CICADA workshop on Hybrid Systems and Model Reduction
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Model Reduction of Hybrid Systems

H. R. Shaker
Hybrid Systems Research Group
Section for Automation and Control
Institute of Electronic Systems
Aalborg University

Outline:

- Hybrid Systems and Complexity
- Model Reduction: Combinatorial Reduction and Order Reduction
- Hybrid Systems Classification : Model Reduction Viewpoint
- Future Works

Intuitive Introduction to Hybrid Systems

- Discrete program with an analog environment.

What does it mean?

Sequence of discrete steps – in each step the system evolves continuously according to some dynamical law until a transition occurs. Transitions are instantaneous.

A Motivating Example: Thermostat

- The heater can be on or off.
- When the heater is on, the temperature increases continuously according to some formula.
- When the heater is off, the temperature decreases.
- Thermostat keeps the temperature within some limit by putting the heater on or off.

Complexity

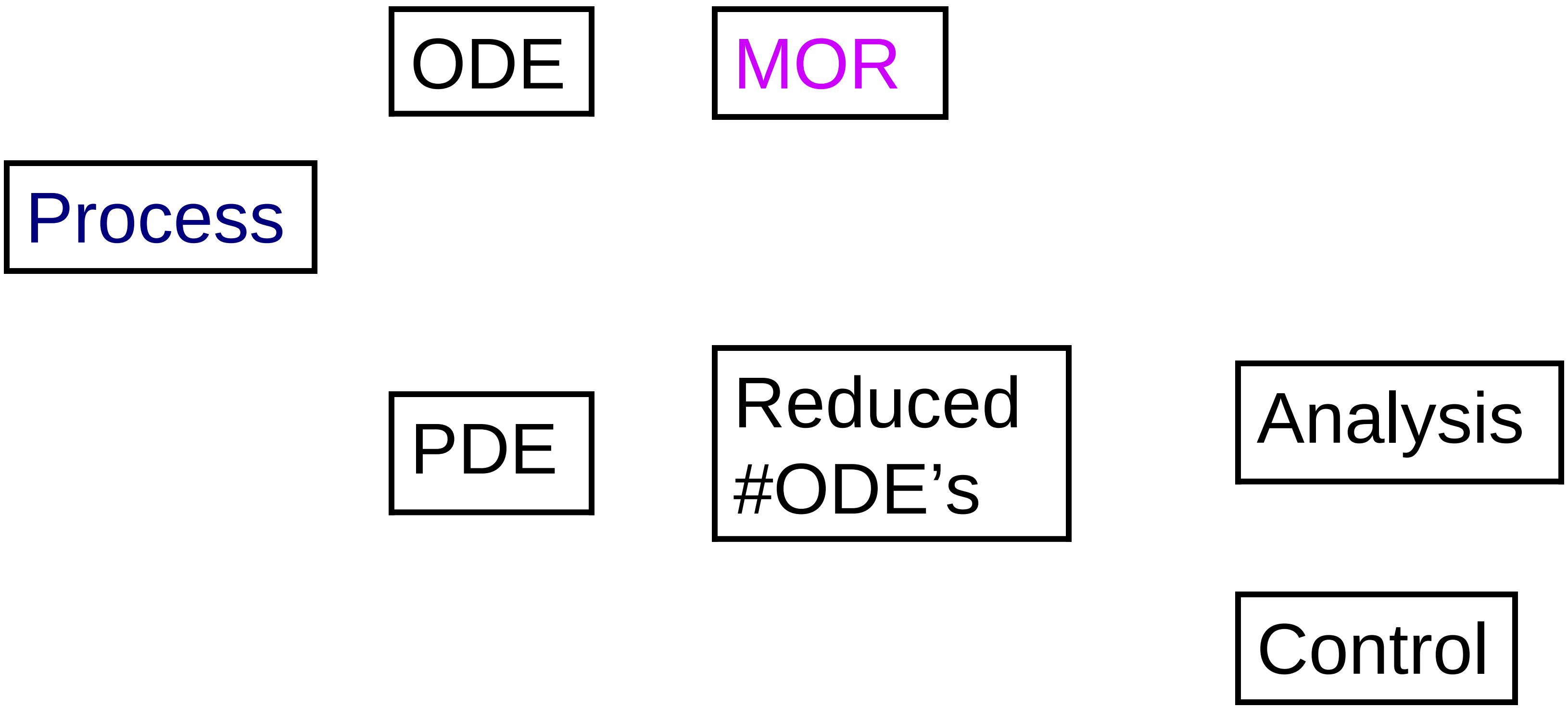
MOR

Complexity, by J. David Sweatt

- Continuous Part
- Combinatorial Part

Model Reduction: strategy for automatic simplification of very complicated systems

Model Order Reduction

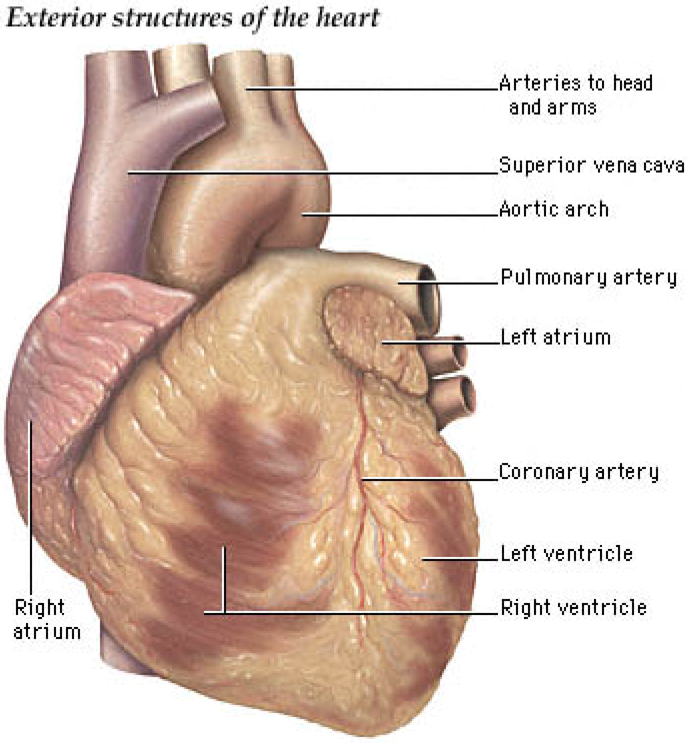


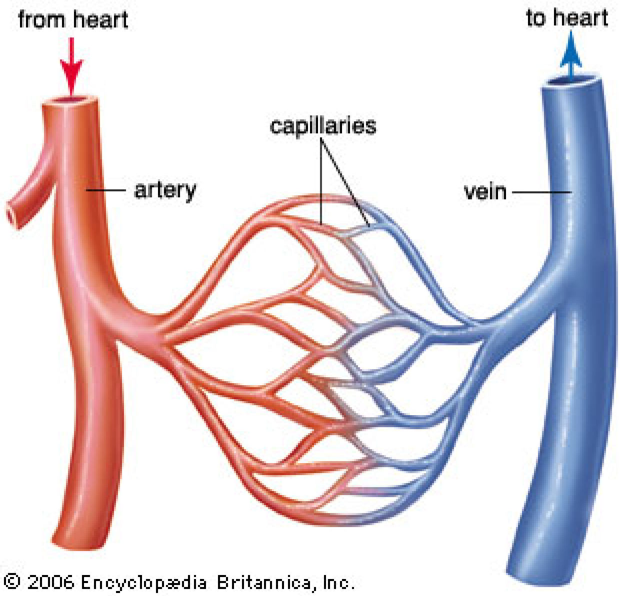
Some reasons :

- A high order system is expensive to simulate and store in computers memory
- Simulation and Analysis is time consuming
- Design procedure and control is complicated

Application Example

Realistic Simulation of Heart





Objective: reducing the order of dynamic systems

Model Reduction Algorithms

Nonlinear MOR

Linear MOR

NOT SO RICH!!

