The University of Manchester

Centre for Interdisciplinary Computational
and Dynamical Analysis (CICADA)

CICADA workshop on Hybrid Systems and Model Reduction

19, 20 March 2009

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Introduction

Many systems of relevance to applications are modelled using continuous and discrete state space transitions. This interplay between continuous and discrete time evolutions leads to hybrid systems. The tools from the field of dynamical systems that allow for the analysis and numerical investigations of continuous and discrete time systems, namely singular perturbation methods and centre manifold reduction, allow the qualitative study of system dynamics using low dimensional systems. However, hybrid systems pose a challenge since these reduction methods cannot be directly applied. The goal of the workshop is to look at the recent developments in the area of dynamical systems and model reduction, and identify these tools used in each field, that could be applied for effective investigations of hybrid systems. To this aim we will look at different formalisms used to tackle hybrid systems, the role of singular perturbations in dynamical systems as a model reduction tool, and at the recent developments in the model reduction theory.

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Programme

08.30 - 09.15am Arrivals and Reception

09.15 - 09.30am Opening (**Paul Glendinning** and **Nick Higham**)

09.30 - 10.30am Plenary talk:

John Guckenheimer: *Complex Oscillations: Mixed Modes and Bursting*

10.30 - 11.00am Coffee/tea

11.00 - 11.30am Contributed talk:

Henry Tan: *Simulating catastrophic interface debonding in energetic materials*

11.30 - 12.30pm Plenary talk:

Younes Chahlaoui: *New ideas for model reduction of hybrid switched systems*

12.30 - 02.00pm Lunch, to be served on premises

02.00 - 03.00pm Plenary talk:

Martin Krupa: *Folded saddle-node and the related delay phenomenon*

03.00 - 04.00pm Contributed talks:

Alan Champneys: *The painleve paradox; slip stick with impact as a hybrid system,*
Mike Jeffrey: *A geometrical perspective on discontinuity-induced bifurcations*

04.00 - 04.30pm Coffee/tea

04.30 - 05.00pm Contributed talk:

H.R Shaker: *On model reduction of hybrid systems*

05.00 - 06.00pm Contributed talks:

Constantinos Theodoropoulos: *A Model Reduction Technique for linear Model Predictive Control for Non-linear Large Scale Distributed Systems*

Ioannis Bonis:*Steady-state constrained optimisation for input/output large-scale systems using model reduction technology*

06.00 - 06.30pm Open discussion and closing (**Dave Broomhead**)

07.30pm Conference dinner (at B Lounge @ The Bridge)

Friday morning will be dedicated to informal discussions

Abstracts

Complex Oscillations: Mixed Modes and Bursting

John Guckenheimer

Periodic time series are sometimes observed in which there is additional 'structure' in the form of mixed modes or bursting. Mixed modes occur when the time series consists of compound oscillations with small and large amplitude; in bursting there is an alternation between slowly varying 'steady states' and oscillations. Both types of complex oscillations occur in generic dynamical systems with multiple time scales. This talk will show some examples of these behaviors and survey the theory that has been used to model mixed mode oscillations and bursting.

New ideas for model reduction of hybrid switched systems

Younes Chahlaoui

A reduced model has to fulfill many demands:
On the one hand, the model reduction procedure has to be very efficient, stable and very easy to handle. On the other hand, the behavior of the reduced model should be very similar to the behavior of the original system.
Model reduction of a general linear tine-invariant system is well developed. But as soon as the original system has some special structures or constraints, the model reduction methods become very difficult to develop.
In this talk, we will review some traditional methods and explain how to generalize these methods to hybrid switched systems.

Folded saddle-node and the related delay phenomenon

Martin Krupa

The talk will begin with the definition and motivation of folded saddle-node. It will be argued that in some cases this singularity plays a role in the formation of mixed-mode oscillations and, in particular, influences the firing of some neurons.
Subsequently, the delayed Hopf bifurcation occurring in the unfolding of folded saddle-node will be discussed. The standard cases of delayed Hopf bifurcation, as studied by Neistadt and others, involve the bifurcation sequence ‘stable focus --> unstable focus’. For the case occurring in the unfolding of folded saddle-node one has to deal with the sequence ‘stable node --> stable focus --> unstable focus --> unstable node’.

Simulating Catastrophic Interface Debonding in Energetic Materials

Henry Tan

Energetic materials such as solid propellants consist of fine and coarse particles, and polymeric binder. The total area per unit volume of particle/binder interfaces is very high due to the high particle volume fraction and a large spectrum of particle size distribution. Under non-impact loading, interface debonding significantly affects the macroscopic behaviour. For an energetic material containing large particles and subject to force-controlled loading, catastrophic interface debonding (i.e., sudden debonding under quasi-static loading) will occur when the applied force reaches a critical value that corresponds to the maximum traction at the interfaces.

