Report on the 11th Leslie Fox Prize meeting Cambridge, 20 June 2003

Melvin Leok, Tatjana Stykel, Boris Vexler, Marc Schweitzer, Adam Oberman and Jared Tanner

The eleventh Leslie Fox prize meeting was held on Friday, 20th June, at the Isaac Newton Institute, Centre for Mathematical Sciences, University of Cambridge.

In their first twenty years, Leslie Fox Prize meetings have developed a set, familiar pattern. To remind readers, this biennial prize was established in 1985 to honour Leslie Fox, a pioneer of modern numerical analysis, upon his retirement from the chair of numerical analysis at Oxford University. It is open to young numerical analysts worldwide. Candidates, should be under 31 years of age in the preceding January, to submit essays for the consideration of a committee of three adjudicators. This committee, which in the current round comprised of Arieh Iserles (Cambridge University) Chair, Alastair Spence (Bath University) and Endre Süli (Oxford University), shortlists the candidates. The lucky six are invited to present a 40-minutes long talk at the meeting and, ultimately, the Adjudicators choose first and second prize winners on the basis of the lectures, rewarding mathematical and algorithmic brilliance in tandem with presentational skills.

The six shortlisted candidates demonstrated this year the breadth of modern computational mathematics and scientific computing: from themes in (very) pure mathematics which set the foundations of computation on a firmer basis, to classical, evergreen themes in numerical analysis, to application areas. They have set a marvellous example of the many exciting developments that make numerical analysis into such a vibrant area of mathematical activity.

The order of talks was determined by ballot and the first to walk to the podium was **Adam Oberman** from University of Texas at Austin. In a wide-ranging talk, Adam commenced from a brief survey of viscosity solutions of partial differential equations, a key concept in modern analysis of nonlinear partial differential equations, and used it to justify his interest in ensuring that discretisations share with the exact solution the elusive feature of monotonicity. While it is relatively straightforward to design monotone methods for very simple partial differential equations, the task becomes very difficult for more realistic PDEs. Adam demonstrated that, commencing from these 'simple methods for simple equations', it is possible to design, similarly to building a complicated structure from simple Lego bricks, monotone methods for difficult, nonlinear PDEs that occur in important applications, e.g. in combustion theory, free boundary problems, differential games and financial mathematics.

The focus of the second speaker, **Melvin Leok** from California Institute of Technology, was setting the theoretical foundations for the study of computational mechanics. Commencing from Lagrangian and Hamiltonian formulation, Melvin soon dazzled the audience with themes that are often unfamiliar to numerical analysts: combinatorial topology and differential geometry. Thus, he used exterior calculus to 'hardwire' discrete counterparts of conservation laws into the very nature of the underlying finite-element space, the purpose being to formulate a comprehensive *discrete* counterpart of classical differential geometry. One can only wonder how would Sir Michael Atiyah, the founding director of the Isaac Newton Institute, have reacted to the application of his very own Atiyah connections in numerical analysis.

The third speaker, **Boris Vexler** from University of Heidelberg, devoted his talk to the interface of two subject-areas of great contemporary interest: *a posteriori* error estimates and identification of parameters. The point of departure is PDEs which, unlike in undergraduate courses but as in real life, contain unknown parameters. The idea is to determine parameters from the knowledge of the solution at some discrete values: this corresponds to 'tweaking' the mathematical model in response to observation. Since the procedure necessarily involves numerical solution of PDEs and optimization in tandem, it represents a formidable computational challenge. Standard methods are exceedingly expensive and prone to deliver the wrong result. Boris showed that, disregarding small terms and carefully measuring the numerical error, it is possible to design efficient methods that deliver high-quality solutions for a relatively modest cost.