SVD Based

Moment Matching Based

Model Reduction of Hybrid Systems

- Luc C.G.J.M. Habets and Jan H. van Schuppen. Reduction of affine systems on polytopes. *In Proceedings of the International Symposium MTNS* ,2002.
- H. Gao, J. Lam, C. Wang “Model simplification for switched hybrid systems” *Systems & Control Letters*, Vol. 55, No. 12. 2006, pp. 1015-1021.
- E. Mazzi, A. S. Vincentelli, A. Balluchi, and A. Bicchi. “Hybrid system model reduction” *IEEE Int. Conf. on Decision and Control*, 2008.
- Y. Chahlaoui , “Model reduction of switched dynamical systems”, 4th Conference on Trends in Applied mathematics in Tunisia, Algeria and Morocco., Morocco, Kenitra,2009.

Hybrid System Classification

- Autonomous Switched Systems

-switching time/frequency is known.

-switching signal is known

- State-based Switches
- Switched systems with known switching sequence

Model Reduction of Hybrid Systems: Autonomous Switched Systems

- For some classes of Autonomous Switched Systems : Ordinary MOR Techniques.

Reduction in one shot?!

$$\dot{x} = \begin{cases} f_1(x,u) & u < 1 \\ f_2(x,u) & 1 < u < 10 \\ f_3(x,u) & u > 10 \end{cases}$$

Model Reduction of Hybrid Systems: Autonomous Switched Systems

- For some classes of Autonomous Switched Systems : Some MOR Techniques are preferred.

Accuracy Enhancement

One Shot Reduction

$$\dot{x} = \begin{cases} A_1 x + B_1 u & t < 1 \\ A_2 x + B_2 u & 1 < t < 10 \\ A_3 x + B_3 u & t > 10 \end{cases}$$

Model Reduction of Hybrid Systems: Autonomous Switched Systems

- For Autonomous Switched Systems with known switching signal we can treat the signal as a parameter : PVL Model Reduction can be used

It is usually not feasible or very restrictive

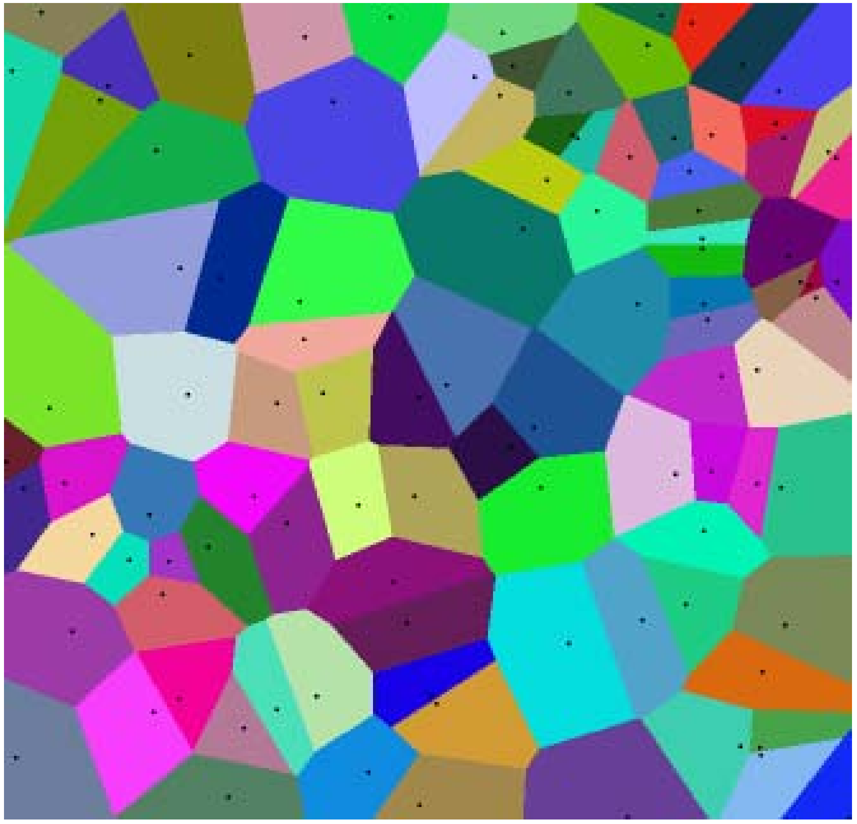
Model Reduction of Hybrid Systems: State based Switches