Mathematically this is due to the cusp catastrophe of the nonlinear static stress-strain curve. Sudden debonding of the particle/binder interfaces is important since it affects the sensitivity to external disturbances during the production, transportation, and storage. Understanding and modelling of catastrophic interface debonding is essential concerning the safety of energetic materials. Previously, the effects of binder and interfaces viscoelasticity on the sudden debonding are not well characterized. A robust simulation is greatly needed due to the difficulties in obtaining interface data.

The challenge here is to simulate the sudden debonding (which needs the ability to account for the multiple time-scales) of particles with large spectrum of size distribution (which needs the ability to account for the multiple resolutions in space).

We are developing a hierarchical, adaptive, Material Point Method to tackle the challenges. The Material Point Method discretizes the material as a collection of particles, and tracts the movements of particles and interactions among them. Our results demonstrate that the method has advantages in handling viscoelastic behaviours, fracture and dynamic processes. The simulation results is compared with analytical solutions and verified experimentally.

The painleve paradox; slip stick with impact as a hybrid system

Alan Champneys

The motion of a rigid body subject to Coulomb friction and newtonian restitution is known to lead to paradoxes where one cannot ascribe unique forward evolution. In this talk, which is joint work with Arne Nordmark and Harry Dankowicz, we show how introducing a complient formulation and taking the limit of infinite stiffness can resolve the paradox in almost all cases. The one case that remains is the possibility of "reverse chatter" which are sequences of impacts that converge in negative time.

A geometrical perspective on discontinuity-induced bifurcations

Mike Jeffrey

Whether hybrid, piecewise-smooth, or slow-fast in nature, the dynamics underlying discontinuity-induced bifurcations has at its heart a simple geometry. Such bifurcations of limit cycles and equilibria at discontinuities result often in violent changes in stability, and although a bifurcation theory can provide a basis for understanding them, it is sometimes unintuitive and highly specific to given modeling conditions. The theory centers around grazing points, where a flow becomes tangent to a discontinuity, poised between smooth and nonsmooth dynamics. A geometrical perspective takes the focus off an ideal `switching-surface' and instead looks at the anisotropy introduced by discontinuities, providing the dynamics underlying incidences of grazing, and a nonsingular alternative to singular perturbation theory. This offers new insights into the way different kinds of discontinuity (hybrid, piecewise-smooth, etc.) are united by common singularities. It also reveals what bifurcations are possible, including novel catastrophic bifurcations and occurrence of nondeterministic chaos to be introduced here.

On model reduction of hybrid systems

H. R. Shaker and R. Wisniewski

The ever-increasing need for accurate mathematical modelling of physical as well as artificial processes for simulation and control leads to models of high complexity. The complexity appears as high order describing dynamical system or complex nonlinear structure. This problem demands efficient computational prototyping tools to replace such complex models by approximate simpler models, which are capable of capturing dynamical behaviour and preserving essential properties of the complex one. Due to this fact model reduction methods have become increasingly popular over the last two decades. On the other hand, most of the methods that are proposed so far for control and analysis of hybrid and switched systems are suffering from high computational burden when dealing with large-scale dynamical systems. Because of the weakness of nonlinear model reduction techniques and due to pressing needs for efficient analysis and control of large-scale dynamical hybrid and switched systems; it is essential to study model reduction of hybrid and switched systems in particular. This fact has motivated the researchers in hybrid systems to study model reduction. To the best of our knowledge just few research works on model reduction of hybrid systems have been reported in the literature [1]-[4]. Therefore this problem is still largely open. We will start the talk with giving a brief walk-trough the previous methods, their advantages and disadvantages concerning computations and conservatism and also from system and control theoretic technical viewpoint. We will proceed with the classification of hybrid and switched systems and we will mainly concentrate on switched systems with autonomous switching and state-based switched systems. The generalized gramian framework for model reduction of switched systems with autonomous switches will be presented then. This framework is based on the generalized gramians instead of gramians [5].

