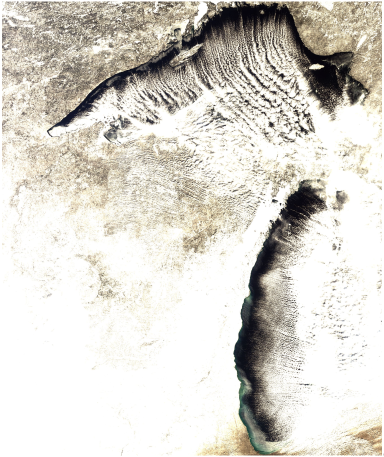
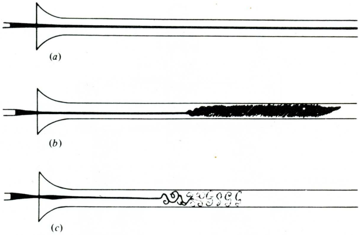
Shear Turbulence

Fabian Waleffe

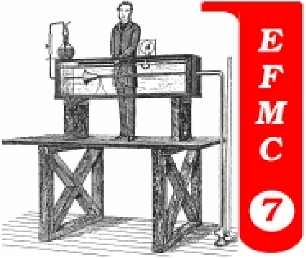
Depts. of Mathematics and Engineering Physics

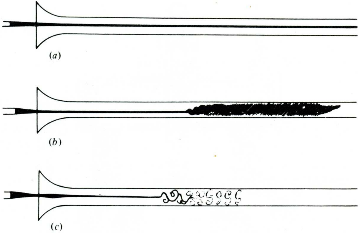


University of Wisconsin, Madison



Mini-Symposium on
Subcritical Flow Instability
'for training of early stage researchers'





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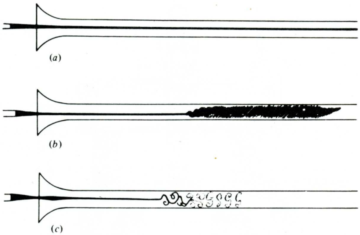
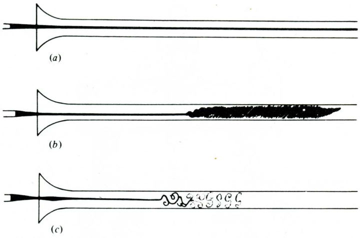
Instability?





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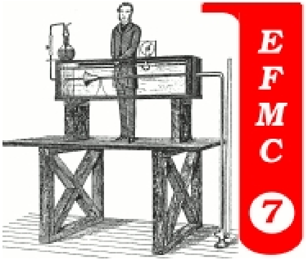
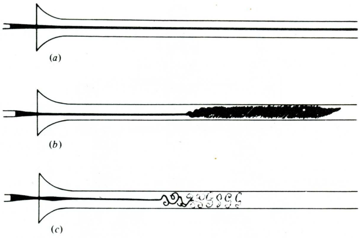
Instability?

Subcritical instability?





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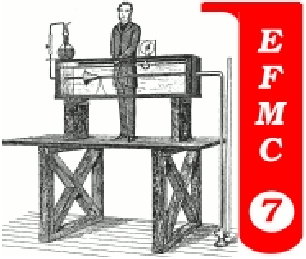
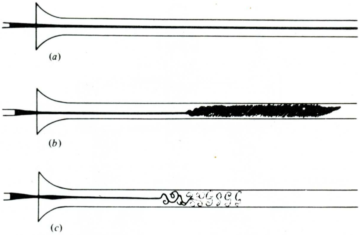
Instability?

Subcritical instability?

Transition to turbulence?





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Instability?

Subcritical instability?

Transition to turbulence?

What is turbulence?

Incompressible flow: Navier-Stokes equations

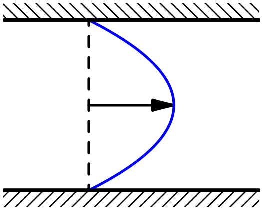
3D velocity field $\mathbf{u}(\mathbf{x}, t)$:

$$\boldsymbol{\nabla} \cdot \mathbf{u} = 0$$

$$\partial_t \mathbf{u} + \boldsymbol{\nabla} \cdot (\mathbf{u}\mathbf{u}) + \boldsymbol{\nabla} p = \frac{1}{R} \nabla^2 \mathbf{u}$$

R: Reynolds number

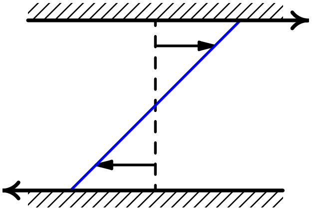
Simple geometries, simple flow?



Pipe, Channel

$\mathbf{u} = (1 - y^2)\,\hat{\mathbf{x}}$

$R \lesssim 2,000$



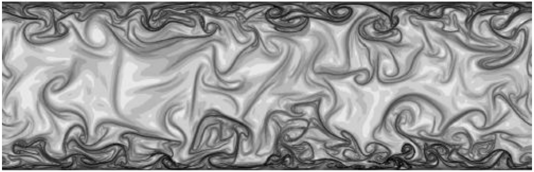
Plane Couette

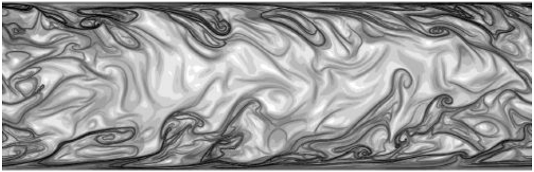
$\mathbf{u} = y\,\hat{\mathbf{x}}$

$R \lesssim 350$

Channel flow for $R \gtrsim 2,000$ . . . not simple!

Front





Side

Top

Green, M. A., Rowley, C. W. & Haller, G.
Detection of Lagrangian coherent structures in three-dimensional turbulence,
J. Fluid Mech., **572**, 2007, 111-120.

Instability?

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NSE: $$\frac{d\mathbf{u}}{dt} = f(\mathbf{u}; R)$$

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Linear stability: $$\frac{d(\mathbf{u}-\mathbf{u}_0)}{dt} = \left(\frac{\partial f}{\partial \mathbf{u}}\right)_0 (\mathbf{u}-\mathbf{u}_0)$$

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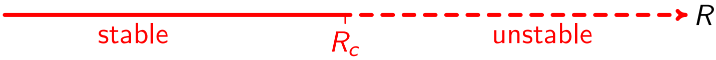
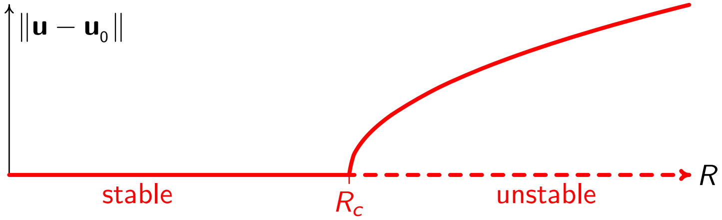
$$\Rightarrow \mathbf{u} - \mathbf{u}_0 = \mathbf{v}e^{\lambda t}, \quad \left(\frac{\partial f}{\partial \mathbf{u}}\right)_0 \mathbf{v} = \lambda \mathbf{v},$$

Instability if $\Re(\lambda) > 0$, typically for $R > R_c$

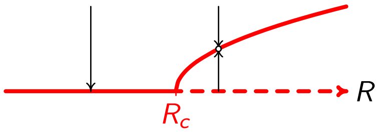
Linear instability



$\Rightarrow$ Nonlinear bifurcation



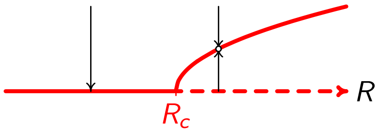
Bifurcations



Supercritical, NL stabilizes

$$\frac{dA}{dt} = (R - R_c)A - A^3$$

Bifurcations



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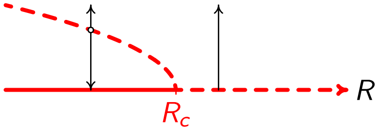

Subcritical, NL *de*-stabilizes,

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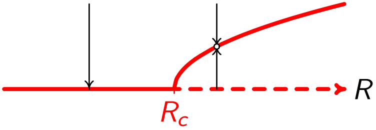

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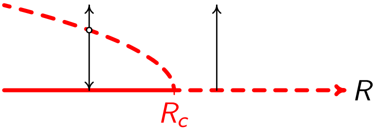

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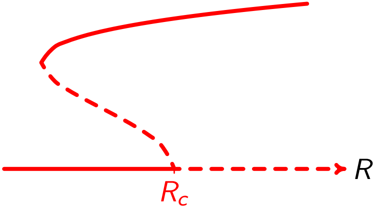
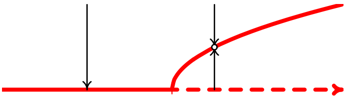
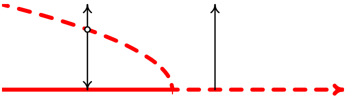


Higher order NL stabilization?

$$\frac{dA}{dt} = (R - R_c)A + A^3 - A^5 \cdots$$

Supercritical or subcritical?





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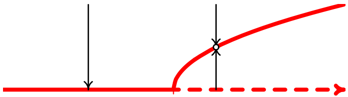

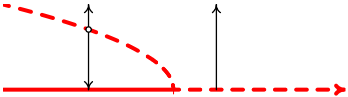

Nonlinearity decides: $$\frac{dA}{dt} = (R - R_c)A \pm A^3$$

Supercritical or subcritical?