- Switched Systems with Guard: If the states are in switching inequality.
- Challenge 1: How to Approximate The Guard
- Challenge 2: Which Approximation is suitable

General form:

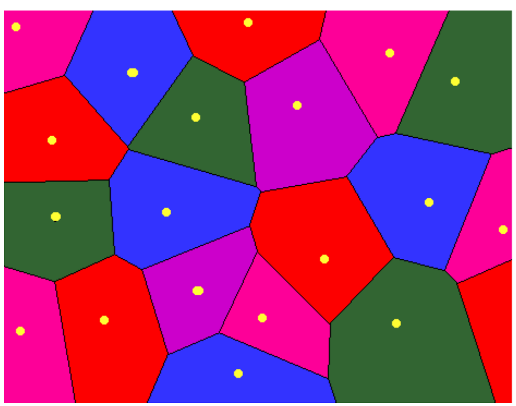
$$\dot{x} = f_i(x,u) : x \in R_i$$

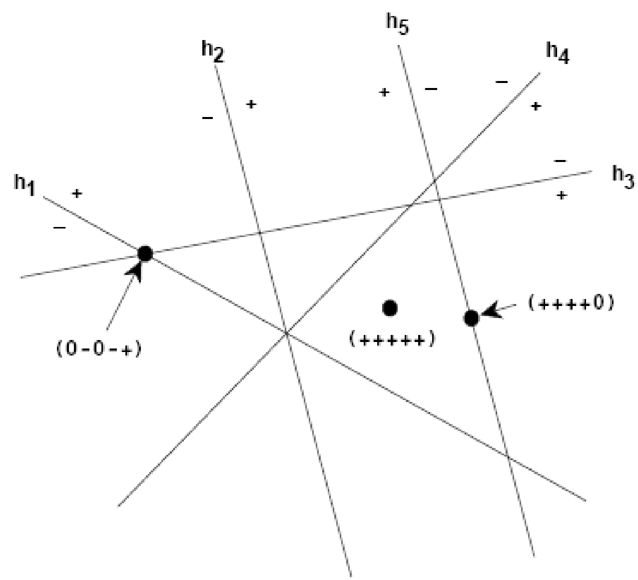
$$1 \le i \le n$$



Model Reduction of PWA systems:

$$S_k : \begin{cases} \dot{x}_k(t) = A_k x_k + B_k u_k + E_k \\ y_k = C_k x_k + D_k u_k \end{cases} \qquad x_k \in P_k, u_k \in U, y_k \in Y \qquad P_k \subset \mathbb{R}^d$$





Model Reduction of PWA systems:

$$P_k \subset \mathbb{R}^d$$

$$S_k : \begin{cases} \dot{x}_k(t) = A_k x_k + B_k u_k + E_k \\ y_k = C_k x_k + D_k u_k \end{cases} \qquad x_k \in P_k,\ u_k \in U,\ y_k \in Y$$

Goal:

$$\tilde{S}_k : \begin{cases} \dot{\tilde{x}}_k(t) = \tilde{A}_k \tilde{x}_k + \tilde{B}_k u_k + \tilde{E}_k \\ \tilde{y}_k = \tilde{C}_k \tilde{x}_k + \tilde{D}_k u_k \end{cases} \qquad \tilde{x}_k \in \tilde{P}_k, \quad d_r < d,\ y_k \in \tilde{Y}$$