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In order to compute the generalized gramians , one should solve Lyapunov inequalities instead of Lyapunov equations. This method is used to devise a technique for structure preserving model reduction methods in [6].We first show that
the generalized method in [5] can be extended to various gramian based reduction methods. We have modified the original method in [5] to avoid numerical instability and to achieve more numerical efficiency by building Petrov-Galerkin projection based on generalized gramians. A method based on the balanced model reduction within frequency bound will be presented in this framework. It is applied for model reduction of switched system by solving system of Lyapunov inequalities to find common generalized gramian. We show that the proposed framework for model reduction of switched system is stability preserving. The numerical results followed by a brief discussion on feasibility and error bound of the method will be presented. Latter, we will sketch a stability preserving framework which is based on local (generalized) gramians instead of common generalized gramian.

We will move to another category of switched systems for the rest of the talk. One of the most important classes of hybrid systems which has been studied extensively in the literature is a class of piecewise affine systems. It is very general and equivalent to other classes of hybrid systems e. g. mixed logical dynamical systems, linear complementary systems, and maxmin-plus-scaling systems. To our knowledge the only available study in the context of reduction of affine systems in the literature is the work done by Habets and Schuppen [2] which has considered the problem of the exact reduction due to non-observability. Our presented work is a generalization of the method in [2]. The technique which will be presented is based on the transformation of affine dynamical systems inside the cells to a new structure and it can be applied to both exact reduction and also approximate model reduction. The information regarding local input-output behaviour
and also switching information are embedded in this structure. In this framework both controllability and observability of the affine system within the polytopes are considered for reduction purpose. Numerical results show that in general for exact reduction this framework works well but in the case of approximate reduction some other issues should be taken into account such as stability preservation. Although the accuracy of the method within the cell is quite well, depending on the dynamics outside of the cell it might happen that the approximation in the neighbourhood and outside of the cell is not satisfactory. These problems need further investigation and research to be done. We will conclude the talk with the direction for future research and some
ideas for further modifications and generalizations.

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A Model Reduction Technique for linear Model Predictive Control for Non-linear Large Scale Distributed Systems

W. Xie and C. Theodoropoulos

MPC (Model Predictive Control) is widely used in the process industries. Over the last two decades, linear MPC has become a popular and effective advanced control strategy. However, linear models often do not adequately describe the dynamics of the (complex) process to be controlled except near the point at which the model was identified [1]. In general, nonlinear large-scale distributed system models lead to expensive computations, which also restrict the application of MPC in this area. Therefore, nonlinear MPC is mostly used in batch operations, while linear MPC is more often used in continuous operations [2].

Model reduction techniques have been used in conjunction with nonlinear MPC for distributed systems (e.g. [3]). The main purpose of this is to develop a model reduction-based methodology to efficiently apply linear MPC techniques for non-linear distributed-parameter systems. A technique combining the POD (proper orthogonal decomposition) method the finite element method and TPWL (Trajectory piecewise-linear) approximations has been developed. The linearisation of even very high-dimensional systems is effectively reduced to a set of 1-dimensional linearisations with respect to time. This technique can have significant impact in the applicability of mutli-parametric MPC to large-scale systems. The methodology is discussed with the aid of illustrative continuous and discrete illustrative benchmarking examples.

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Steady-state constrained optimisation for input/output large-scale systems using model reduction technology

I. Bonis and C. Theodoropoulos

Keywords: model reduction, projection methods, black-box simulators, gradient-based optimization, reduced Hessian

The optimisation of complex industrial systems is based on models, typically consisting of, or including, sets of Partial Differential Equations (PDEs). These are discretized over a computational mesh, leading to large-scale systems. For the solution of such systems of equations direct or iterative methods can be employed, the latter being more efficient as the size of the system increases. The simulator delineated may either be open source (e.g. home-made) or input/output (e.g. commercial) and can be exploited for optimisation, using either deterministic, or stochastic/meta-heuristic methods. Stochastic methods are more suitable for moderate-sized problems [1], as they typically include a large number of function evaluations, i.e. system simulations. On the other hand, deterministic methods have increased requirements in computing power and memory size and as a result, their application to large-scale systems with constraints is often problematic or even unrealistic. This task is even more formidable in the case of black-box simulators, since the system equations and sensitivity information are generally not available to the user.

We have recently developed a model reduction-based framework for steady-state [2] and dynamic [3] deterministic optimisation using input/output dynamic simulators. In this work we extend this methodology by presenting a novel optimisation algorithm that employs black-box steady-state simulators based on solvers using iterative linear algebra methods. The proposed technique is a reduced Hessian one [4] and includes only low-dimensional Jacobian and reduced Hessian matrices. Those are adaptively computed using a basis which corresponds to the dominant modes of the system, calculated through subspace iterations. The reduced Jacobian matrices are computed through a few numerical perturbations to the direction of the dominant modes of the system, whereas the reduced Hessian matrices are computed according to a 2-step projection scheme, firstly onto the dominant subspace of the system and secondly onto the subspace of the independent variables. The performance of the proposed algorithm, as well as its efficiency in handling large-scale input/output simulators is demonstrated through illustrative realistic applications using the state-of-the-art massively parallel finite element code MPSALSA developed at SANDIA National Laboratories [5]. We discuss the convergence of the proposed framework and its efficiency in handling equality and inequality constraints.