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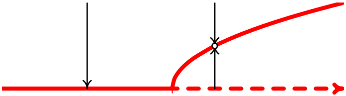

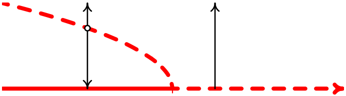

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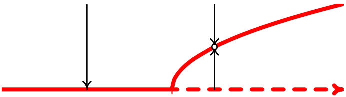

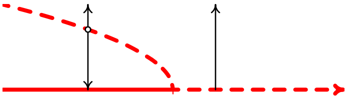

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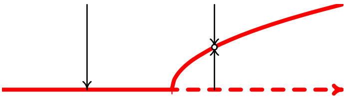

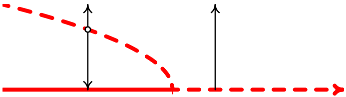

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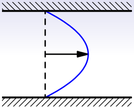
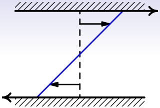
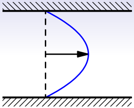
$\Rightarrow$ necessary for *subcritical*: $L \neq L^T$



Back to shear flows



- Linear stability: $L \neq L^T$ ...



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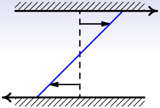

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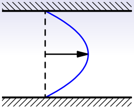

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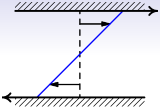

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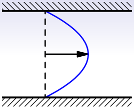

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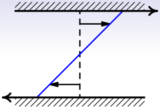

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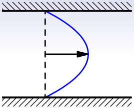

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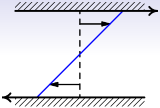

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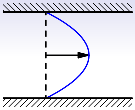

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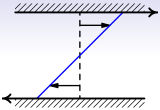

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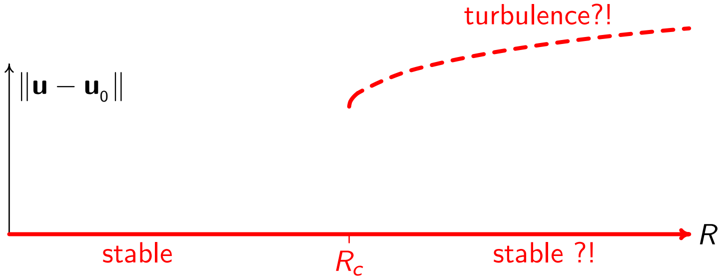
- Who said: *"Subtle is the Lord, but malicious s/he is not."* ?!

Shear turbulence



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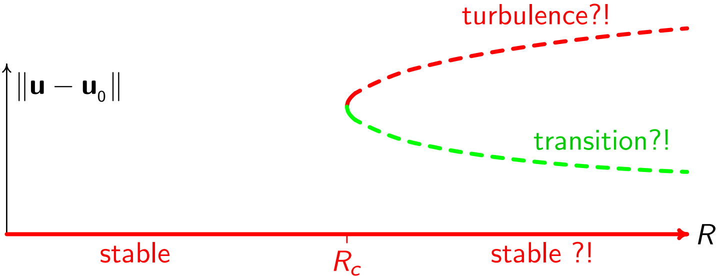
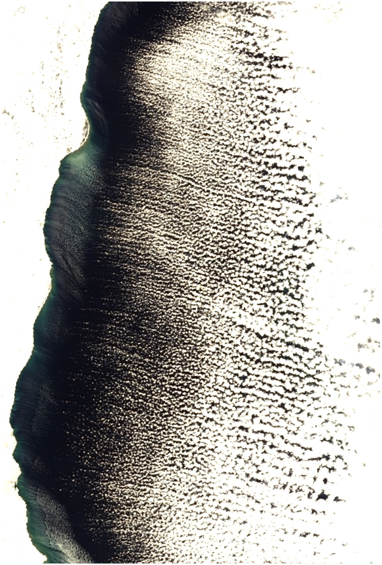
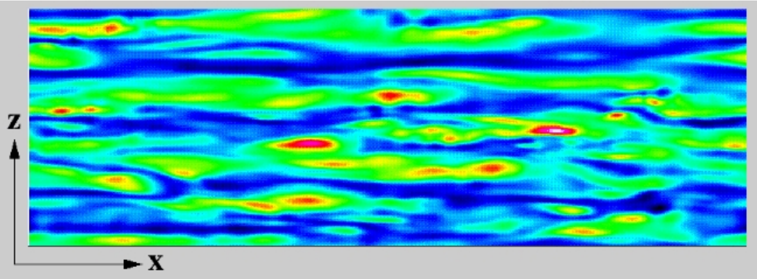


Streaks in turbulent shear flows . . .



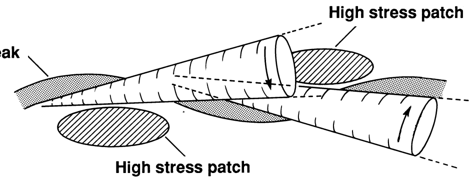
$R \approx 10^9$? (OK, some convection too . . .)

Streaks in turbulent channel flow . . .



Skin friction (Kim, Moin, Moser JFM 2007)

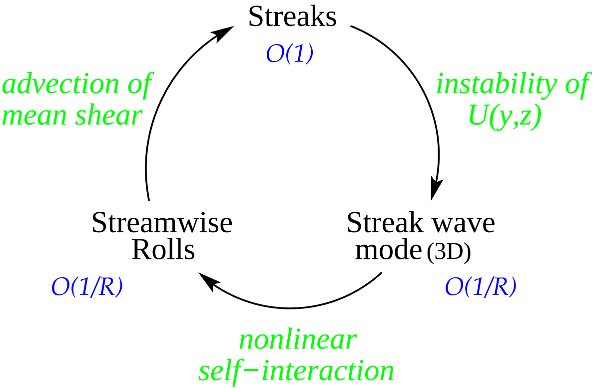
Characteristic near-wall coherent structure



Derek Stretch, CTR Stanford, 1990

Streaks + staggered quasi-streamwise vortices: why?

Self-Sustaining Process (SSP)



WKH 1993, HKW 1995, W 1995, 1997

SSP theory $\longrightarrow$ SSP method

(1) Add $\dfrac{F}{R^2}$ forcing of streamwise rolls to NSE

$\Rightarrow O(\frac{1}{R})$ rolls, $O(1)$ streaks

('Full' NSE, Newton's method)

PRL 1998, JFM 2001, PoF 2003

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(4) (*optional*) Recall:

"Subtle is the Lord, but malicious s/he is not."

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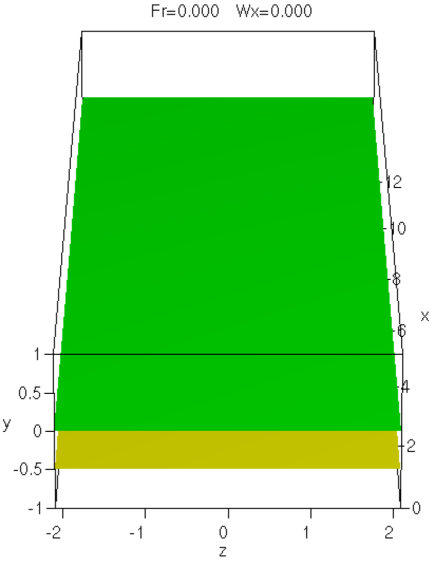
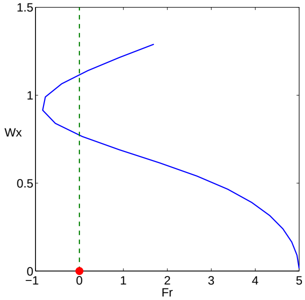
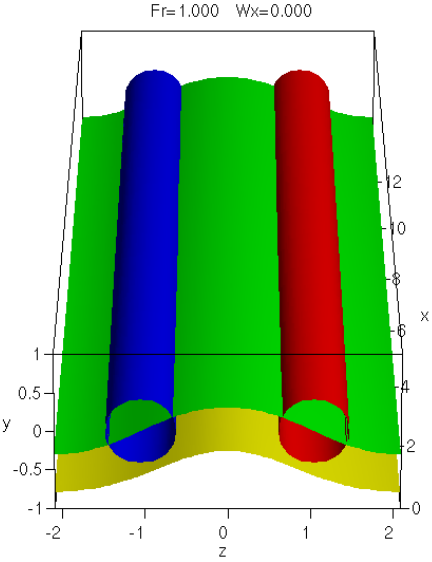
PRL 1998, JFM 2001, PoF 2003

