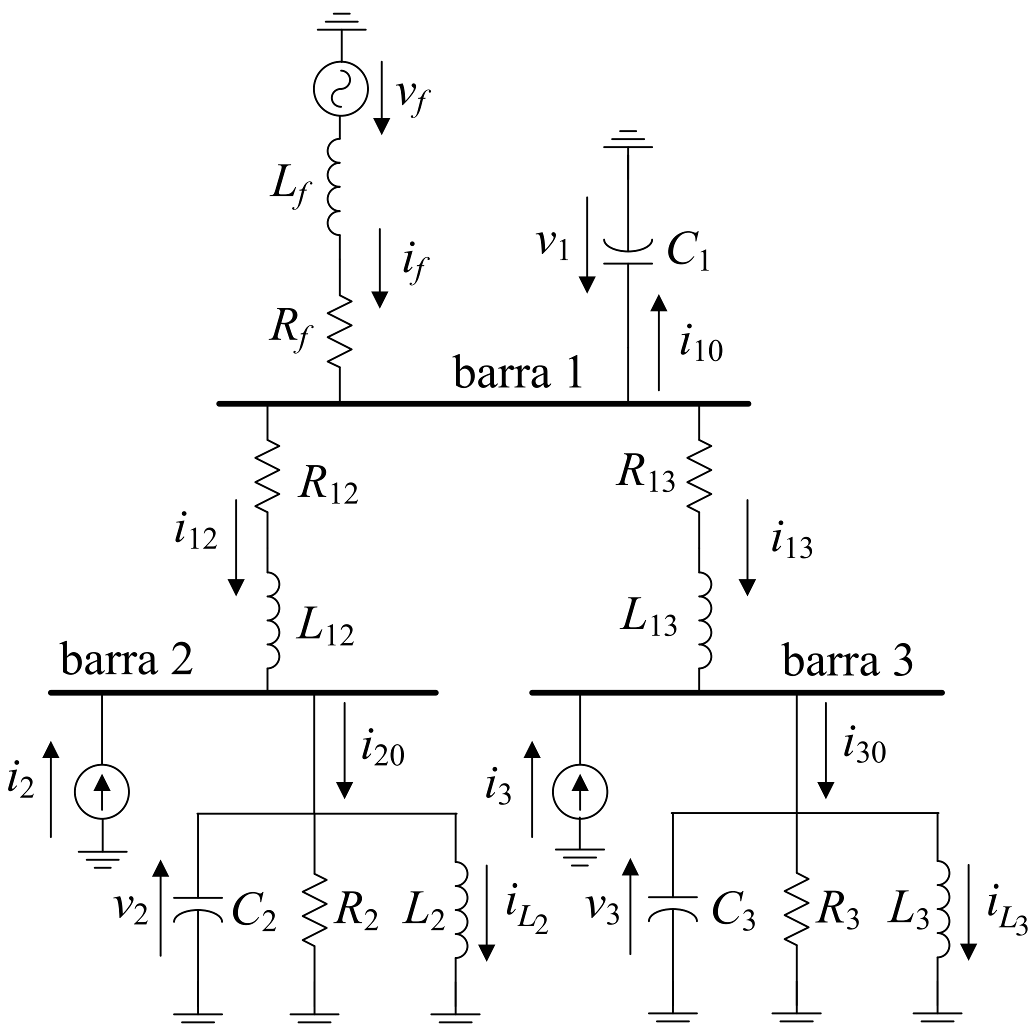
•Voltage Source



$$v_k - v_j = v_f - R_f\, i_f - L_f \frac{di_f}{dt}$$

Electrical Network Modeling

Descriptor System - Example



- Parallel *C* connected to node 1

$$C_1 \frac{dv_{C_1}}{dt} = i_{10} \quad (1) \qquad v_1 - v_{C_1} = 0 \quad (2)$$

- Parallel *RLC* connected to node 2

$$L_2 \frac{di_{L_2}}{dt} = v_{C_2} \quad (3) \qquad v_2 - v_{C_2} = 0 \quad (5)$$

$$C_2 \frac{dv_{C_2}}{dt} = -i_{L_2} - \frac{1}{R} v_{C_2} + i_{20} \quad (4)$$

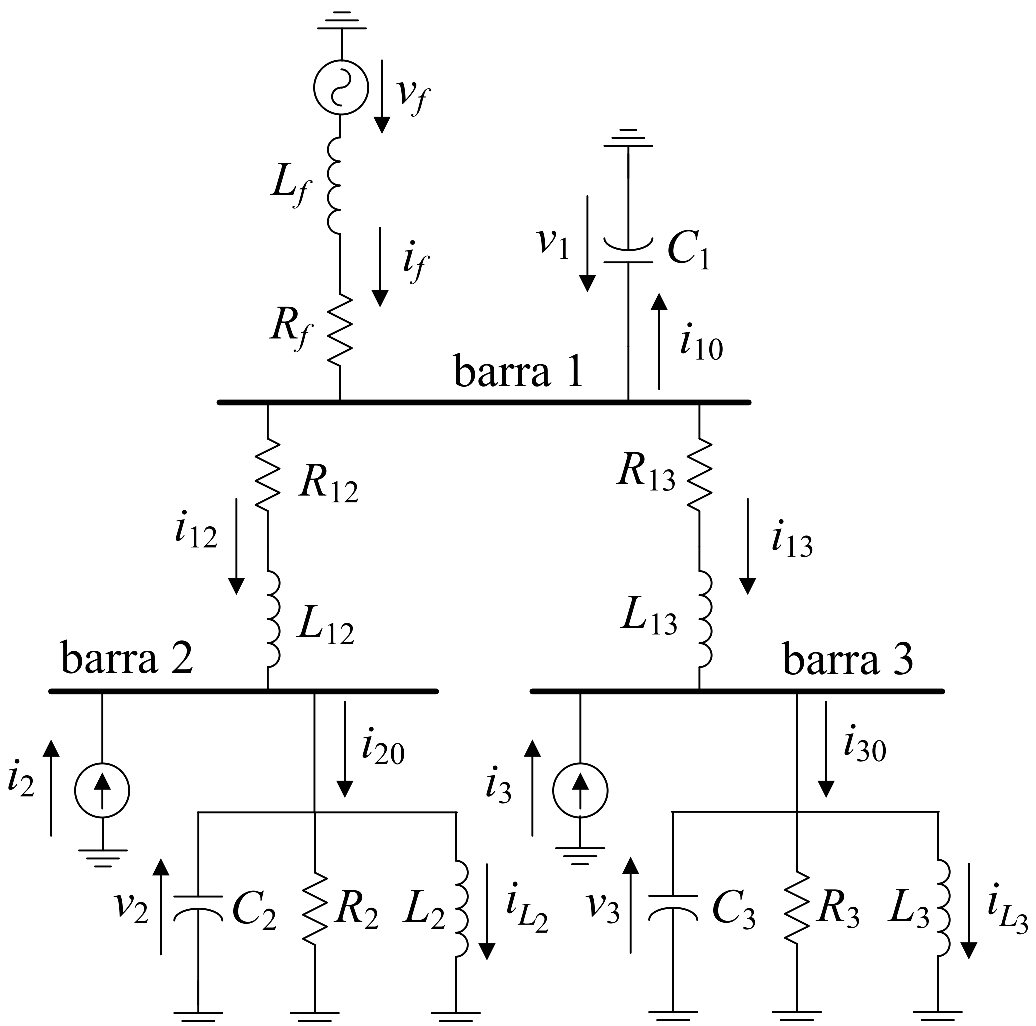
- Parallel *RLC* connected to node 3

$$L_3 \frac{di_{L_3}}{dt} = v_{C_3} \quad (6) \qquad v_3 - v_{C_3} = 0 \quad (8)$$

$$C_3 \frac{dv_{C_3}}{dt} = -i_{L_3} - \frac{1}{R} v_{C_3} + i_{30} \quad (7)$$

Electrical Network Modeling

Descriptor System - Example



- Series *RL* between nodes 1 and 2

$$L_{12}\frac{di_{12}}{dt}=-R_{12}\,i_{12}+v_1-v_2 \quad (9)$$

- Series *RL* between nodes 1 and 3

$$L_{13}\frac{di_{13}}{dt}=-R_{13}\,i_{13}+v_1-v_3 \quad (10)$$

- Voltage source at node 1

$$L_f\frac{di_f}{dt}=-R_f\,i_f-v_1+v_f \quad (11)$$

- Kirchhoff Current Law

- node 1 $\rightarrow -i_{10}-i_{12}-i_{13}+i_f=0 \quad (12)$
- node 2 $\rightarrow -i_{20}+i_{12}+i_2=0 \quad (13)$
- node 3 $\rightarrow \; i_{30}-i_{13}+i_3=0 \quad (14)$

$$
\begin{array}{l}
(1)\rightarrow\\ \\ \\ \\ \\ \\ (7)\rightarrow\\ \\ \\ \\ \\ \\ (13)\rightarrow\\ \\
\end{array}
\begin{bmatrix}
C_1 & & & & & & & & & & & & & \\
 & 0 & & & & & & & & & & & & \\
 & & L_2 & & & & & & & & & & & \\
 & & & C_2 & & & & & & & & & & \\
 & & & & 0 & & & & & & & & & \\
 & & & & & L_3 & & & & & & & & \\
 & & & & & & C_3 & & & & & & & \\
 & & & & & & & 0 & & & & & & \\
 & & & & & & & & L_{12} & & & & & \\
 & & & & & & & & & L_{13} & & & & \\
 & & & & & & & & & & L_f & & & \\
 & & & & & & & & & & & 0 & & \\
 & & & & & & & & & & & & 0 & \\
 & & & & & & & & & & & & & 0
\end{bmatrix}
\frac{d}{dt}
\begin{bmatrix} v_{C_1} \\ i_{10} \\ i_{L_2} \\ v_{C_2} \\ i_{20} \\ i_{L_3} \\ v_{C_3} \\ i_{30} \\ i_{12} \\ i_{13} \\ i_f \\ v_1 \\ v_2 \\ v_3 \end{bmatrix}
=
$$

$$
\begin{array}{l}
(1)\rightarrow\\ \\ \\ \\ \\ \\ (7)\rightarrow\\ \\ \\ \\ \\ \\ (13)\rightarrow\\ \\
\end{array}
\begin{bmatrix}
 & 1 & & & & & & & & & & & & \\
-1 & & & & & & & & & & 1 & & & \\
 & & & 1 & & & & & & & & & & \\
 & & -1 & -1/R_2 & 1 & & & & & & & & & \\
 & & & -1 & & & & & & & & 1 & & \\
 & & & & & & 1 & & & & & & & \\
 & & & & & -1 & -1/R_3 & 1 & & & & & & \\
 & & & & & & -1 & & & & & & 1 & \\
 & & & & & & & & -R_{12} & & 1 & -1 & & \\
 & & & & & & & & & -R_{13} & 1 & & -1 & \\
 & & & & & & & & & & R_f & -1 & & \\
 & -1 & & & & & & & -1 & -1 & 1 & & & \\
 & & & & -1 & & & & 1 & & & & & \\
 & & & & & & & 1 & & -1 & & & &
\end{bmatrix}
\begin{bmatrix} v_{C_1} \\ i_{10} \\ i_{L_2} \\ v_{C_2} \\ i_{20} \\ i_{L_3} \\ v_{C_3} \\ i_{30} \\ i_{12} \\ i_{13} \\ i_f \\ v_1 \\ v_2 \\ v_3 \end{bmatrix}
+
\begin{bmatrix}
 & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\ & & \\
1 & & \\
 & 1 & \\
 & & 1
\end{bmatrix}
\begin{bmatrix} v_f \\ i_2 \\ i_3 \end{bmatrix}
$$

$$C_1 \frac{dv_{C_1}}{dt} = i_{10} \quad (1)$$

$$C_3 \frac{dv_{C_3}}{dt} = -i_{L_3} - \frac{1}{R_3} v_{C_3} + i_{30} \quad (7)$$

$$0 = -i_{20} + i_{12} + i_2 \quad (13)$$

• Considering the nodal voltages as output variables, we have:

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \left[\begin{array}{ccccccccccc|ccc} & & & & & & & & & & & 1 & & \\ & & & & & & & & & & & & 1 & \\ & & & & & & & & & & & & & 1 \end{array}\right] \begin{bmatrix} v_{C_1} \\ i_{10} \\ i_{L_2} \\ v_{C_2} \\ i_{20} \\ i_{L_3} \\ v_{C_3} \\ i_{30} \\ i_{12} \\ i_{13} \\ i_f \\ \hline v_1 \\ v_2 \\ v_3 \end{bmatrix} + \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \begin{bmatrix} v_f \\ i_2 \\ i_3 \end{bmatrix}$$

• Compact form:

$$\mathbf{T}\,\dot{\mathbf{x}}(t) = \mathbf{A}\,\mathbf{x}(t) + \mathbf{B}\,\mathbf{u}(t)$$

$$\mathbf{y}(t) = \mathbf{C}\,\mathbf{x}(t) + \mathbf{D}\,\mathbf{u}(t)$$

Electrical Network Modeling
Y(*s*) Matrix - Equations

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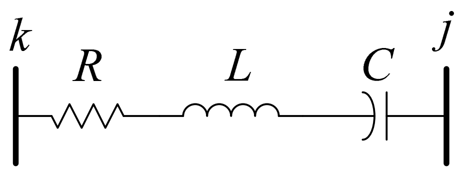
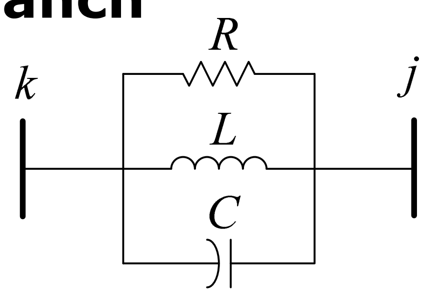
•RLC Series Branch



$$y_{series} = \frac{1}{R + sL + \frac{1}{sC}}$$

$$\frac{dy_{series}}{ds} = \frac{-L + \frac{1}{s^2 C}}{\left(R + sL + \frac{1}{sC}\right)^2}$$

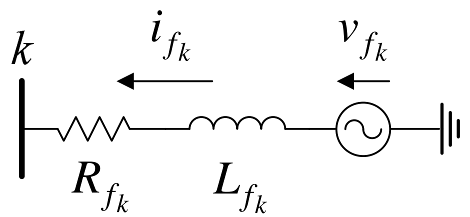
•RLC Parallel Branch



$$y_{parallel} = \frac{1}{R} + \frac{1}{sL} + sC$$

$$\frac{dy_{parallel}}{ds} = C - \frac{1}{s^2 L}$$

•Voltage Source



$$\sum_{j=1}^{n} y_{kj}\, v_j - i_{f_k} = 0$$

$$v_k + z_{f_k}\, i_{f_k} = v_{f_k}$$

$$z_f = R_f + s\, L_f$$

Electrical Network Modeling

$\mathbf{Y}(s)$ Matrix - Example

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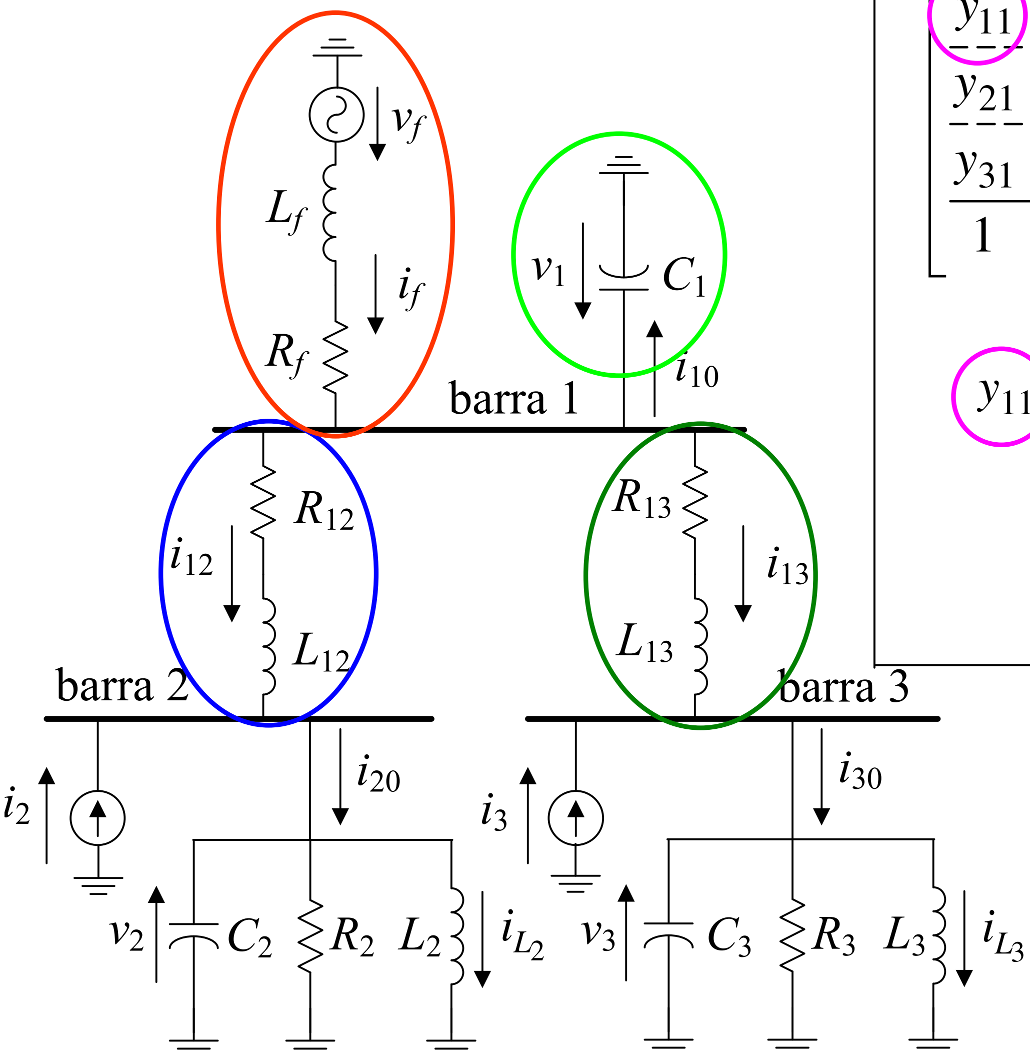


$$\begin{bmatrix} y_{11} & y_{12} & y_{13} & -1 \\ y_{21} & y_{22} & 0 & 0 \\ y_{31} & 0 & y_{33} & 0 \\ 1 & 0 & 0 & z_f \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ i_f \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_2 \\ i_3 \\ v_f \end{bmatrix}$$

$$y_{11} = s\,C_1 + \left(\frac{1}{R_{12} + s\,L_{12}}\right) + \left(\frac{1}{R_{13} + s\,L_{13}}\right)$$

$$y_{13} = y_{31} = -\left(\frac{1}{R_{13} + s\,L_{13}}\right)$$

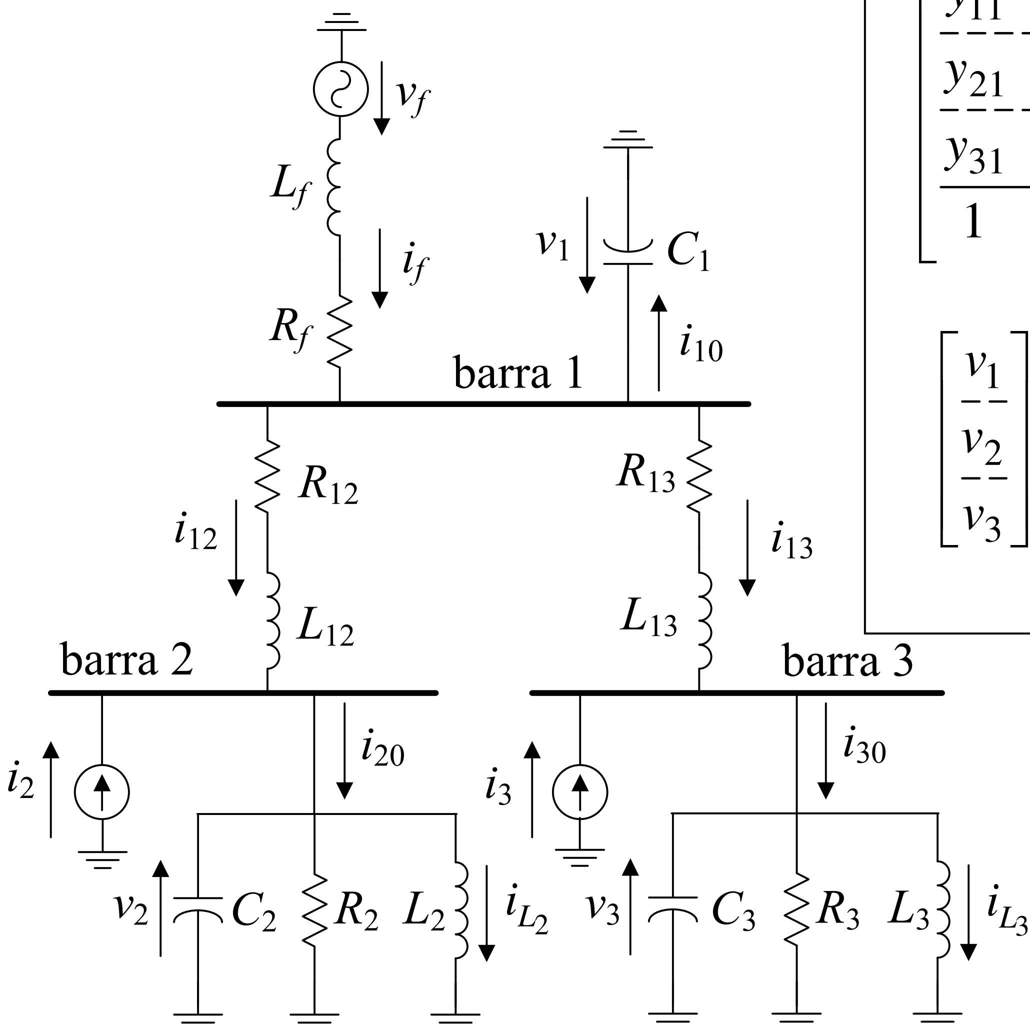
Voltage Source Modeling

KCL $\quad y_{11}\,v_1 + y_{12}\,v_2 + y_{13}\,v_3 - i_f = 0$

KVL $\quad v_1 + z_f\,i_f = v_f$

Electrical Network Modeling

$\mathbf{Y}(s)$ Matrix - Example



$$\begin{bmatrix} y_{11} & y_{12} & y_{13} & -1 \\ y_{21} & y_{22} & 0 & 0 \\ y_{31} & 0 & y_{33} & 0 \\ 1 & 0 & 0 & z_f \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ i_f \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_2 \\ i_3 \\ v_f \end{bmatrix}$$

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ i_f \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_2 \\ i_3 \\ v_f \end{bmatrix}$$

- Compact form:

$$\mathbf{Y}(s)\,\mathbf{x}(s) = \mathbf{B}\,\mathbf{u}(s)$$

$$\mathbf{y}(s) = \mathbf{C}\,\mathbf{x}(s) + \mathbf{D}\,\mathbf{u}(s)$$

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Details in: [1] S. Gomes Jr., C. Portela, N. Martins – "Detailed Model of Long Transmission Lines for Modal Analysis of ac Networks", Proceedings of IPST'2001, Rio de Janeiro, Brazil, 2001 .