References:

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Conference dinner (at B Lounge @ The Bridge)

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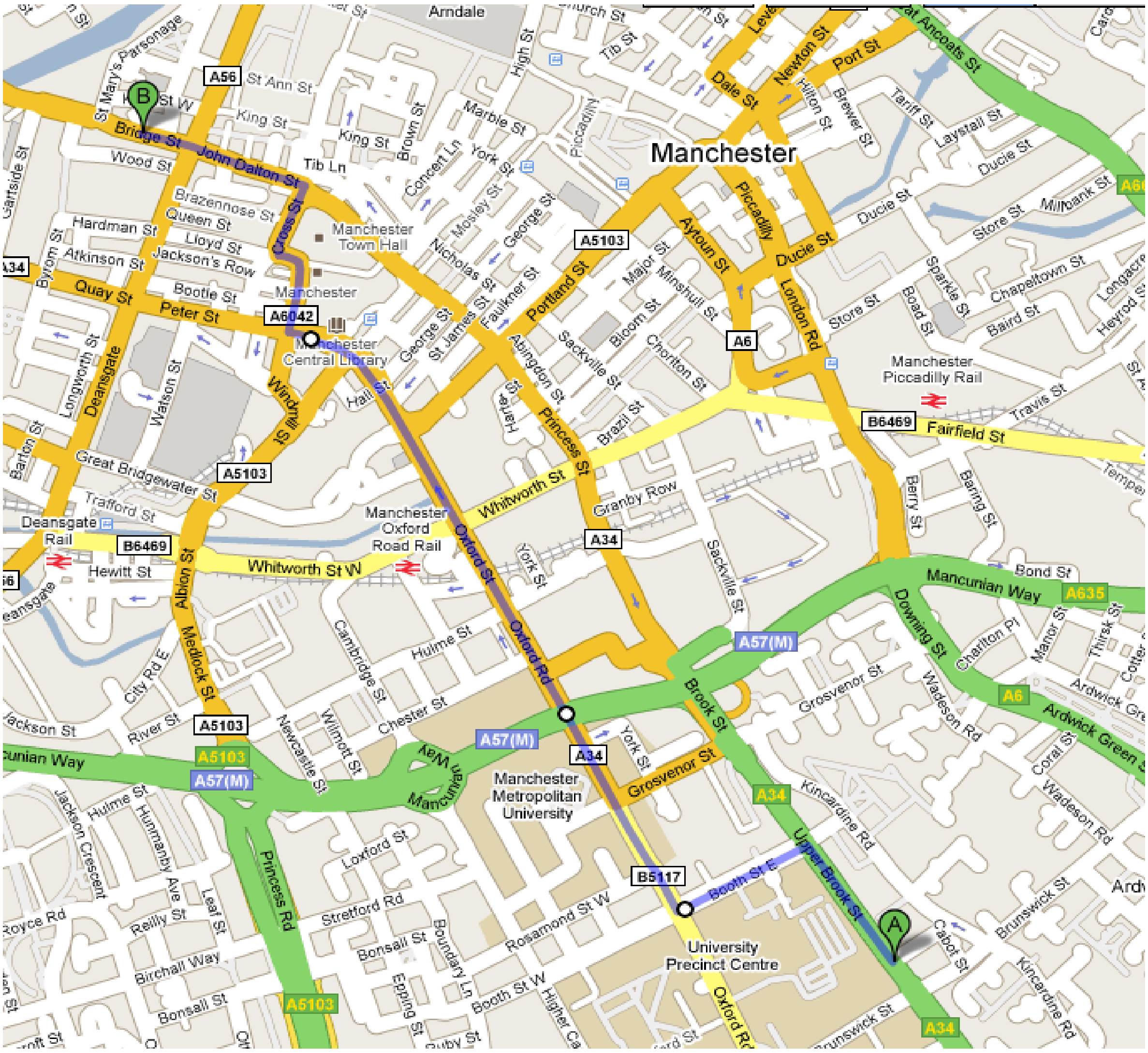
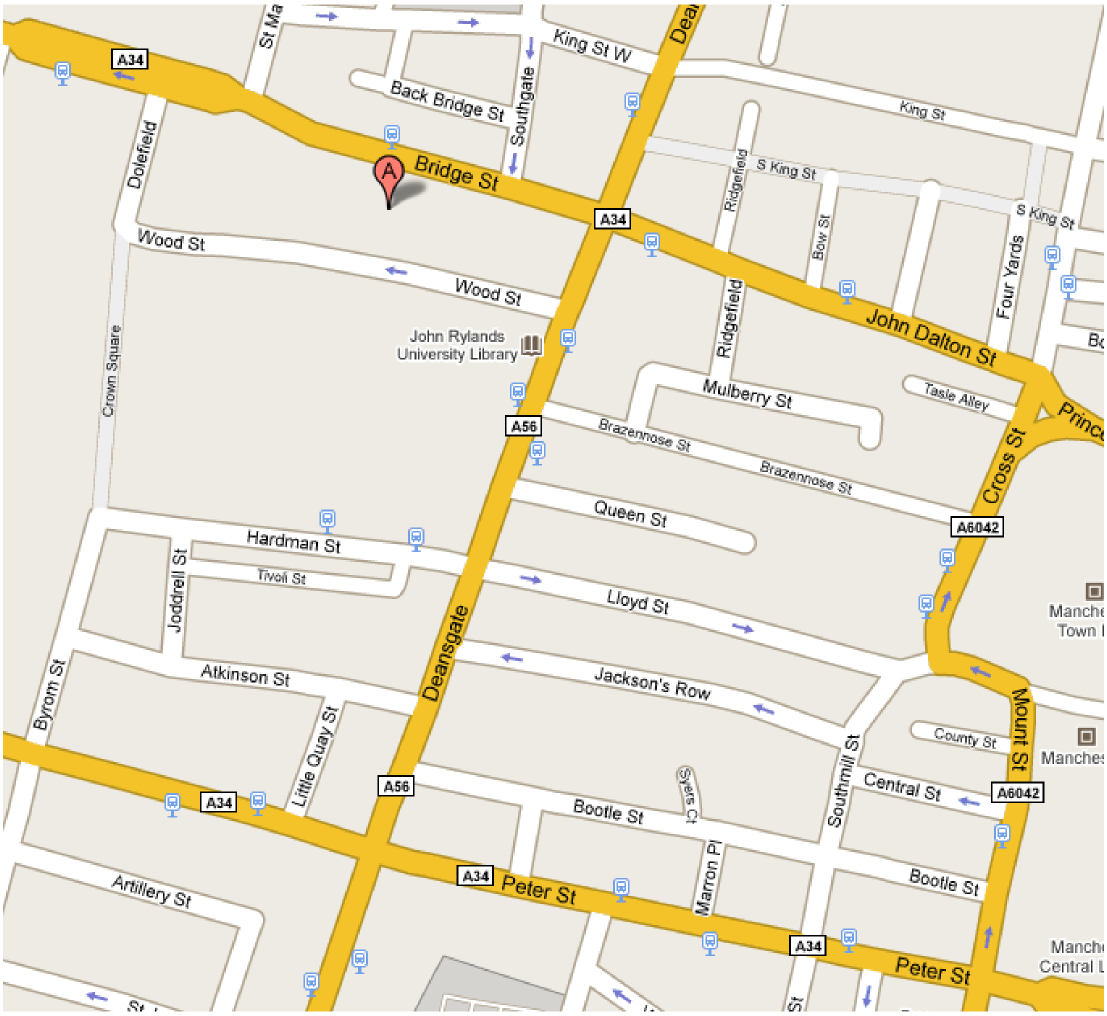


Walking directions: **1.5 mi** – about **30 mins**

1. Head **northwest** on **A34/Upper Brook St toward Booth St E** (0.2 mi)
2. Turn **left** at **Booth St E** (0.2 mi)
3. Turn **right** at **B5117/Oxford Rd.** Continue to follow **Oxford Rd** (400 ft)
4. Continue on **A34/Peter St** (0.7 mi)
5. Turn **right** at **A6042/Mount St** (423 ft)
6. Turn **left** at **A6042/Albert Square.** Continue to follow A6042 (0.1 mi)
7. Turn **left** at **A34/John Dalton St.** Continue to follow A34 (0.2 mi)



A34
Bridge St
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Southgate
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Wood St
Wood St
Crown Square
John Rylands
University Library
A56
Brazennose St
Queen St
Hardman St
Tivoli St
Joddrell St
Lloyd St
Jackson's Row
Atkinson St
Byrom St
Little Quay St
Deansgate
A56
Bootle St
Artillery St
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Syers Ct
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Ridgefield
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