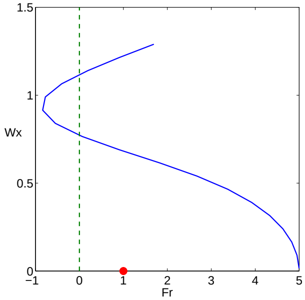
Laminar Couette flow: u=0 & u=-0.5



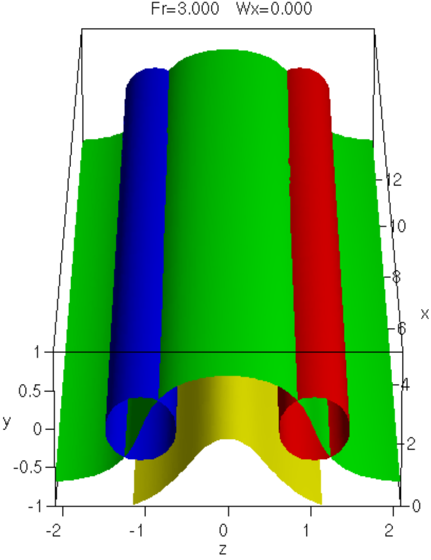


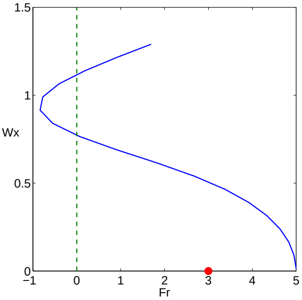
SSP: Streamwise Rolls create Streaks



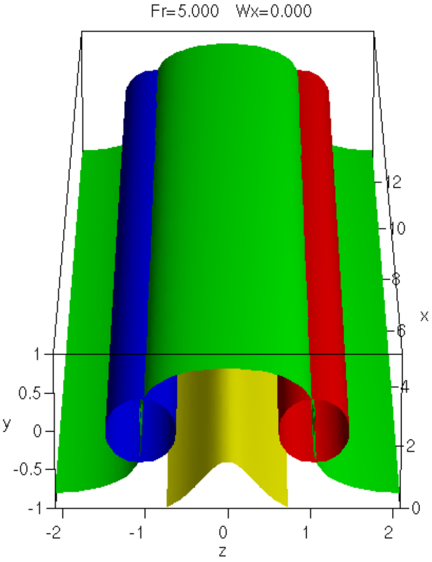


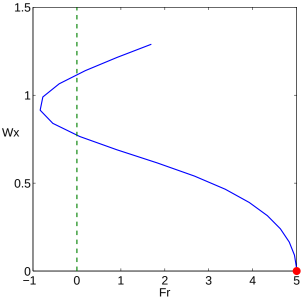
SSP: Rolls create Streaks



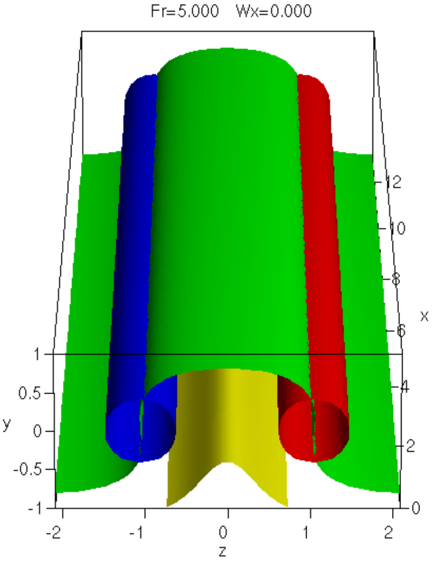


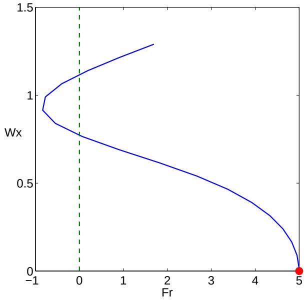
SSP: Rolls create Streaks



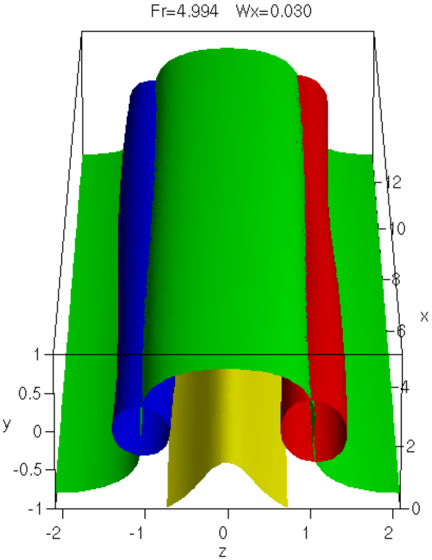


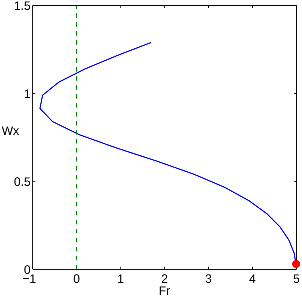
SSP in action: Subcritical Bifurcation from Streaks



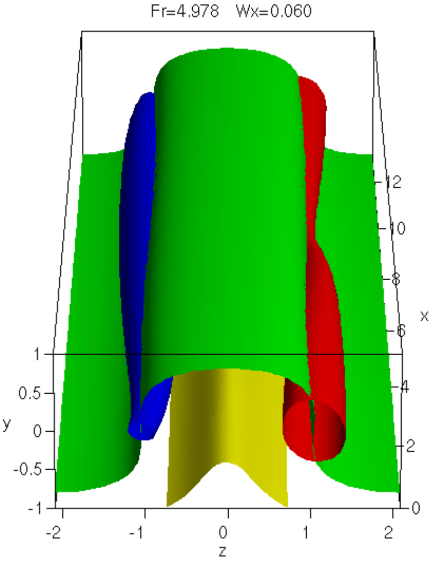


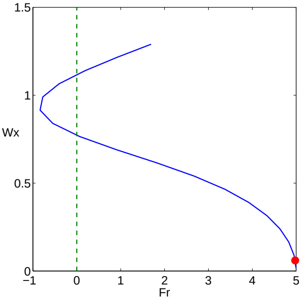
SSP in action: Subcritical Bifurcation from Streaks



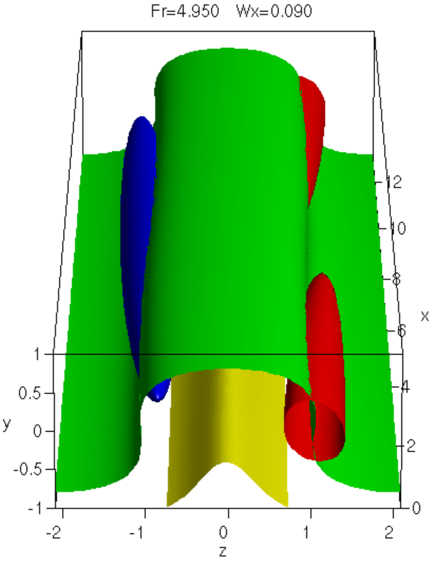


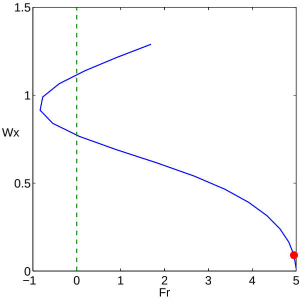
SSP in action: Subcritical Bifurcation from Streaks



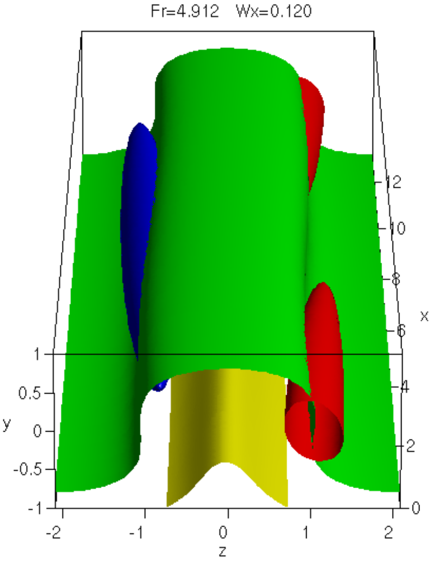


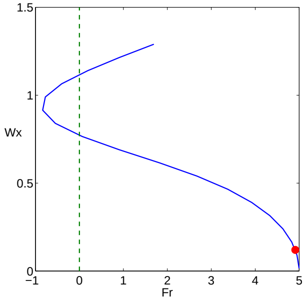
SSP in action: Subcritical Bifurcation from Streaks



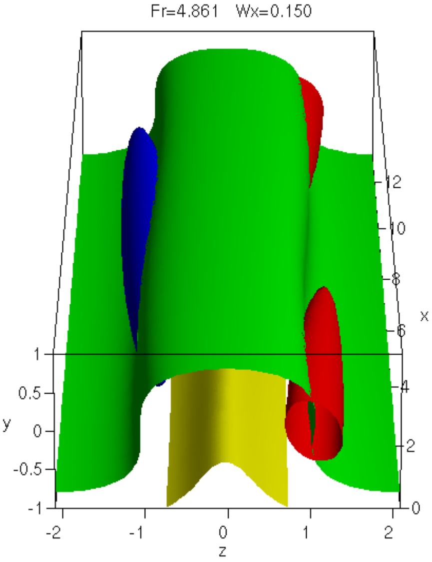


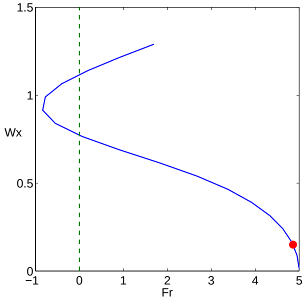
SSP in action: Subcritical Bifurcation from Streaks



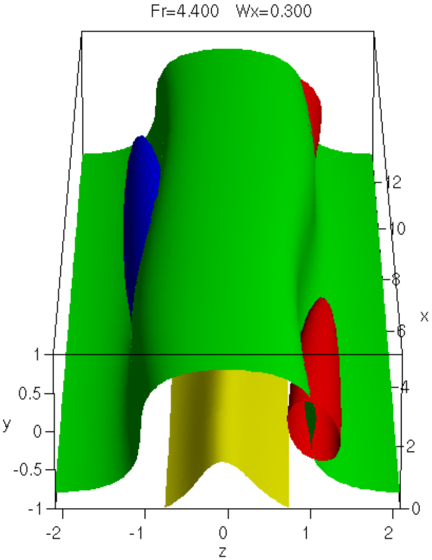


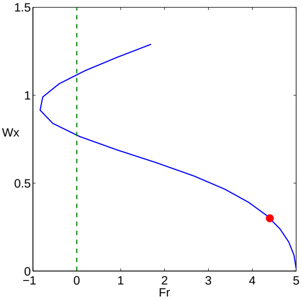
SSP in action: Subcritical Bifurcation from Streaks