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_s & -y_m \\ -y_m & y_s \end{bmatrix} \cdot \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

$$y_s = y_c \cdot \coth(\gamma \cdot l) \qquad y_m = y_c \cdot \mathrm{csch}(\gamma \cdot l)$$

$$\gamma = \sqrt{Z_u(s) \cdot Y_u(s)} \qquad y_c = \sqrt{\frac{Y_u(s)}{Z_u(s)}}$$

Transmission Lines - Derivatives

$$y_s = y_c \cdot \coth(\gamma \cdot l) \qquad \frac{dy_s}{ds} = \frac{dy_c}{ds} \cdot \coth(\gamma l) - y_c \cdot \frac{d\gamma}{ds} \cdot l \cdot \operatorname{csch}(\gamma l)$$

$$y_m = y_c \cdot \operatorname{csch}(\gamma \cdot l) \qquad \frac{dy_m}{ds} = \frac{dy_c}{ds} \cdot \operatorname{csch}(\gamma l) - y_c \cdot \frac{d\gamma}{ds} \cdot l \cdot \operatorname{csch}(\gamma l) \cdot \coth(\gamma l)$$

$$\gamma = \sqrt{Z_u(s) \cdot Y_u(s)} \qquad \frac{d\gamma}{ds} = \frac{1}{2\gamma}\left[Z_u \cdot \frac{dY_u}{ds} + Y_u \cdot \frac{dZ_u}{ds} \right]$$

$$y_c = \sqrt{\frac{Y_u(s)}{Z_u(s)}} \qquad \frac{dy_c}{ds} = \frac{1}{2 \cdot \sqrt{Z_u \cdot Y_u}} \cdot \left[\frac{dY_u}{ds} - y_c^{\,2} \cdot \frac{dZ_u}{ds} \right]$$

Transmission Lines Parameters

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$$\mathbf{Y_u} = s \cdot \mathbf{C} \qquad \mathbf{Z_u} = \mathbf{Z}^{(e)} + \mathbf{Z}^{(i)} + \mathbf{Z}^{(g)} \qquad \mathbf{Z}^{(e)} = s\,\mathbf{L}^{(e)} = s\left(\mu_0\,\varepsilon_0\,\mathbf{C}^{-1}\right)$$

- Conductor Internal Impedance $Z^{(i)}$
  - ✓ Bessel functions
  - ✓ Complex depth penetration

- Ground Effect ($Z^{(g)}$)
  - ✓ Infinite integral of Carson
  - ✓ Complex depth ground return

- The expressions for the s-derivatives, required by the modal analysis algorithms, are presented in the paper

Internal Impedance (Bessel Functions)

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$$\mathbf{Z_u} = \mathbf{Z}^{(\mathbf{e})} + \mathbf{Z}^{(\mathbf{i})} + \mathbf{Z}^{(\mathbf{g})}$$

$$k(s) = \sqrt{\frac{s \cdot \mu_0}{\sigma}} \cdot \frac{1}{2 \cdot \pi \cdot r_e}$$

Diagonal matrix with the following elements:

$$z_i = k(s) \cdot \frac{n(s)}{d(s)}$$

$$n(s) = I_0(\rho_1) \cdot K_1(\rho_0) + I_1(\rho_0) \cdot K_0(\rho_1)$$

$$d(s) = I_1(\rho_1) \cdot K_1(\rho_0) + I_1(\rho_0) \cdot K_1(\rho_1)$$

$$\rho_0 = r_i \cdot \sqrt{s \cdot \mu \cdot \sigma} \qquad \rho_1 = r_e \cdot \sqrt{s \cdot \mu \cdot \sigma}$$

I_0, I_1 are the modified Bessel functions of first kind while K_0 and K_1 are the modified Bessel functions of second kind. Indexes 0 and 1 represent the order of the functions. The parameter σ is the conductor conductivity and μ is the conductor magnetic permeability.

Ground Impedance (Carson Infinite Integral, 1926)

$$\mathbf{Z_u} = \mathbf{Z}^{(\mathbf{e})} + \mathbf{Z}^{(\mathbf{i})} + \mathbf{Z}^{(\mathbf{g})}$$

Matrix with the following elements:

$$z_{i,i}^{(g)} = s \frac{\mu_0}{\pi} \int_0^{\infty} \left[\frac{e^{-2 \cdot h_i \cdot x}}{x + \sqrt{x^2 + s \cdot \mu_0 \cdot \sigma}} \right] \cdot dx$$

$$z_{i,j}^{(g)} = s \frac{\mu_0}{\pi} \int_0^{\infty} \left[\frac{e^{-(h_i + h_j) \cdot x} \cos\left(x \cdot d_{i,j}\right)}{x + \sqrt{x^2 + s \cdot \mu_0 \cdot \sigma}} \right] \cdot dx$$

Solved numerically for each value of s.

Ground Impedance (Complex depth return plane, Semlyen, 1988)

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$$\mathbf{Z_u} = \mathbf{Z}^{(\mathbf{e})} + \mathbf{Z}^{(\mathbf{i})} + \mathbf{Z}^{(\mathbf{g})}$$

Matrix with the following elements:

$$z_{i,i}^{(e,g)} = z_{i,i}^{(e)} + z_{i,i}^{(g)} = s\,\frac{\mu_0}{2\,\pi}\ln\left[\frac{2\,(h_i + p)}{r_e}\right]$$

$$z_{i,j}^{(e,g)} = z_{i,j}^{(e)} + z_{i,j}^{(g)} = s\,\frac{\mu_0}{2\,\pi}\ln\left[\frac{\sqrt{(h_i + h_j + 2\,p)^2 + d_{i,j}^2}}{\sqrt{(h_i - h_j)^2 + d_{i,j}^2}}\right]$$

Where p is:

$$p = \frac{1}{\sqrt{s \cdot \mu_0 \cdot (\sigma + s\,\varepsilon)}}$$

Network Modeling Example

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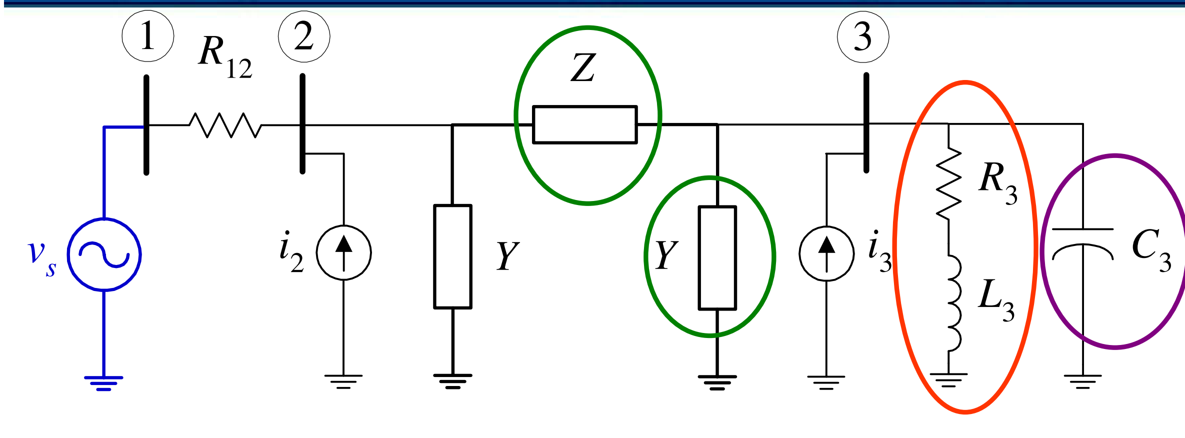

Voltage Source Modeling

$$\mathbf{Y}(s)=\begin{bmatrix} 1 & 0 & 0 \\ y_{21} & y_{22} & y_{23} \\ 0 & y_{32} & y_{33} \end{bmatrix}$$

$$\mathbf{Y}(s)=\begin{bmatrix} y_{11} & y_{12} & 0 \\ y_{21} & y_{22} & y_{23} \\ 0 & y_{32} & y_{33} \end{bmatrix}$$

$$y_{33}(s)=\frac{1}{R_3+s\,L_3}+s\,C_3+Y+\frac{1}{Z}$$

$$Y+\frac{1}{Z} = y_c(s)\coth[\gamma(s)\,l]$$

$$\frac{d\mathbf{Y}(s)}{ds}=\begin{bmatrix} dy_{11}/ds & dy_{12}/ds & 0 \\ dy_{21}/ds & dy_{22}/ds & dy_{23}/ds \\ 0 & dy_{32}/ds & dy_{33}/ds \end{bmatrix}$$

$$\frac{dy_{33}}{ds}=\frac{-L_3}{(R_3+s\,L_{12})^2}+C_3+\frac{d}{ds}\{\,y_c(s)\coth[\gamma(s)\,l]\,\}$$

Y(s) Dominant Pole Algorithm (YDPA)

Transfer Function

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- The transfer function can be written as a summation of partial fractions:

$$G(s) = \mathbf{c}^t \mathbf{Y}(s)^{-1} \mathbf{b} = \sum_{i=1}^{n} \frac{R_i}{s - \lambda_i} + d$$

$\lambda_i \rightarrow$ System pole

$R_i \rightarrow$ Residue associated to λ_i

$d \rightarrow$ direct term of $G(s)$

$n \rightarrow$ Number of system poles

- The direct term is defined by:

$$d = \lim_{s \to \infty} G(s) = \lim_{s \to \infty} \mathbf{c}^t \ \mathbf{Y}(s)^{-1} \ \mathbf{b}$$

- The dominant poles of $G(s)$ are those most relevant to time and frequency responses.
- They are associated with the local maximum values (peaks) of the frequency magnitude plot. The frequencies associated with these peaks are used as initial shifts in the Y(s) DOMINANT POLE ALGORITHM (YDPA).

Y(s) Dominant Pole Algorithm (YDPA)

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- While value of pole shift correction ($\Delta\lambda^{(k)}$) is higher than 10^{-10}:
  - ✓ Calculate, using pole estimate:
    $$\begin{bmatrix} \mathbf{Y}(\lambda^{(k)}) & -\mathbf{b} \\ \mathbf{c}^t & 0 \end{bmatrix} \cdot \begin{bmatrix} \mathbf{v}^{(k)} \\ u^{(k)} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix}$$
    $$\begin{bmatrix} \mathbf{Y}(\lambda^{(k)})^t & \mathbf{c} \\ -\mathbf{b}^t & 0 \end{bmatrix} \cdot \begin{bmatrix} \mathbf{w}^{(k)} \\ u^{(k)} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix}$$
  - ✓ Transfer function Residue:
    $$R_i^{(k+1)} = -\frac{1}{\left[\mathbf{w}^{(k)}\right]^t \cdot \dfrac{d\mathbf{Y}(\lambda^{(k)})}{ds} \cdot \mathbf{v}^{(k)}}$$
  - ✓ Pole Correction: $\Delta\lambda^{(k)} = -u^{(k)} \cdot R_i^{(k+1)}$
  - ✓ New pole estimate: $\lambda^{(k+1)} = \lambda^{(k)} + \Delta\lambda^{(k)}$
  - ✓ Next Iteration: $k = k+1$
- End of While Loop

References on YDPA

- [2] S. Gomes Jr., N. Martins, C. Portela, – “Modal Analysis Applied to s-Domain Models of ac Networks”, Proceedings of IEEE Winter Meeting, Columbus, Ohio, USA, 2001.

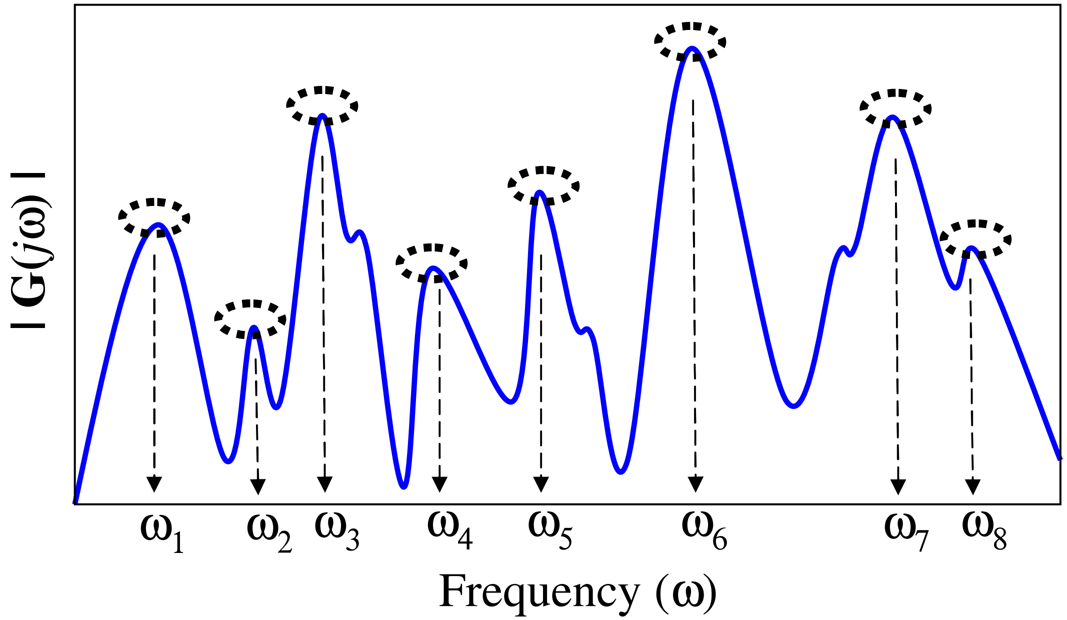
- [3] S. Gomes Jr., N. Martins, S. L. Varricchio, C. Portela, “Modal Analysis of Electromagnetic Transients in ac Networks having Long Transmission Lines”. IEEE Transactions on Power Delivery, v.20, 2005.