AFFINE SYSTEMS to LINEAR SYSTEM:

- new input vector : $W_k := \begin{bmatrix} u_k \\ E_k \end{bmatrix}$

$$S_k : \begin{cases} \dot{x}_k(t) = A_k x_k + \begin{bmatrix} B_k & I \end{bmatrix} W_k \\ y_k = C_k x_k + \begin{bmatrix} D_k & O \end{bmatrix} W_k \end{cases} \qquad x_k \in P_k, \; y_k \in Y$$

Polytopes:

$$\dot{x} = A_i x + B_i u_i \qquad M_i x > d_i$$

$$y_i = C_i x \qquad i = \{1, 2, \ldots, m\}$$

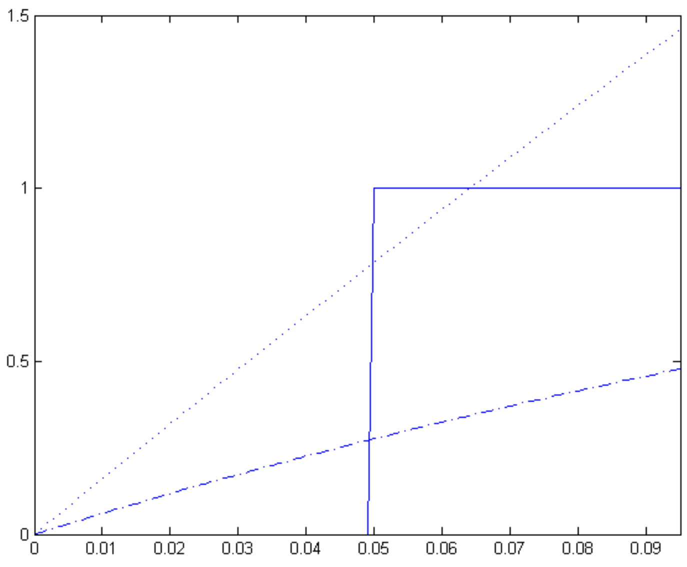
New Output: $Y_i := \begin{bmatrix} y_i \\ M_i x \end{bmatrix} = \begin{bmatrix} C_i \\ M_i \end{bmatrix} x = W_i x$

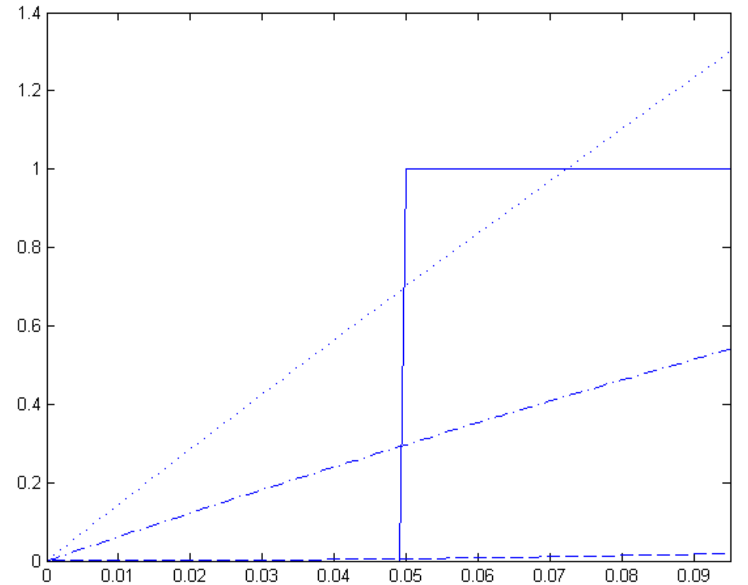
- Now we can use linear techniques for both exact and approximate reduction

example:

$$
\begin{cases}
\dot{x} = \begin{bmatrix} -1.119 & 0.2557 & -0.01542 \\ 0.2557 & -1.892 & 0.1438 \\ -0.01542 & 0.1438 & -1.889 \end{bmatrix} x + \begin{bmatrix} 1.444 \\ 0 \\ 0.6232 \end{bmatrix} u \\
y = \begin{bmatrix} 0.799 & 0.9409 & -0.9921 \end{bmatrix} x
\end{cases}
$$