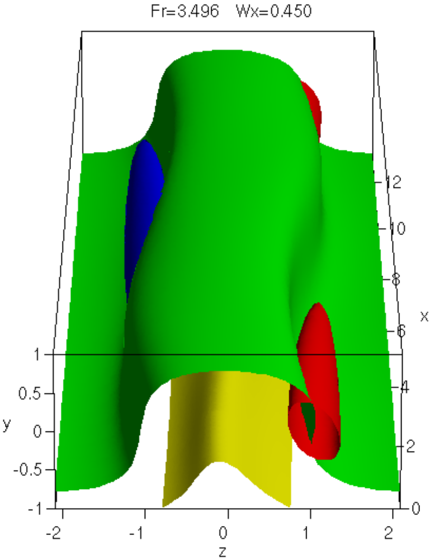


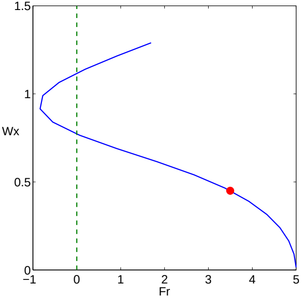
SSP in action: Subcritical Bifurcation from Streaks



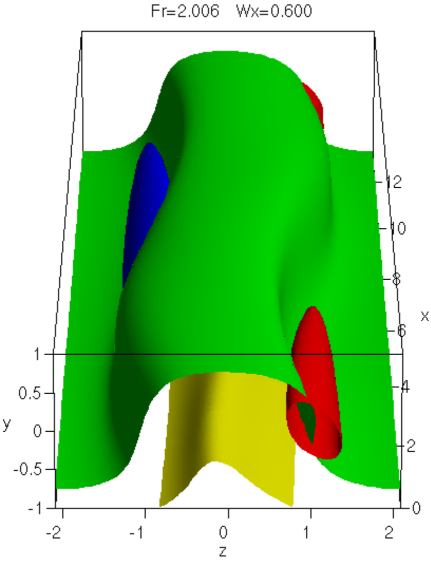


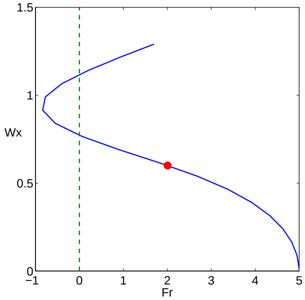
SSP in action: Subcritical Bifurcation from Streaks



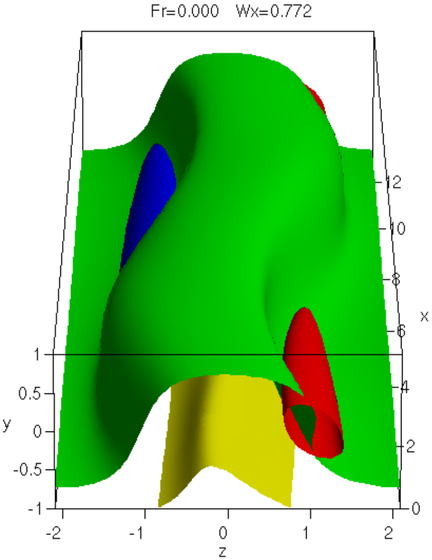


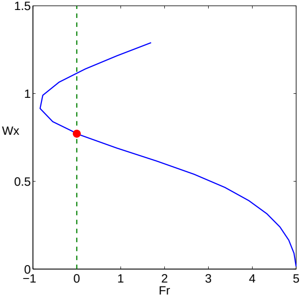
SSP in action: Subcritical Bifurcation from Streaks



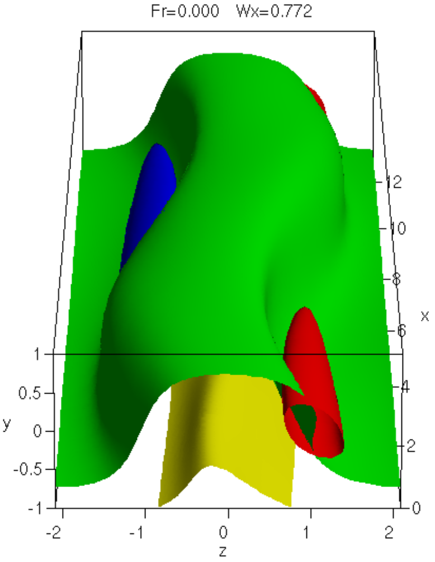


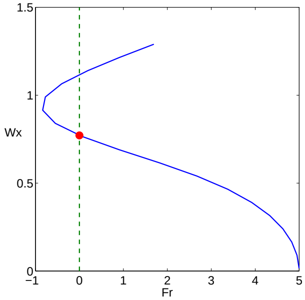
SSP: Self-Sustained! 3D Lower branch



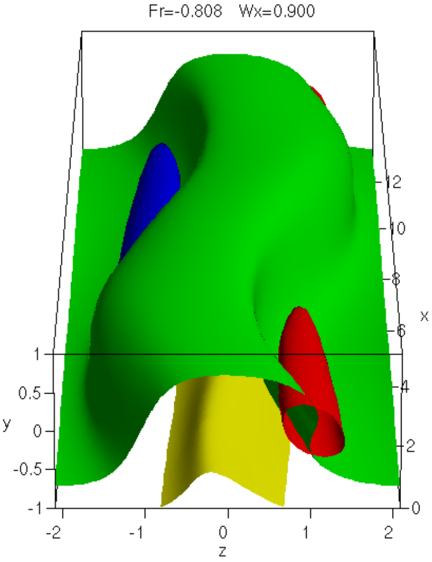


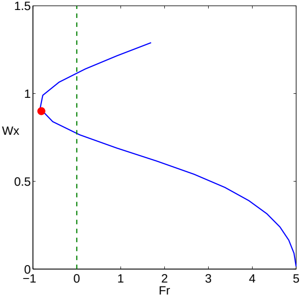
SSP: Self-Sustained! 3D Lower branch



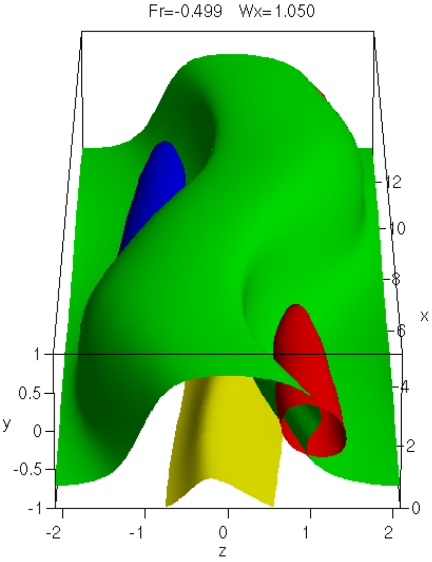


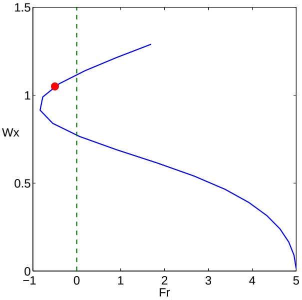
SSP: Bifurcation from Streaks



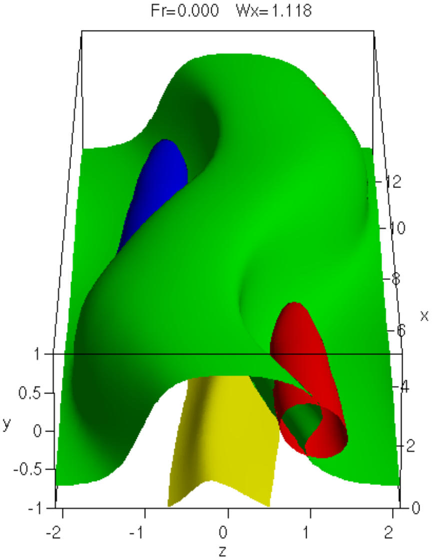


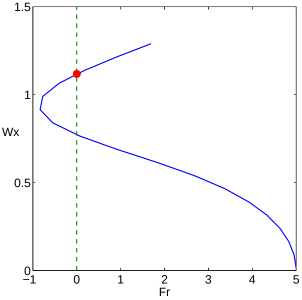
SSP: Bifurcation from Streaks



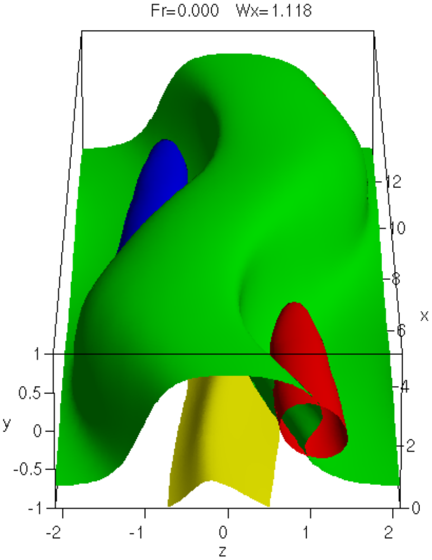


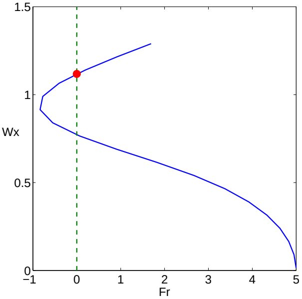
SSP: Self-Sustained! 3D Upper branch





SSP: Self-Sustained! 3D Upper branch





Homotopy

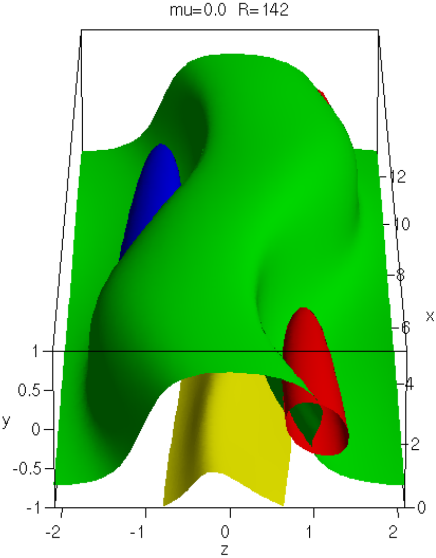
Free-Free Couette (FFC) $\rightarrow$ Rigid-Free Poiseuille (RFP)