- [4] S. L. Varricchio, S. Gomes Jr., N. Martins “Modal analysis of industrial system harmonics using the s-domain approach”, IEEE Transactions on Power Delivery, v.19, 2004

Multiple Dominant Pole Algorithm (YMDPA)

Y(s) Multiple Dominant Poles (YMDPA)

- The dominant poles of $G(s)$ are those most relevant to time and frequency responses.
- The moduli of the residues of the dominant poles are larger when compared to those of the remaining poles.
- They are associated with the local maximum values of the frequency response magnitude plot. These values can be used as initial estimates for the Y(s) MULTIPLE DOMINANT POLE ALGORITHM (YMDPA).



$$G(j\omega) = \mathbf{c}^t \mathbf{Y}(j\omega)^{-1}\mathbf{b}$$

$$\lambda^{(0)} = j\,[\,\omega_1 \quad \omega_2 \quad \cdots \quad \omega_7 \quad \omega_8\,]$$

Initial estimates for YMDPA

Description of Algorithm (YMDPA)

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- For an estimate $\lambda^{(k)}$ the matrix $\mathbf{Y}(\lambda^{(k)})$ and its derivative are built.
- The function $f(s)$ and its derivative are determined using the previously calculated poles and residues $(\lambda_i,\ R_i)$.

$$f\left(\lambda^{(k)}\right)=\sum_j \frac{R_j}{\lambda^{(k)}-\lambda_j} \qquad \frac{df\left(\lambda^{(k)}\right)}{ds}=-\sum_j \frac{R_j}{\left(\lambda^{(k)}-\lambda_j\right)^2}$$

- Solving (1) and (2), one obtains the vectors $\mathbf{v}$, $\mathbf{w}$ and the scalar u.

$$\begin{bmatrix} \mathbf{Y}\left(\lambda^{(k)}\right) & -\mathbf{b} \\ \mathbf{c}^t & -d \end{bmatrix}\begin{bmatrix} \mathbf{v}^{(k)} \\ u^{(k)} \end{bmatrix}=\begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (1) \qquad \begin{bmatrix} \mathbf{Y}\left(\lambda^{(k)}\right)^t & \mathbf{c} \\ -\mathbf{b}^t & -d \end{bmatrix}\begin{bmatrix} \mathbf{w}^{(k)} \\ u^{(k)} \end{bmatrix}=\begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (2)$$

- Using the values of u, $\mathbf{v}$, $\mathbf{w}$, $d\mathbf{Y}/ds$, f and df/ds, the pole correction value is obtained:

$$\Delta\lambda^{(k)}=\frac{u^{(k)}-f\left(\lambda^{(k)}\right)\left(u^{(k)}\right)^2}{\left(\mathbf{w}^{(k)}\right)^t \dfrac{d\mathbf{Y}\left(\lambda^{(k)}\right)}{ds}\mathbf{v}^{(k)}-\dfrac{df\left(\lambda^{(k)}\right)}{ds}\left(u^{(k)}\right)^2}$$

Modal Analysis – MDPA

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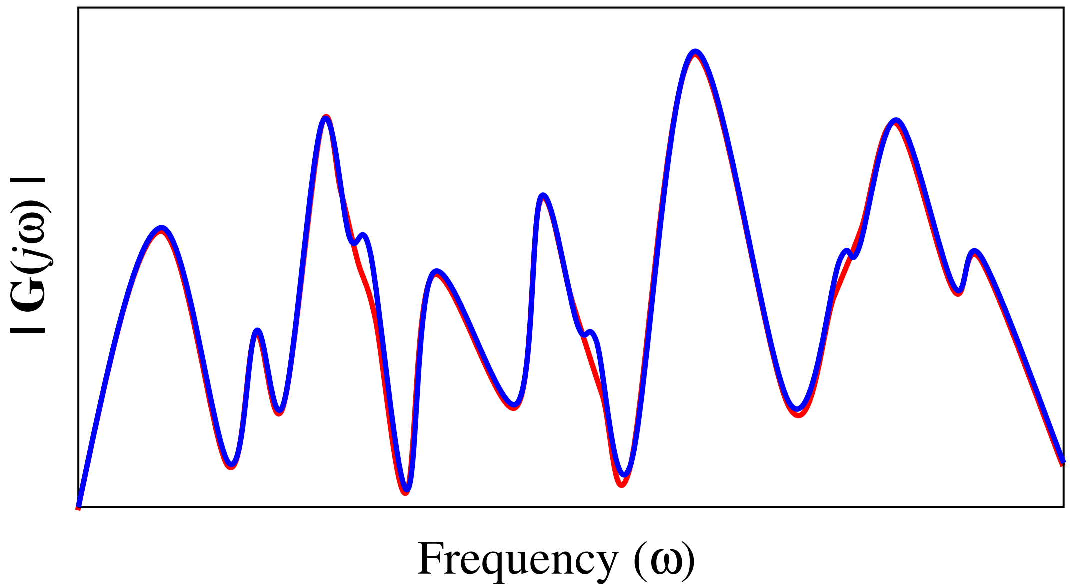
- Updated value of the pole for the next iteration: $\lambda^{(k+1)} = \lambda^{(k)} + \Delta\lambda^{(k)}$

- Associated residue: $$R^{(k+1)} = -\frac{\left[1 - u^{(k)}\ f\left(\lambda^{(k)}\right)\right]}{u^{(k)}}\Delta\lambda^{(k)}$$



$$G(j\omega) = \mathbf{c}^t\mathbf{Y}(j\omega)^{-1}\mathbf{b}$$

$$G(j\omega) \cong \sum_{i\in\Omega}\frac{R_i}{j\omega - \lambda_i} + d$$

$\Omega \rightarrow$ set of dominant poles

- It must be pointed out that the YMDPA is designed to avoid successive convergences to the same pole value.

Test Systems

References

Small Test System:

- S. Gomes Jr., N. Martins, S. L. Varricchio, C. Portela, “Modal Analysis of Electromagnetic Transients in ac Networks having Long Transmission Lines”. IEEE Transactions on Power Delivery, v.20, 2005.

Medium Scale Test System (Submitted)

- S. Gomes Jr., N. Martins, C. Portela, “Computing Multiple Dominant Poles for Modal Analysis of s-Domain Models”, submited to IEEE Transactions on Power Systems, 2007

Electromagnetic Transient Problem
Small Test System

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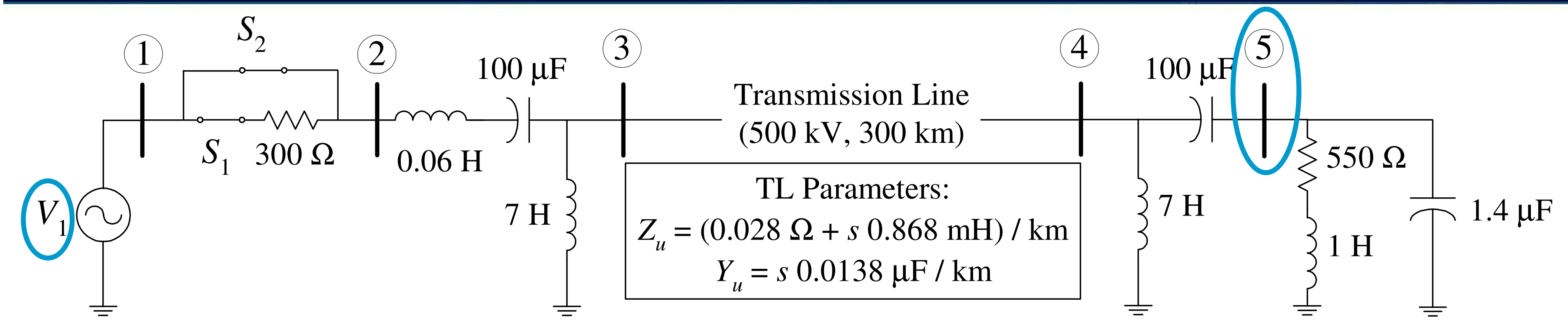