Navier–Stokes equations are fundamental to the theory of fluid flow and they are at the focus of much theoretical and computational attention. Their elusive, yet ever important,

discretisation motivated the next talk, by **Tatjana Stykel** from University of Calgary. Once Navier–Stokes equations are discretised, they result in a descriptor system: a large set of differential-algebraic equations. Since solving such a system is exceedingly expensive, the standard recourse of reduced model theory is to replace it by a substantially smaller system whilst retaining its main features. This task is complicated by singularity of underlying matrices. Tatjana approached it by a range of algorithmic tools: Hankel singular values and ADI iteration for Lyapunov equations, thereby dramatically reducing the size of the problem without incurring large error.

The next speaker, **Jared Tanner** from University of California at Davis, attacked one of the oldest and wiliest enemies of a numerical analyst, the Gibbs effect. Recovering functions from their spectral data is crucial to a very wide range of applications, because 'nice' functions can be recovered at an exponential speed. Most functions, alas, are not 'nice' and they display a range of phenomena, in particular discontinuities that arrest the speed of convergence and render recovery either ineffective or inordinately expensive. While it is usual to alleviate the Gibbs effect in the spectral space, using filters, Jared approached the problem in a different manner altogether, using mollifiers in the physical space. This approach has many intrinsic advantages and allows paying closer attention to the 'nastier' parts of the function (whose nastiness manifests itself in the physical space!), yet it requires careful use of approximation theory and harmonic analysis techniques.

The sixth and last speaker, **Marc Schweitzer** from University of Bonn, addressed himself to one of the most exciting developments in modern numerical analysis, meshless PDE methods. Such methods avoid the many difficulties associated with designing and adapting a multivariate grid, typically by assigning each discretisation point its own shape function. In several dimensions the solution is thus represented as a linear combination of 'simple' patches. Of course, while avoiding many problems associated with grids, meshless methods confront a numerical analyst with other challenges, in particular in the solution of large, structured sets of algebraic equations. Marc approached this task by a sophisticated medley of techniques: multilevel solvers, smoothers, block Gauss–Seidel iteration and L_2 projections.

As the audience and the speakers were drinking tea and touring the new Centre for Mathematical Sciences, the Adjudicators debated the award of the prizes. Having carefully considered all the contributions, we decided to award the First Fox Prize to **Jared Tanner** and to award the Second Fox prize to all remaining five candidates. The Adjudicators wish to pay tribute to the high level of mathematics and innovative numerics in all the talks, and in particular to the major effort of all six finalists to present the fruits of their research in a lively and highly comprehensible manner.

The formal proceedings concluded with the award ceremony, the participants being presented with the prizes, membership of the IMA and book and journal prizes generously donated by Cambridge University Press, Oxford University Press and Springer Verlag. Informally, the day concluded with a sumptuous dinner at Selwyn College.

The Leslie Fox Prize has become an institution, an opportunity for the numerical community to acknowledge its brightest young members. Scanning the list of past winners reveals a roll call of young researchers that went on to become world authorities in all branches of numerical mathematics. It is the privilege of the UK numerical analysis community to be the custodians of this prize. Yet, privileges carry responsibilities and, as one adjudicator completing his term and free to air his views, I feel obliged to introduce a jarring note to an otherwise-celebratory article. The reader would have noticed that none of the finalists was from the UK. In the previous Leslie Fox prize just one finalist was from a UK university. The reader probably would not have noticed the sparsity of the audience. Unfortunately, it appears that the very community that is formally behind the Leslie Fox Prize is making very little effort to support it, to encourage its brightest students to take part and to attend the meeting.❑

ARIEH ISERLES
DAMTP CAMBRIDGE

Deputy Executive Director appointed

Malcolm Davies

Malcolm Davies is a Chartered Engineer with a professional background in the research and development of telecommunications systems. To complement his lifelong interest in designing products that aid communications between people everywhere, he brings his experience in project management to the IMA team at Catherine Richards House. Malcolm joined the team on the 9th June 2003 and is looking forward to actively supporting the Institute's aims and objectives as well as managing a range of activities that are so important to the ongoing success of any well run organisation.❑