$$\mu = 0 \rightarrow 1$$

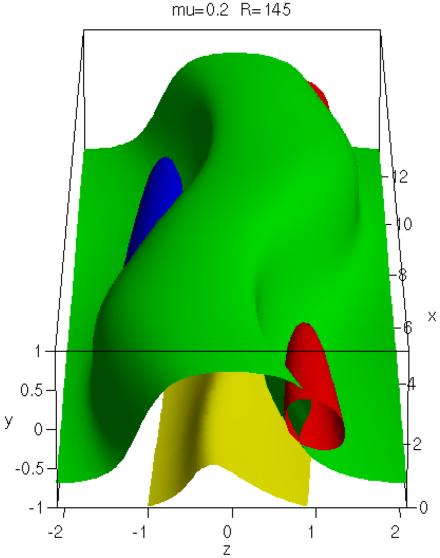
$$BC: \quad (1-\mu)\frac{du}{dy} + \mu u = 0$$

$$Flow: \quad U_L(y) = y + \mu\left(\frac{1}{6} - \frac{y^2}{2}\right)$$

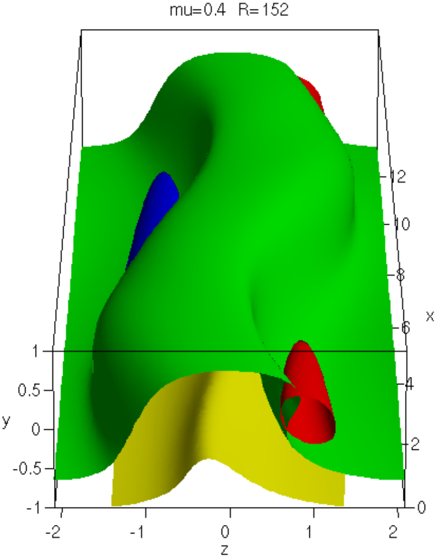
FFC $\longrightarrow$ RFP



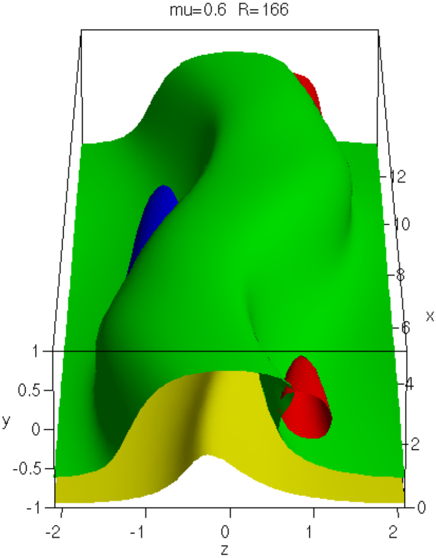
FFC ⟶ RFP



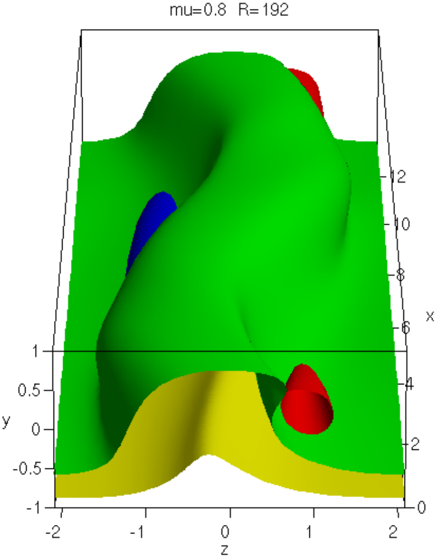
FFC $\longrightarrow$ RFP



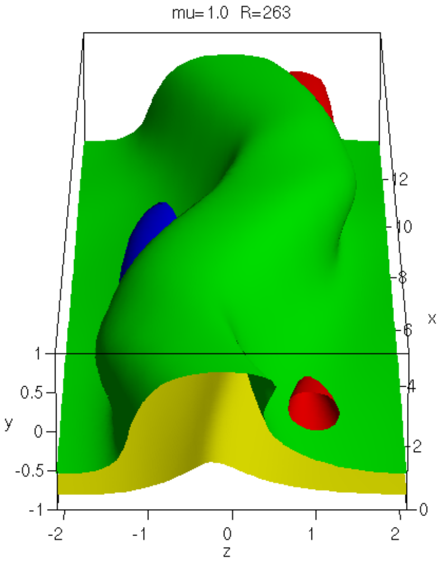
FFC $\longrightarrow$ RFP



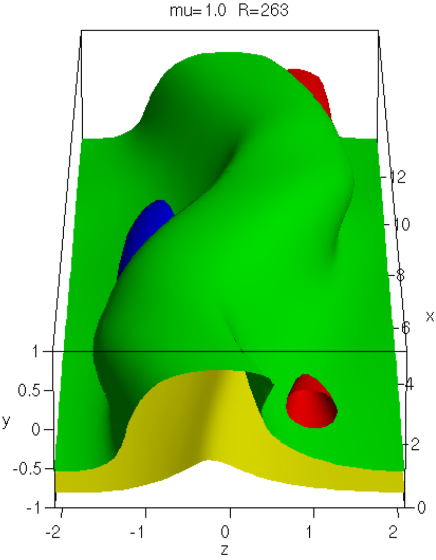
FFC $\longrightarrow$ RFP



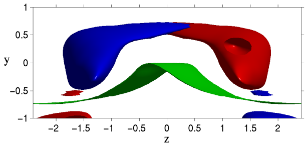
Poiseuille traveling wave!

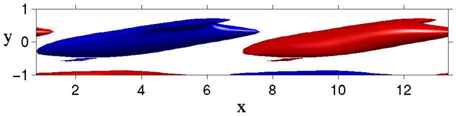


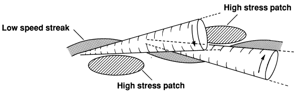
Poiseuille traveling wave!

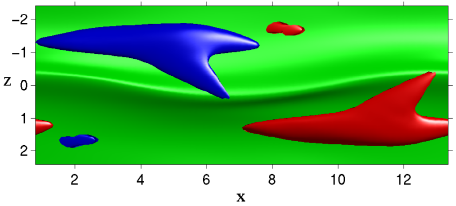


Optimum Traveling Wave: 100^+ !





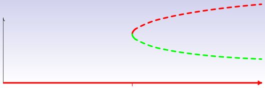


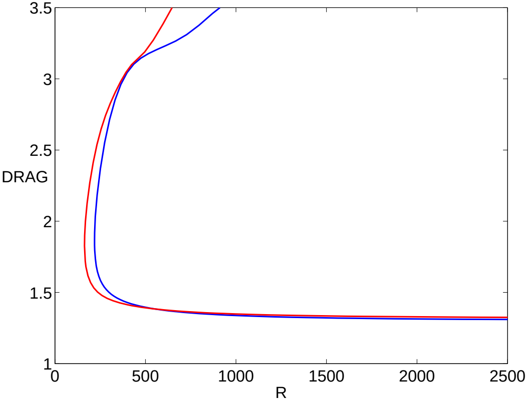


$$\min\ R_\tau = 2h^+ = 44 \quad \text{for } L_x^+ = 274,\ L_z^+ = 105$$

just right!

‘Out-of-the-blue-sky’

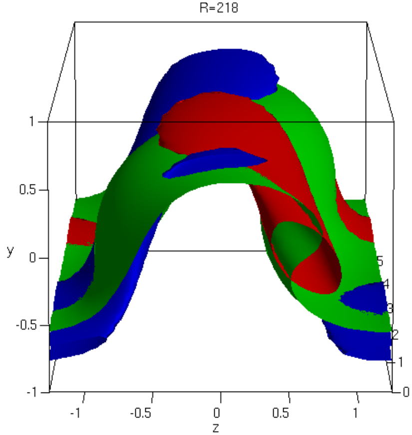




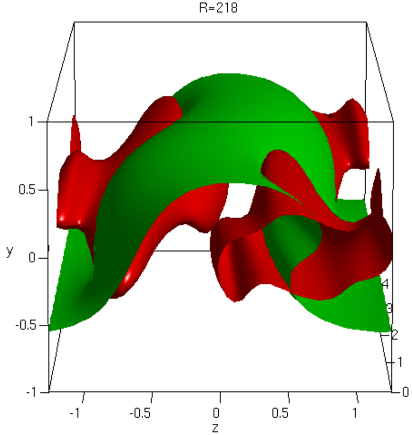
Lower branch does NOT bifurcate from laminar flow!