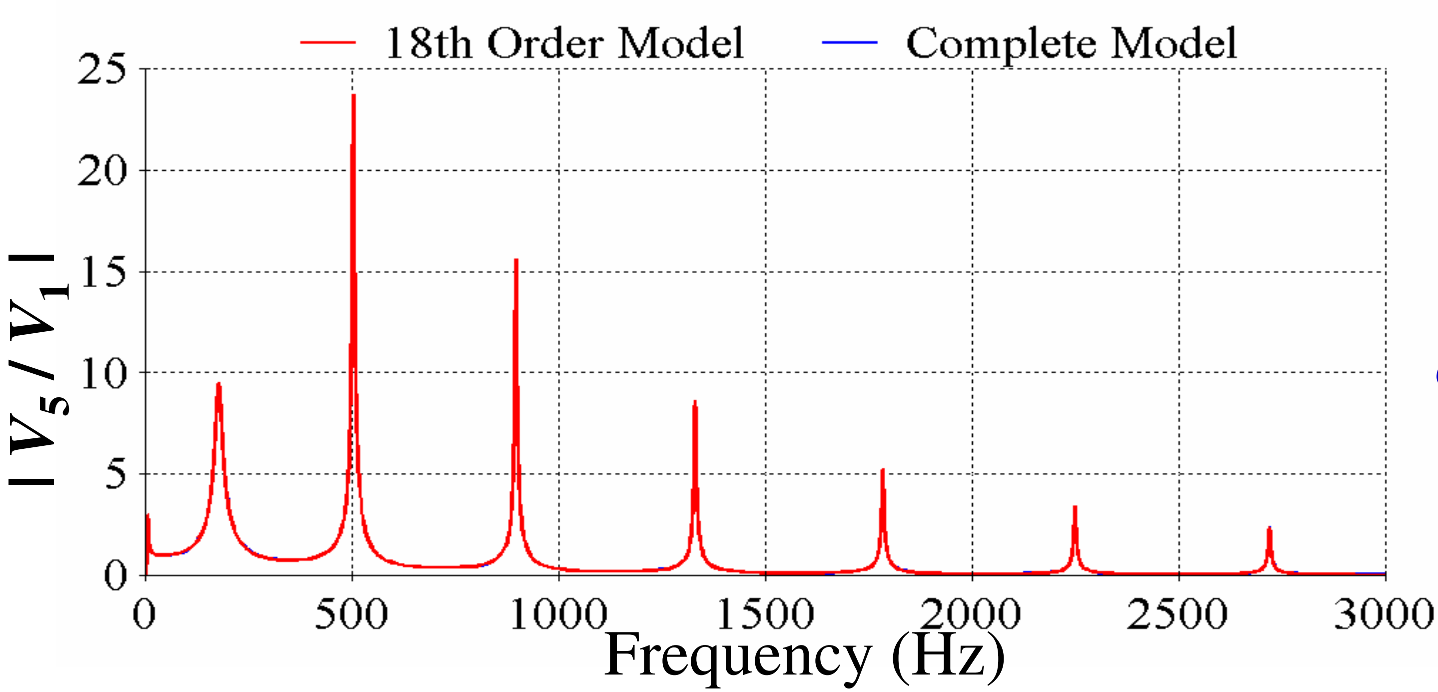

$$G_{-R}(j\omega) = \sum_{i=1}^{18} \frac{R_i}{j\omega - \lambda_i}$$

$$G_{-R}(j\omega) = \mathbf{c}^t \ \mathbf{Y}(j\omega)^{-1} \ \mathbf{b}$$

New England Medium Scale Test System

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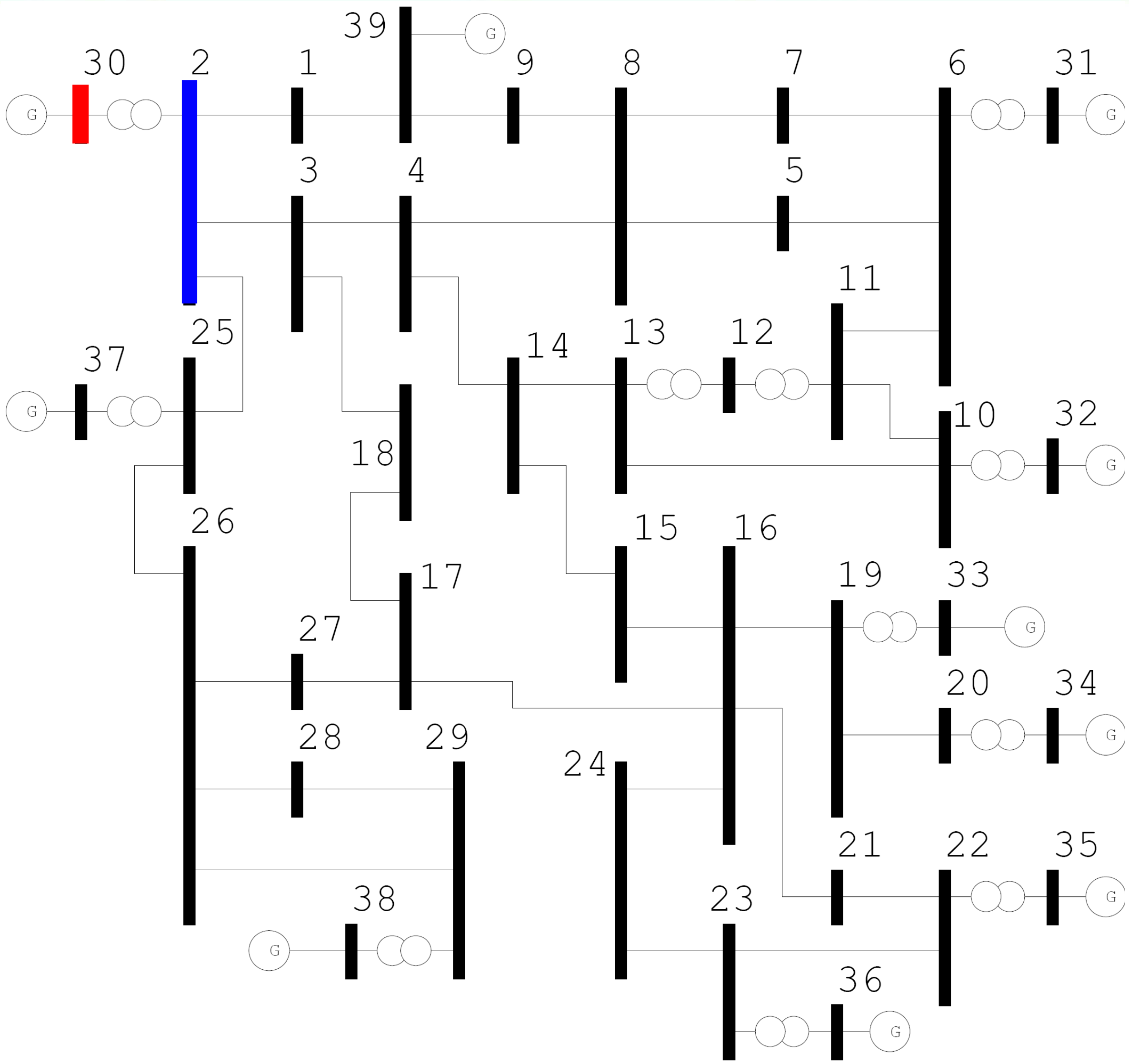


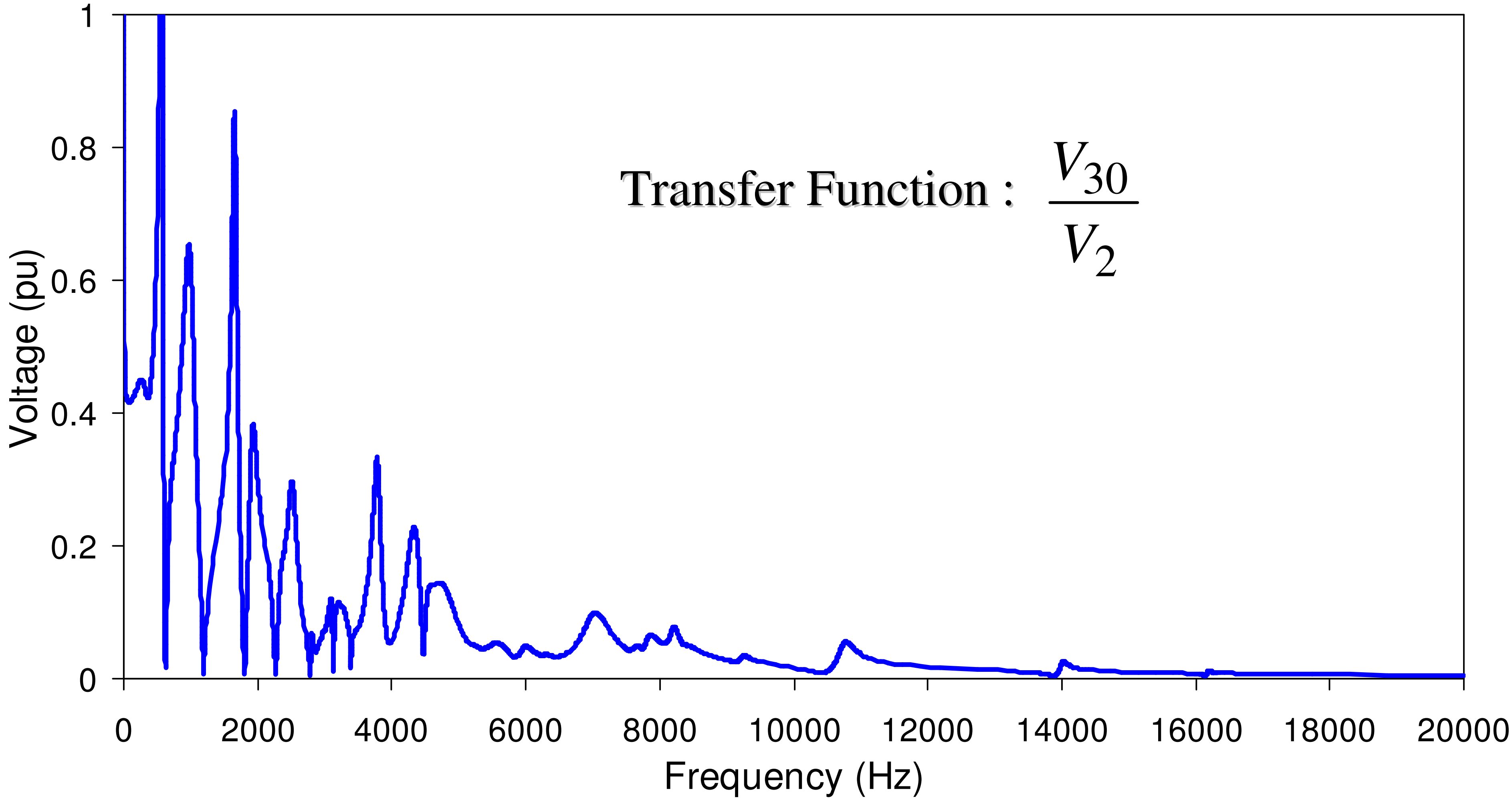
Input: V_{30}

Output: V_2

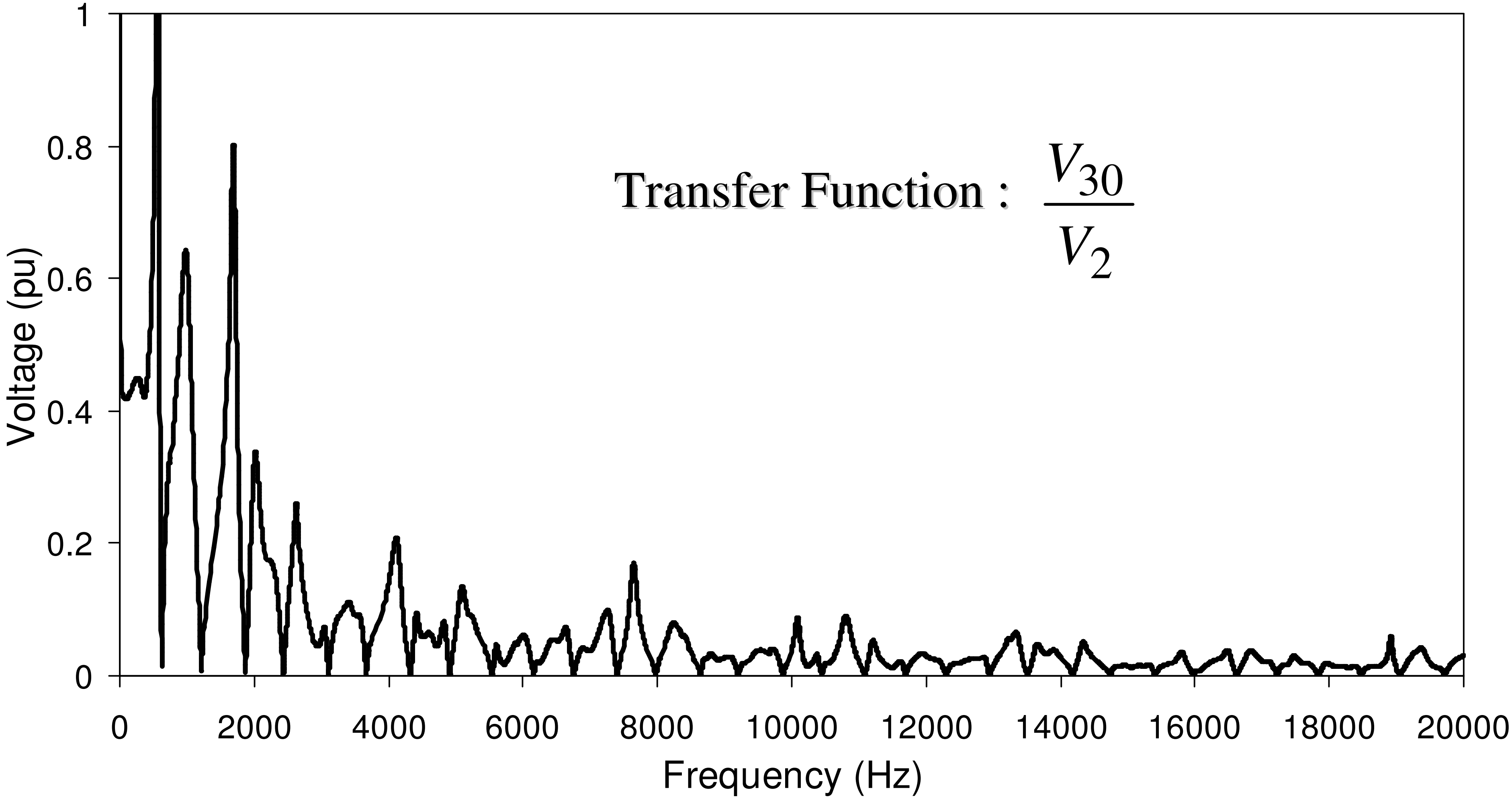
Transfer Function :

$$\frac{V_{30}}{V_2}$$

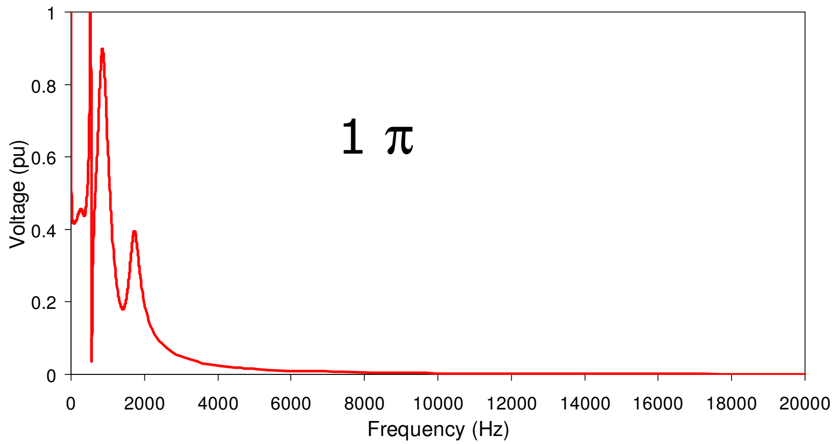
TL represented by lumped elements (5 π's) Descriptor System or Y(s) model

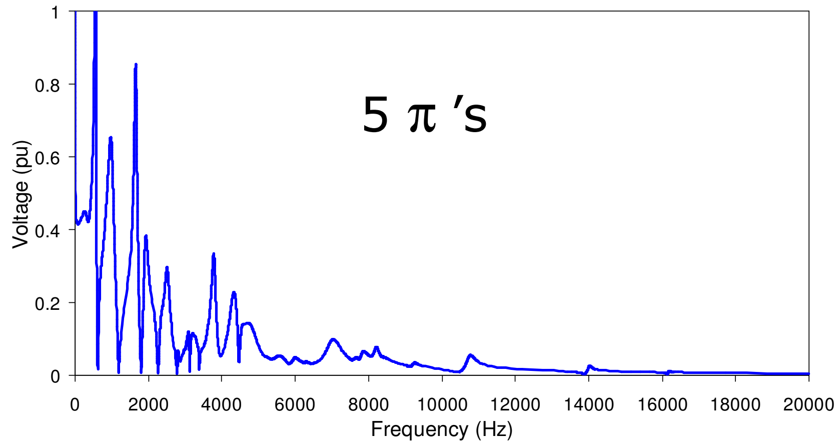


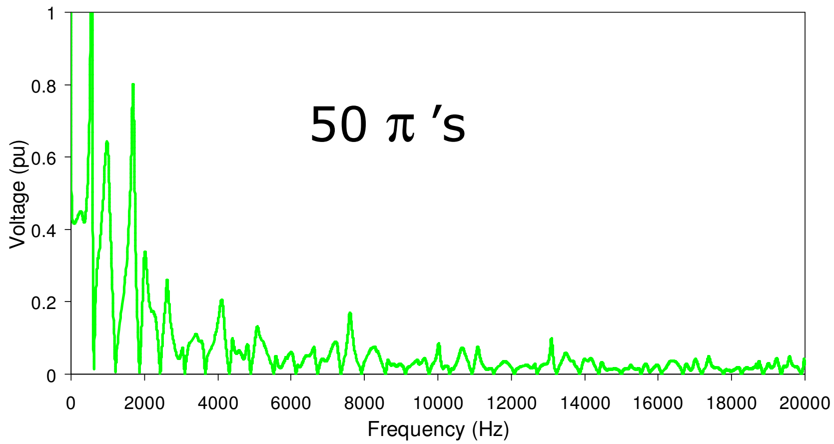
TL represented by distributed elements – only Y(s) model

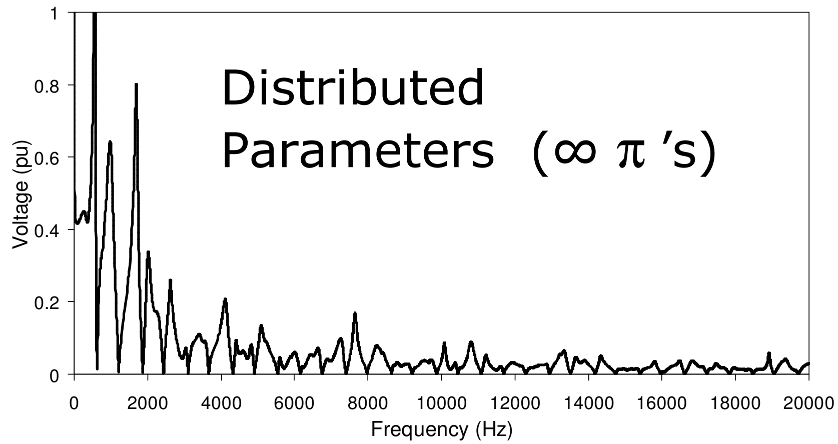


Frequency Response Comparison









Frequency Response Comparison

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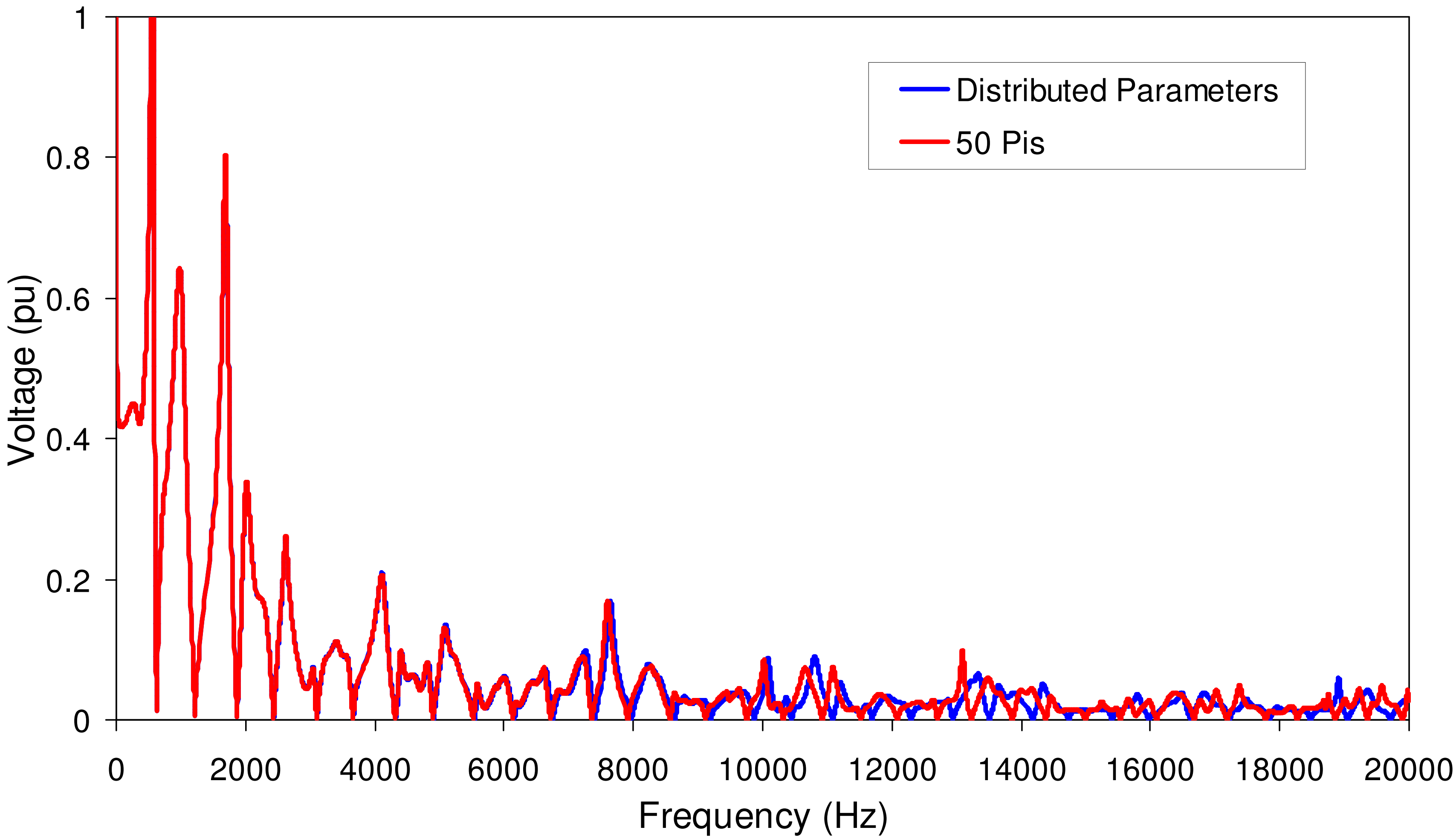

Comparing of Matrix Dimensions

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Model	Lines	Non-zeros	States	Differential Equations
1 π	291	747	91	133
			42 poles at infinity	
5 π's	571	1583	363	409
			46 poles at infinity	
50 π's	3721	10,988	≈3450	3514
Distributed Parameters	49	151	Infinite	s-Domain

TL represented by lumped elements (1Pi)

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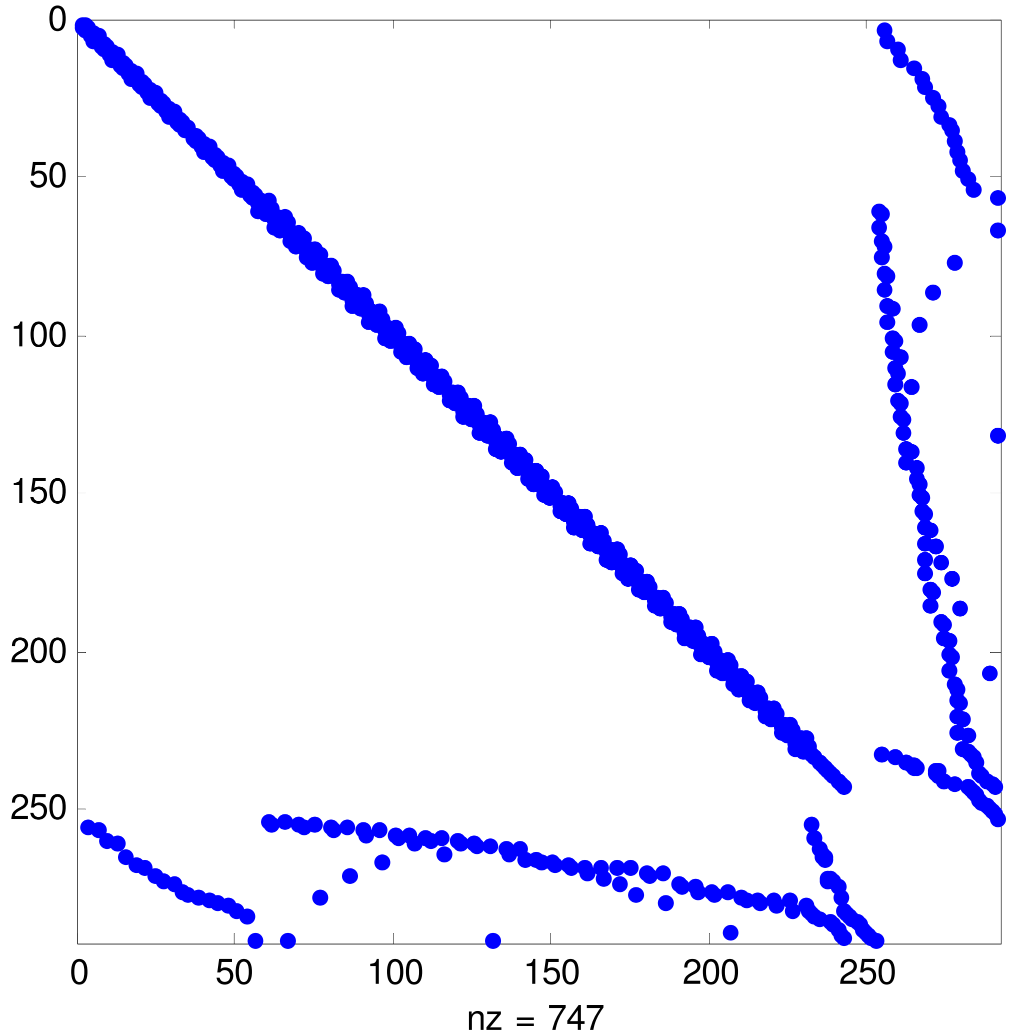

TL represented by lumped elements (5Pi)