RRC $(\alpha, \gamma) =$ (1, 2), (1.14, 2.5), up to R≈ 60 000 + asymptotics

Vortex visualization

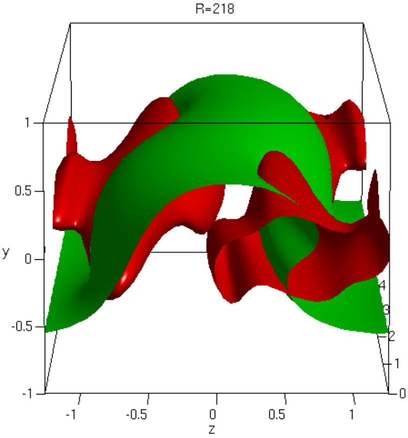


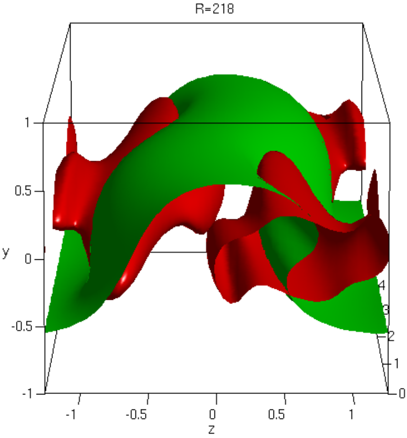
ω_x



$2Q = \nabla^2 p = \Omega_{ij}\Omega_{ij} - S_{ij}S_{ij}$

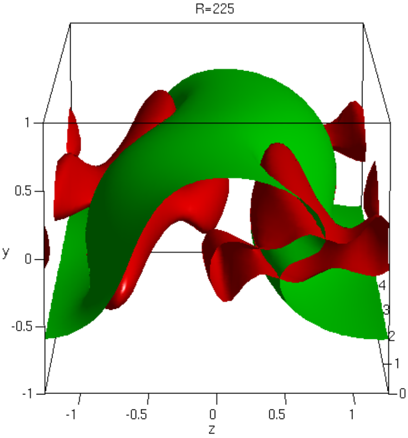
Upper and Lower branches

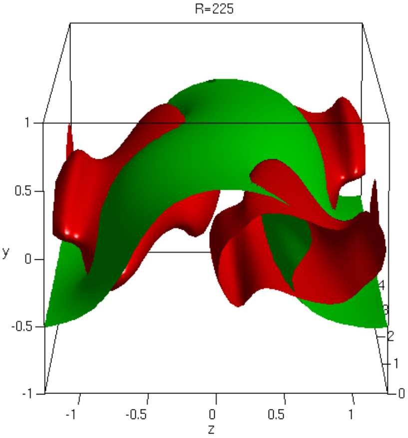




0.6 max(Q), $(\alpha, \gamma) = (1.14, 2.5)$.

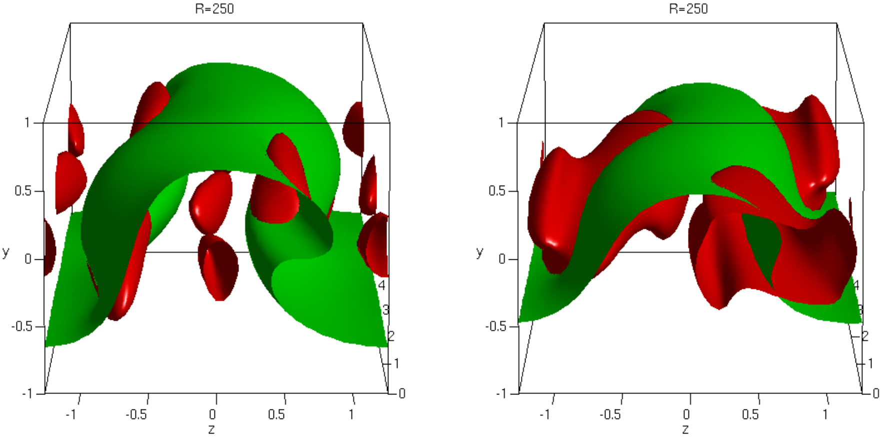
Upper and Lower branches





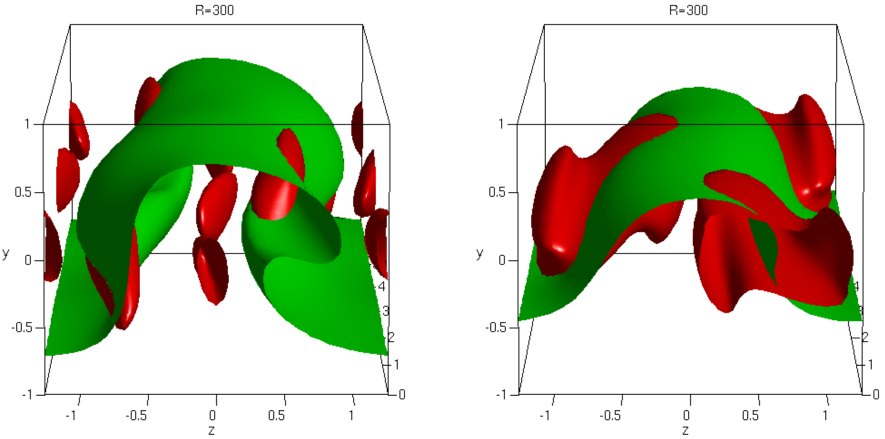
0.6 max(Q), $(\alpha, \gamma) = (1.14, 2.5)$.

Upper and Lower branches



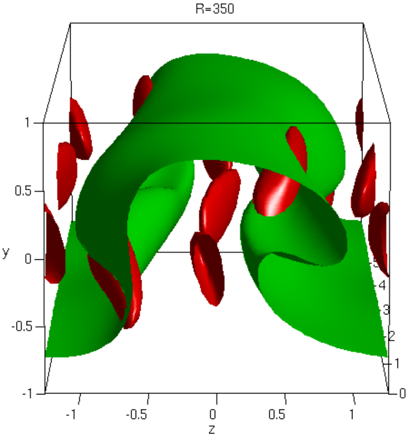
0.6 max(Q), $(\alpha, \gamma) = (1.14, 2.5)$.

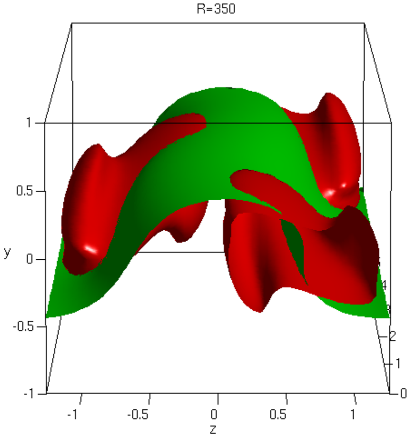
Upper and Lower branches



0.6 max(Q), $(\alpha, \gamma) = (1.14, 2.5)$.

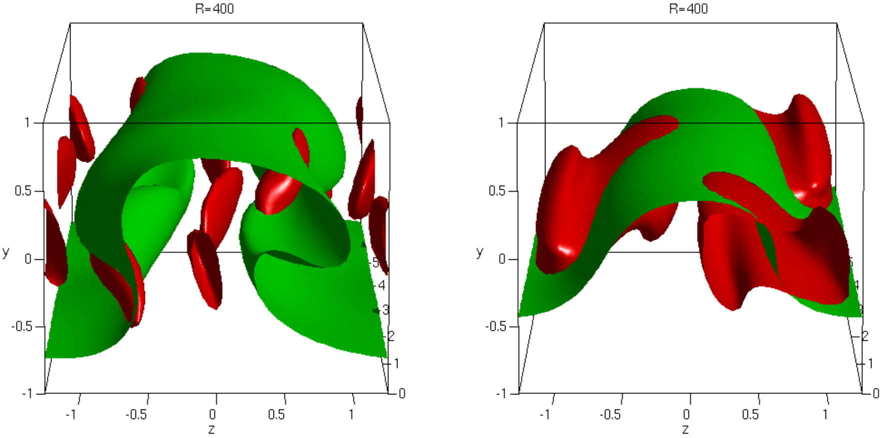
Upper and Lower branches





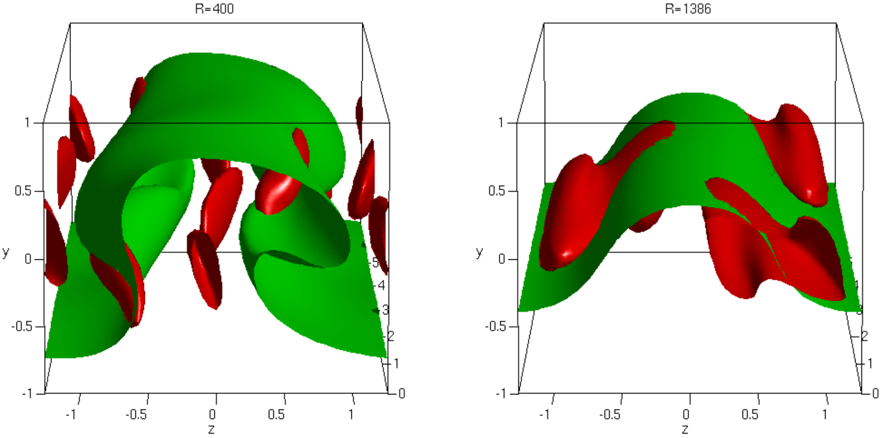
0.6 max(Q), $(\alpha, \gamma) = (1.14, 2.5)$.

Upper and Lower branches



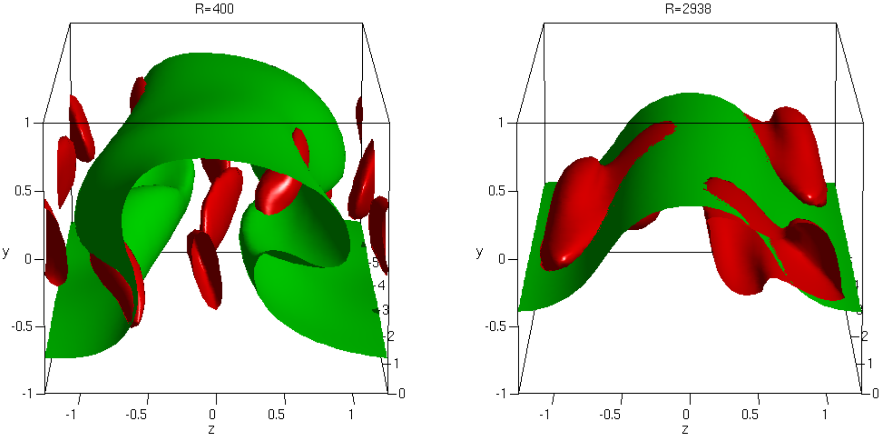
0.6 max(Q), $(\alpha, \gamma) = (1.14, 2.5)$.