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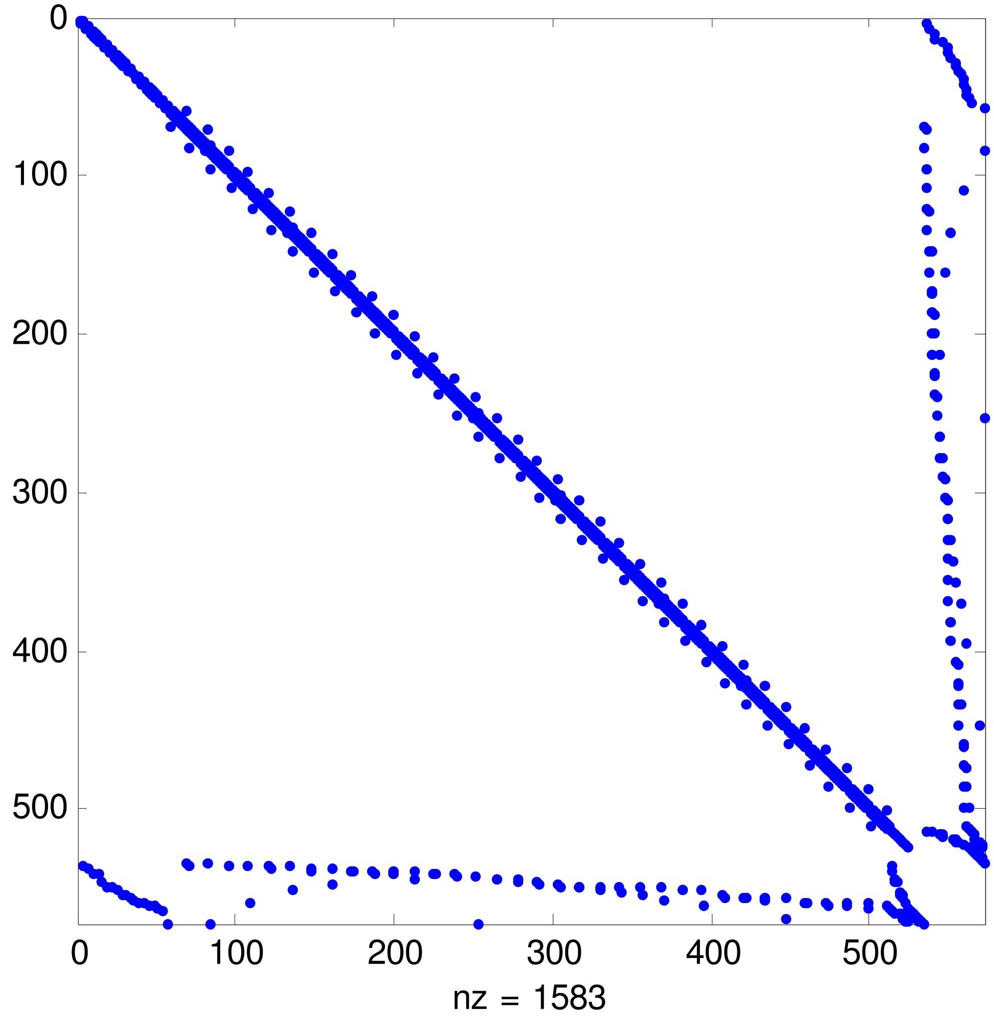

TL represented by lumped elements (50Pi)

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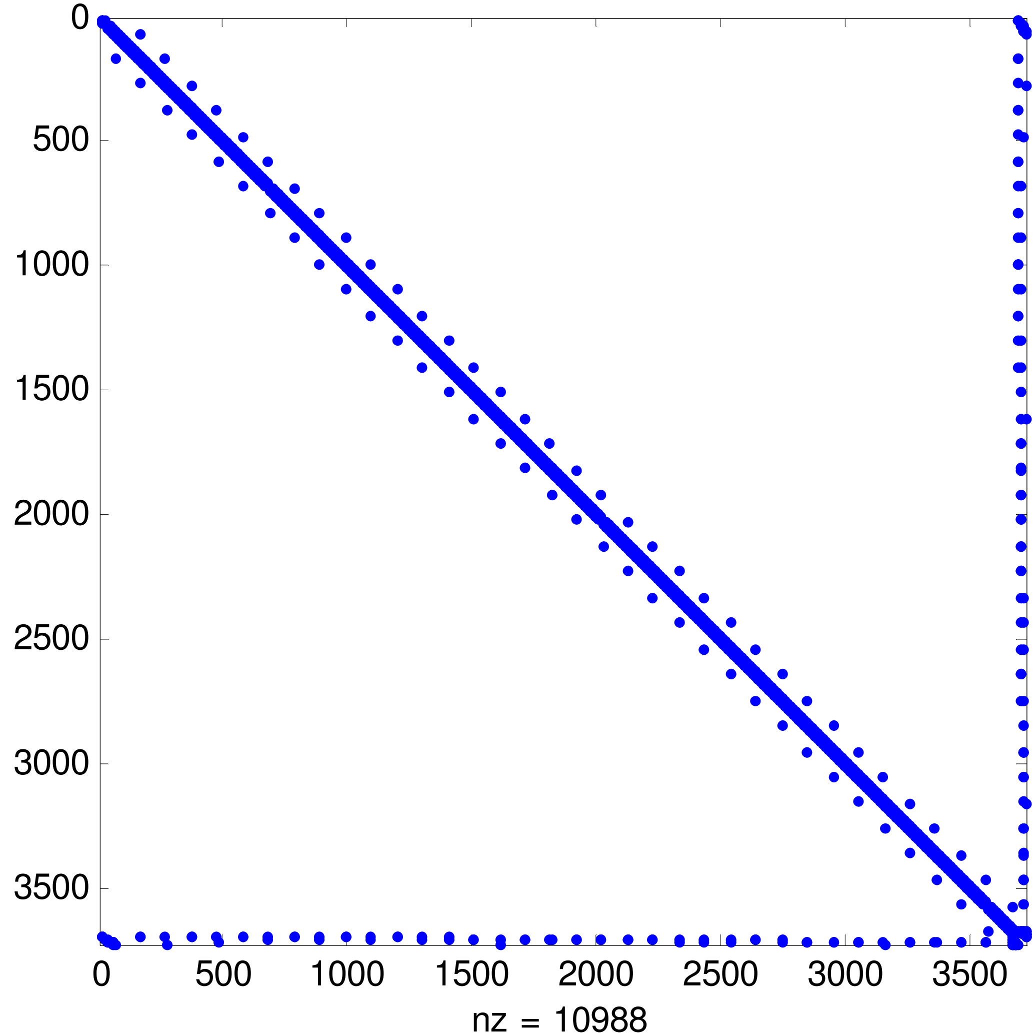
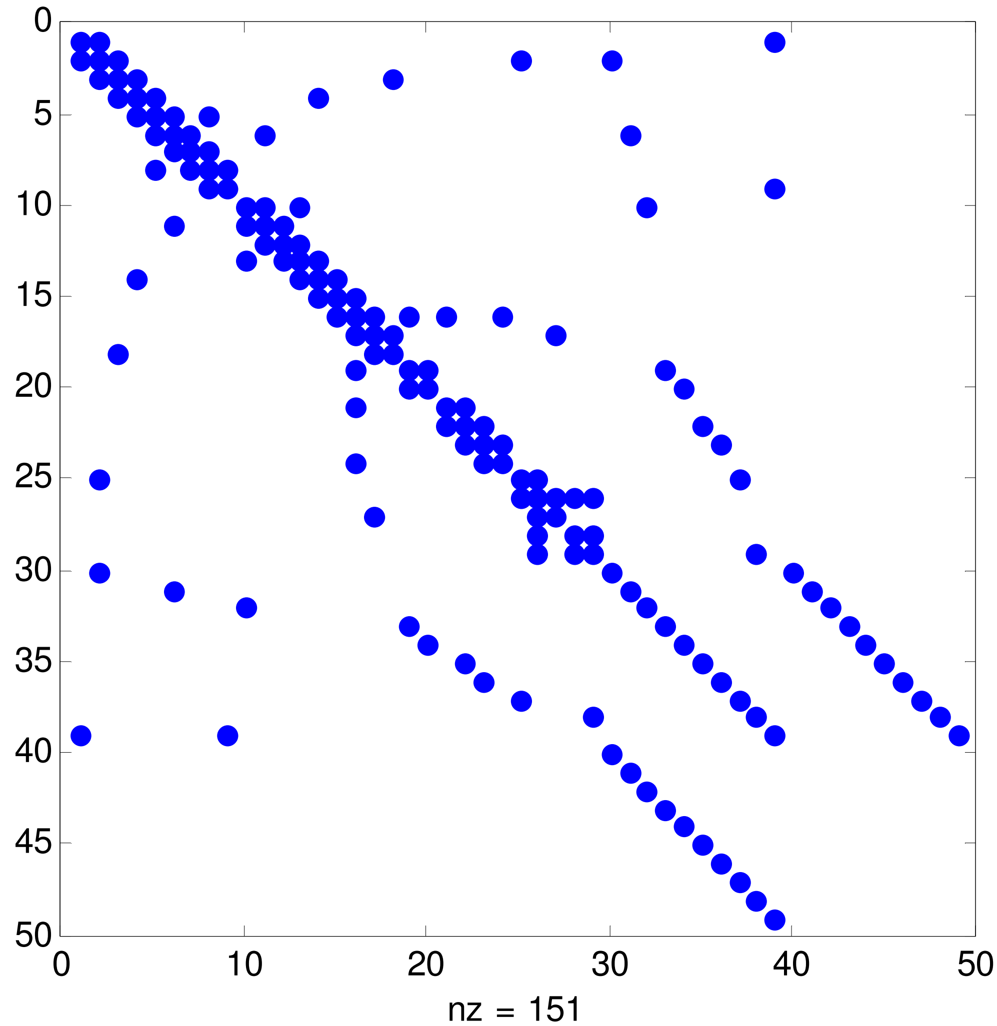


TL represented by Distributed Parameters – Y(s) model



Initial Estimates (65 poles) Maxima of Frequency Response

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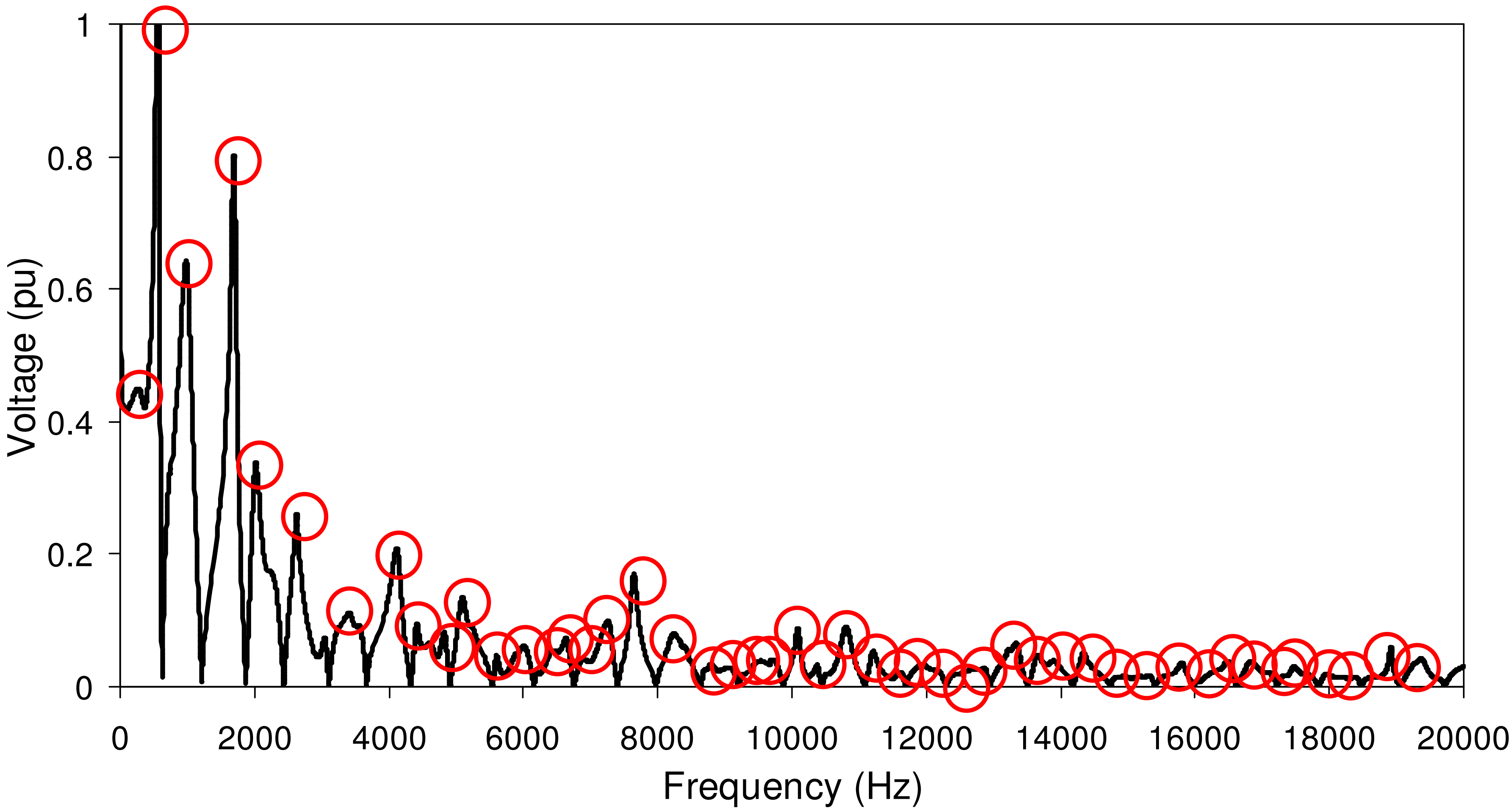

Convergence Trajectories

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1) 0.0000 + j 3529.4711
-97.7156 + j 3530.0323
-92.3038 + j 3551.5737
-92.5777 + j 3550.5634
-92.5785 + j 3550.5609
-92.5785 + j 3550.5609

2) 0.0000 + j 10612.3382
-310.3376 + j 10612.1829
-298.5317 + j 10661.3549
-297.3249 + j 10660.7282
-297.3253 + j 10660.7296
-297.3253 + j 10660.7296

3) 0.0000 + j 6155.0852
-666.8774 + j 6166.9126
-675.1764 + j 6326.7929
-651.5844 + j 6329.6281
-652.3466 + j 6329.9020
-652.3468 + j 6329.9029
-652.3468 + j 6329.9029

4) 0.0000 + j 1671.3037
-1353.3000 + j 1195.1432
-14.4765 + j 2259.3130
139.8079 + j 1784.0788
-907.2363 + j 703.6984
255.9207 + j 3245.4372
2291.8512 + j 4814.2130

Diverges!

Convergence Trajectories

CEPEL Grupo Eletrobrás

5) 0.0000 + j 16474.1872
-394.3475 + j 16346.4561
-565.8726 + j 16148.8898
-594.2694 + j 16390.8764
-612.8858 + j 16314.1086
-621.3457 + j 16313.6911
-621.2512 + j 16313.7738
-621.2513 + j 16313.7738
-621.2513 + j 16313.7738

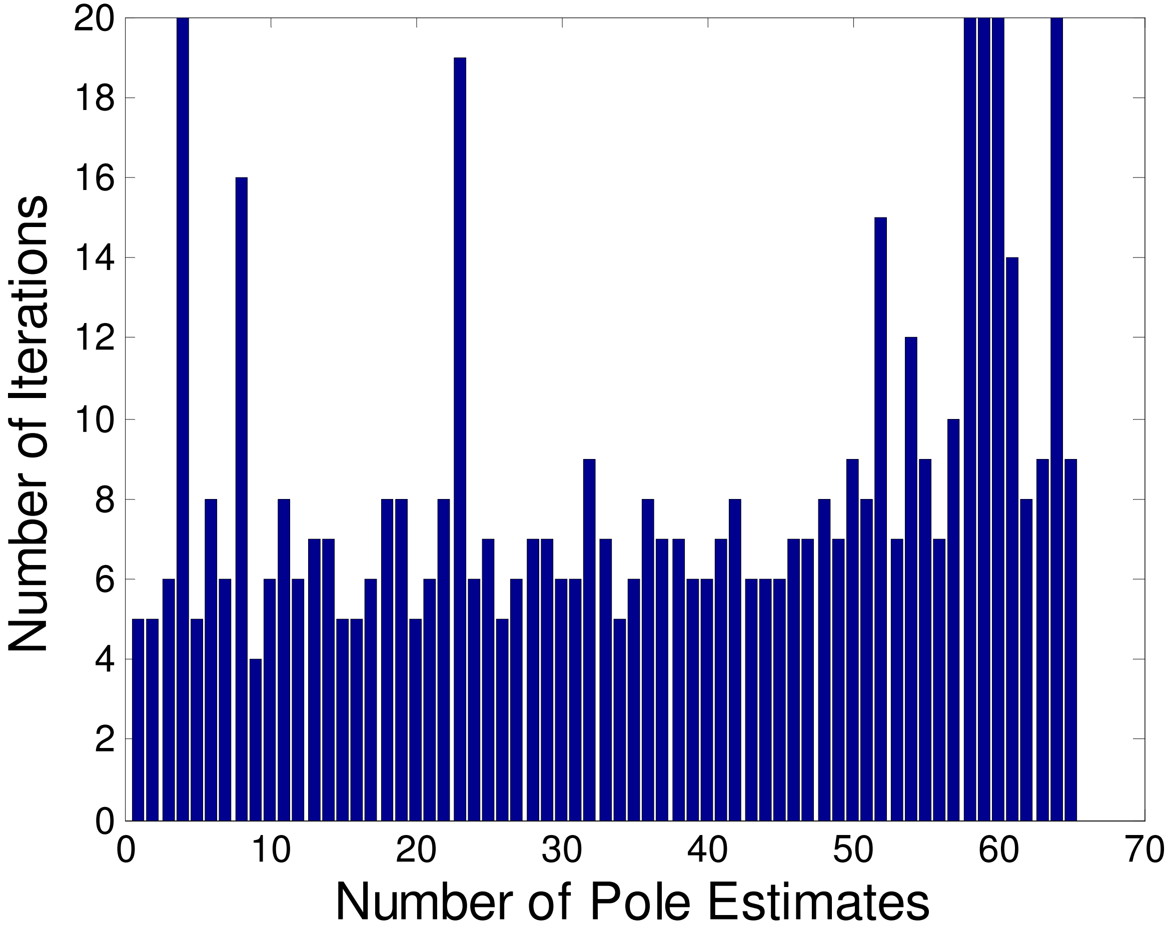
6) 0.0000 + j 25838.3619
-524.3319 + j 25758.6854
-447.1832 + j 25973.4738
-466.0145 + j 25947.2204
-466.7910 + j 25946.8793
-466.7914 + j 25946.8796
-466.7914 + j 25946.8796

7) 0.0000 + j 14009.1523
-999.6891 + j 13754.1664
-2202.9826 + j 18151.3126
-1509.4668 + j 8367.5991
-910.0498 + j 7359.5485
-810.3676 + j 7322.0306
-523.5739 + j 7231.6206
-485.9699 + j 5155.3132 Diverges!

8) 0.0000 + j 31992.2676
-578.7945 + j 31903.8852
-433.5513 + j 31905.0448
-423.3669 + j 31951.9051
-425.9337 + j 31951.0445
-425.9302 + j 31951.0531
-425.9302 + j 31951.0531

Convergence Report

65 Initial Shifts, 60 Converged Poles



Choosing Adequate Initial Shift

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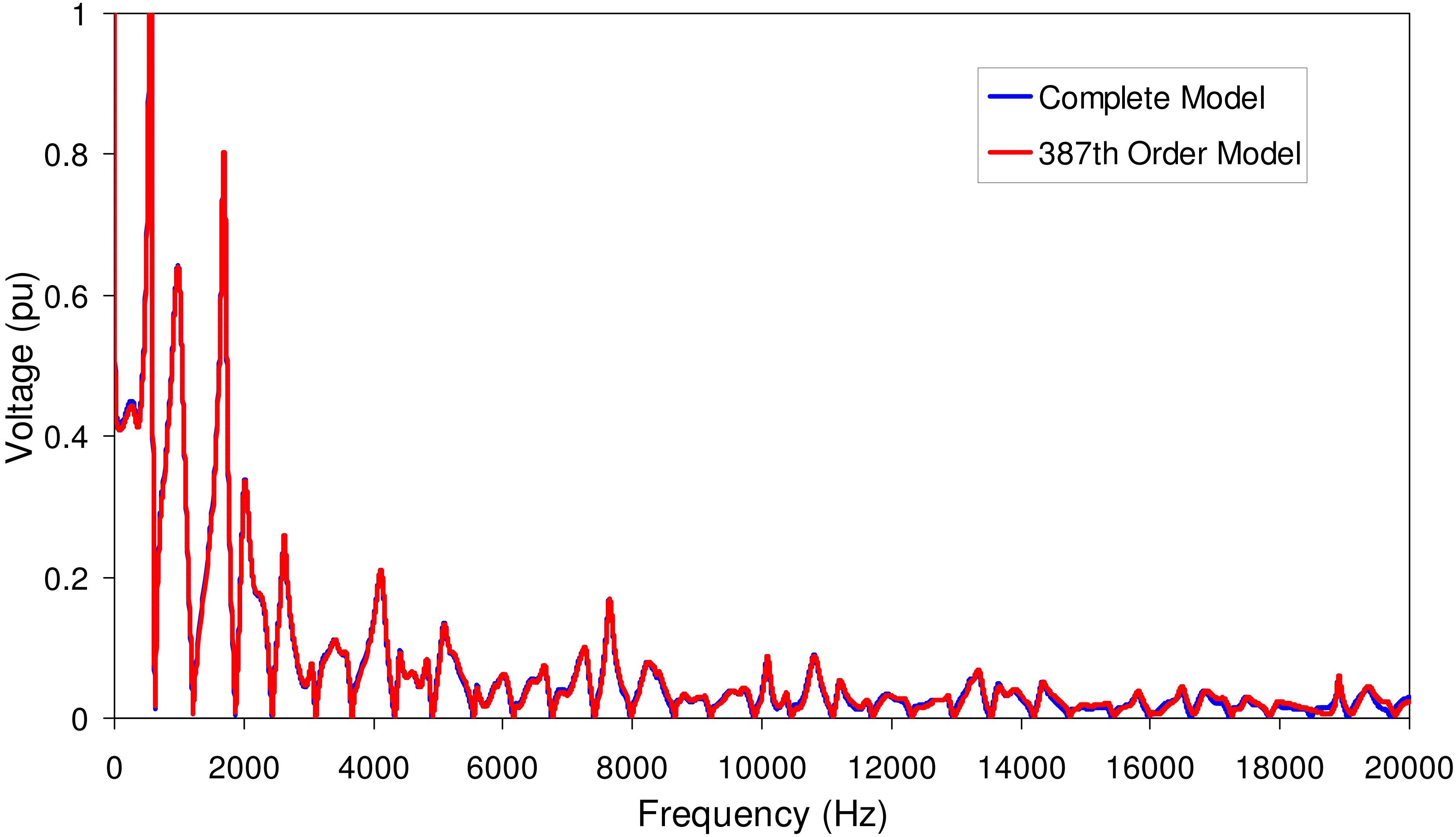
- Imaginary part of initial shifts made equal to the frequencies where the modal error is maximum (peak)
- Real Part Estimate of Shift:
  - ✓ Zero
  - ✓ Search maximum of model error in s-plane, for a fixed imaginary part and varying real part of initial shift.
- 201 poles were calculated (order 387)
  - ✓ 15 real poles
  - ✓ 186 complex conjugated pairs

Reduced Order Model



Conclusions

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- Developed Algorithms: YDPA & YMDPA
  - ✓ Full Newton Algorithms (quadratic convergence)
  - ✓ Efficient and Robust for large scale systems
  - ✓ Requires fairly good initial shifts
  - ✓ Allows eigenvalue analysis of infinite systems
    - *Distributed parameters*
    - *Frequency Dependent Elements*
    - *Transport Delay*
- YMDPA (Y(s) Multiple Dominant Poles)
  - ✓ Sequential
  - ✓ Deflation by Full Newton subtraction of combined effects of converged poles
  - ✓ No associated numerical difficulties
  - ✓ Initial shifts extracted from frequency response plots

THANK YOU FOR YOUR ATTENTION !

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