Upper and Lower branches



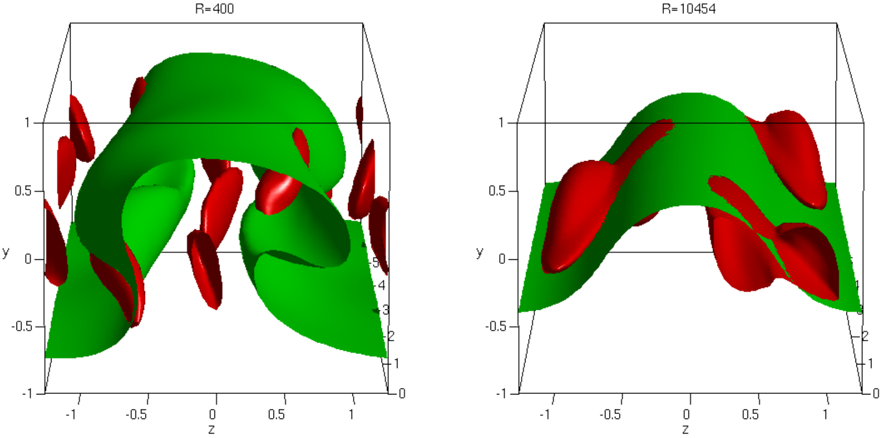
0.6 max(Q), $(\alpha, \gamma) = (1.14, 2.5)$.

Upper and Lower branches



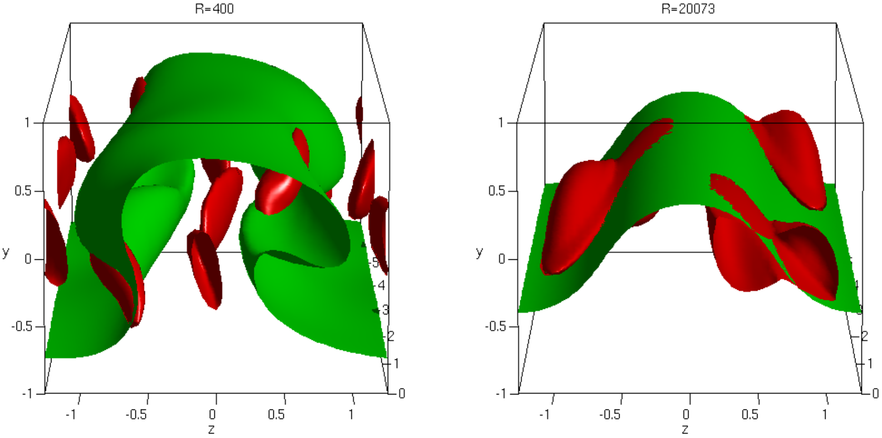
0.6 max(Q), $(\alpha, \gamma) = (1.14, 2.5)$.

Upper and Lower branches



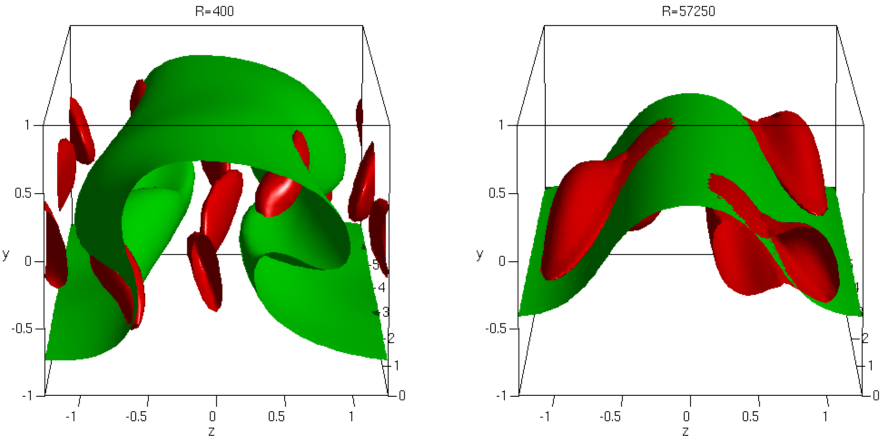
0.6 max(Q), $(\alpha, \gamma) = (1.14, 2.5)$.

Upper and Lower branches



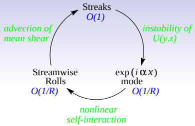
0.6 max(Q), $(\alpha, \gamma) = (1.14, 2.5)$.

Upper and Lower branches

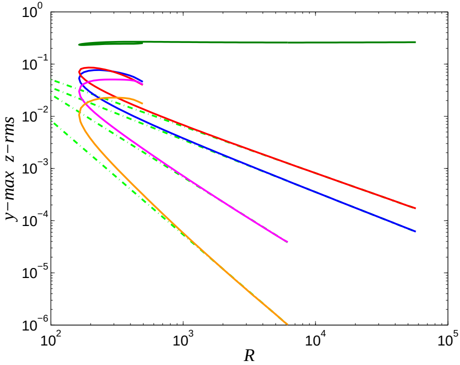


0.6 max(Q), $(\alpha, \gamma) = (1.14, 2.5)$.

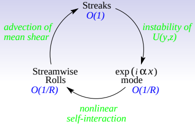
Structure of LBS, $R \to 60\,000$



$$\mathbf{u} - \bar{u}\,\hat{\mathbf{x}} = [u_0 - \bar{u}, v_0, w_0](y,z) \quad + \quad e^{i\alpha x}\,\mathbf{u}_1(y,z) + \cdots$$

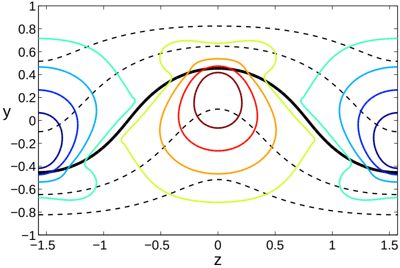


Structure of LBS, $R = 50\,171$

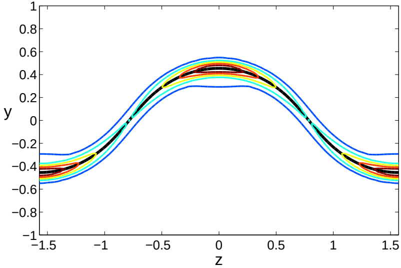


$$\mathbf{u} = [u_0, v_0, w_0](y,z) \qquad + \qquad e^{i\alpha x}\,[u_1, v_1, w_1](y,z) + \cdots$$

$u_0(y,z), v_0(y,z)$

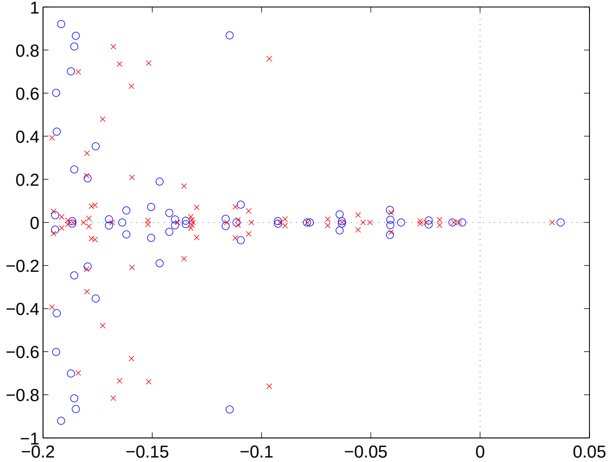


$|w_1|(y,z)$



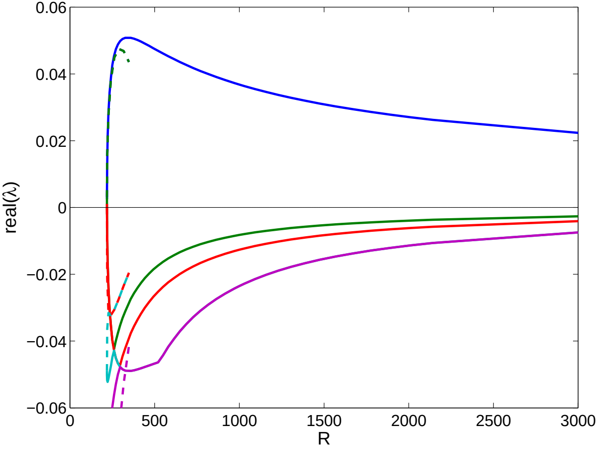
critical layer!

LB eigenvalues, $(\alpha, \gamma) = (1.14, 2.5)$, $(1, 2)$, $R = 1000$

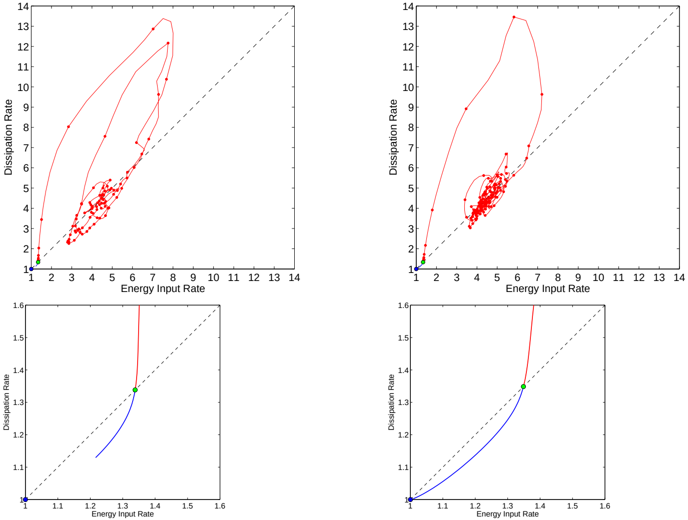


Only 1 unstable eig!

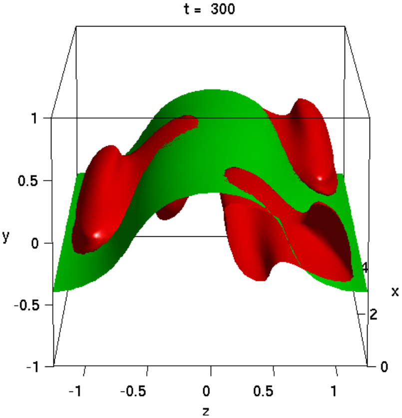
Stability of LBS

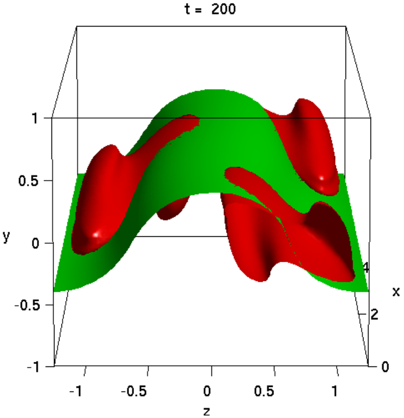


LB $\longleftrightarrow$ Transition $(\alpha, \gamma) = (1.14, 2.5), (1, 2),$ $R = 1000$



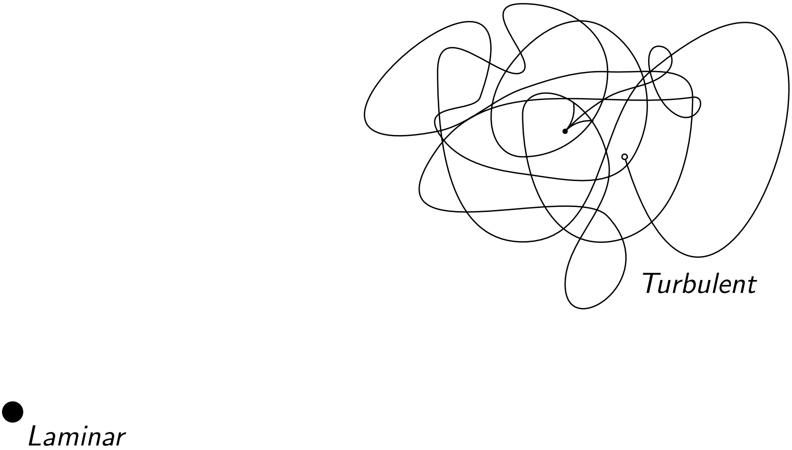
Lower branch $R = 1000$



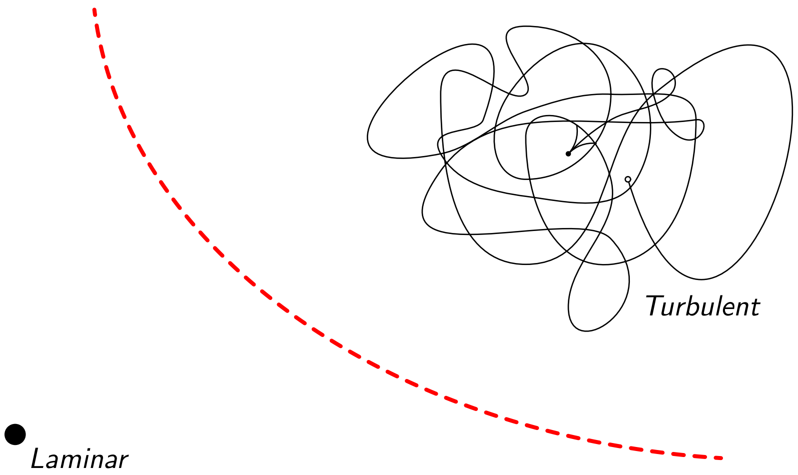


0.6 max(Q), $R = 1000$, $(\alpha, \gamma) = (1.14, 2.5)$.

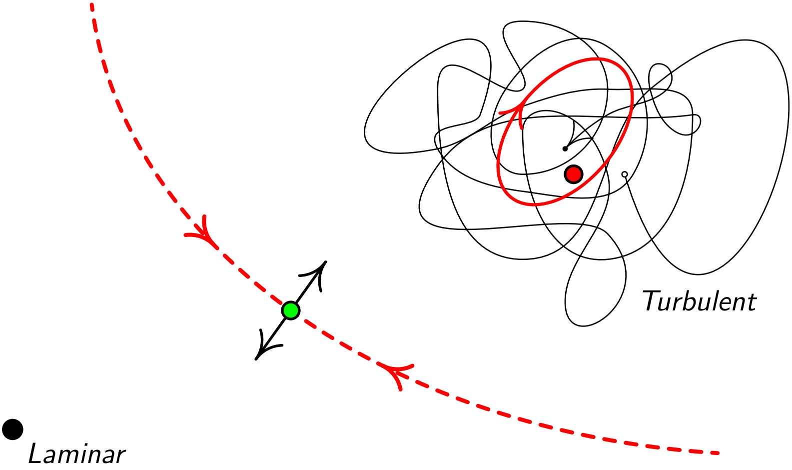
Two states of fluid flow?



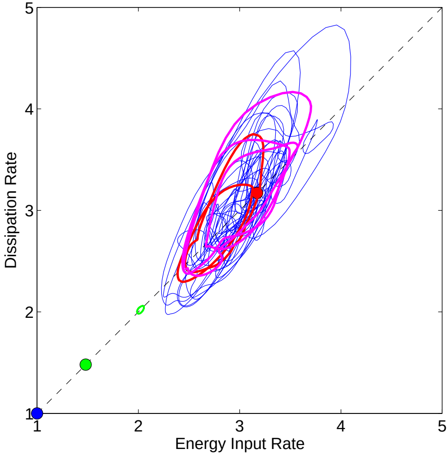
Separatrix, transition threshold



Unstable Coherent States!

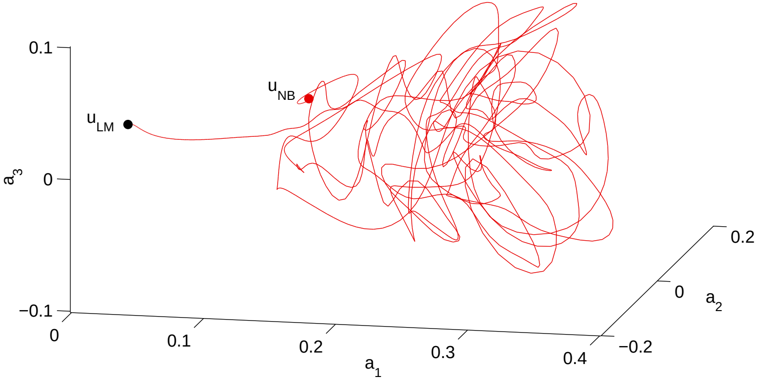


PCF data $(R = 400)$



Periodic solutions in HKW $(1.14, 1.67)$ by *Viswanath, JFM 2007* & *Gibson (TBA)*

Visualizing State Space (10^5 dof's)



RRC, R=400, *Gibson, Halcrow, Cvitanovic*, *JFM to appear* `arxiv.org/0705.3957`

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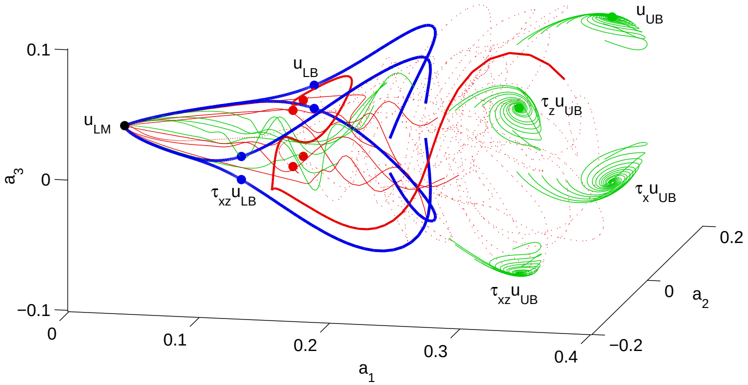

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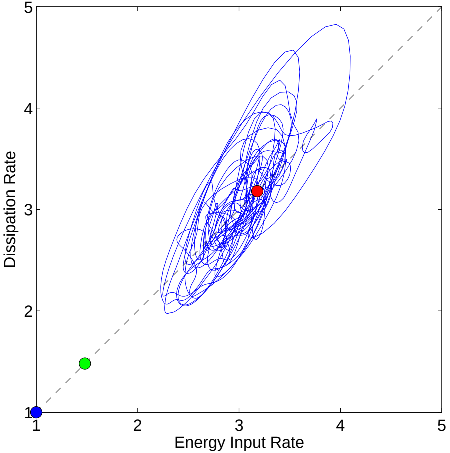
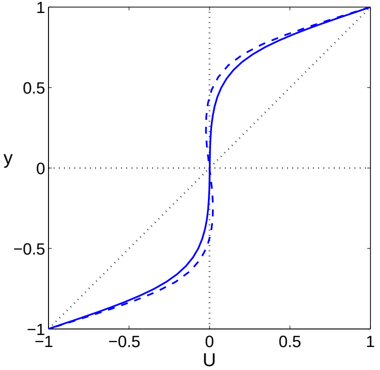
- Lower Branch = Gate to Turbulence!

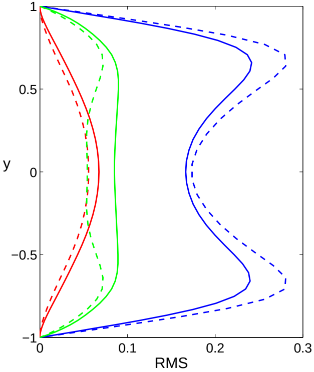
Upper and Lower branches   no-slip Couette



Steady State & 'turbulent' (by Jue Wang & John Gibson) in RRC, $R = 400$, (α, γ)=(0.95,1.67)

Upper branches ⟷ Turbulence (no-slip Couette)





solid: Turbulent (avg t=2000), dash: fixed point

Mean and RMS velocity profiles