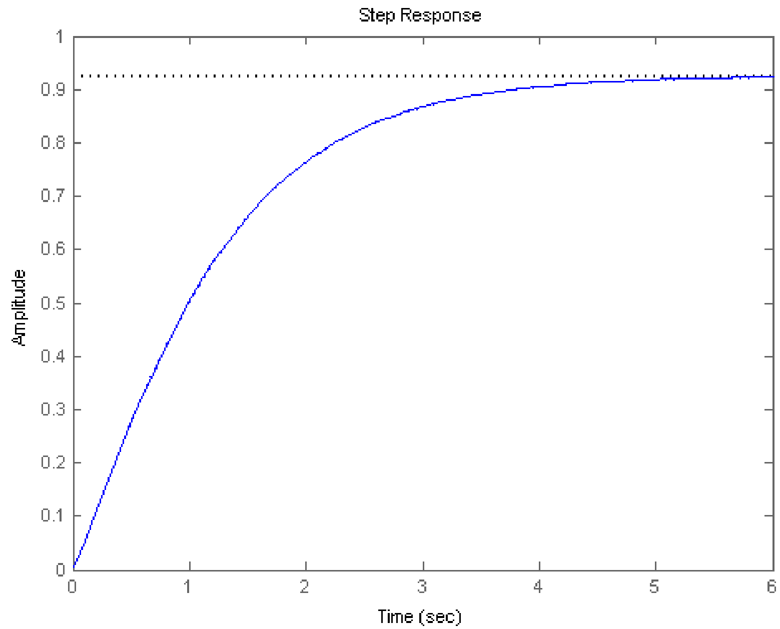
$$
\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1 & -1 \end{bmatrix} x \le \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix}
$$





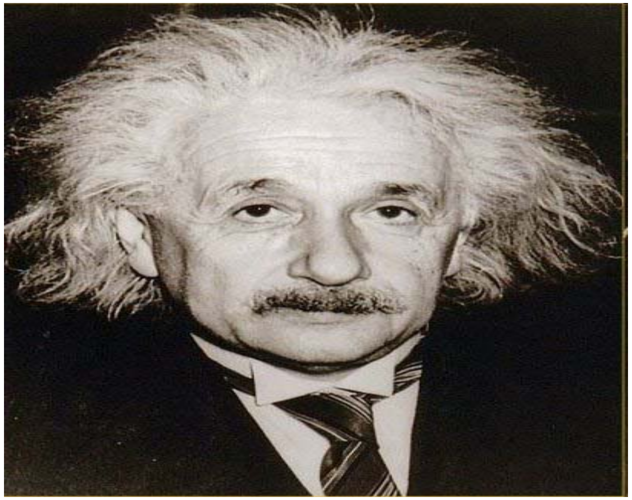
$$\begin{cases} \dot{x} = \begin{bmatrix} -1.034 & 0.01587 \\ -0.7875 & -2.046 \end{bmatrix} x + \begin{bmatrix} 1.614 \\ 0.6135 \end{bmatrix} u \\ y = \begin{bmatrix} 0.5064 & -0.46 \end{bmatrix} x \end{cases}$$

$$\begin{bmatrix} 0.8679 & 0.0655 \\ 0.0892 & -0.2289 \\ 0.2731 & 0.2993 \\ -1.2302 & -0.1359 \end{bmatrix} x \le \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$



Try to simplify as much as possible but not more

Stability



Generalized Gramian Framework for Model Reduction of Switched Systems

$$AP + PA^* + BB^* = 0$$
$$A^*Q + QA + C^*C = 0$$

Gramian

$$AP_g + P_g A^* + BB^* \le 0$$
$$A^*Q_g + Q_g A + C^*C \le 0$$

Generalized Gramian

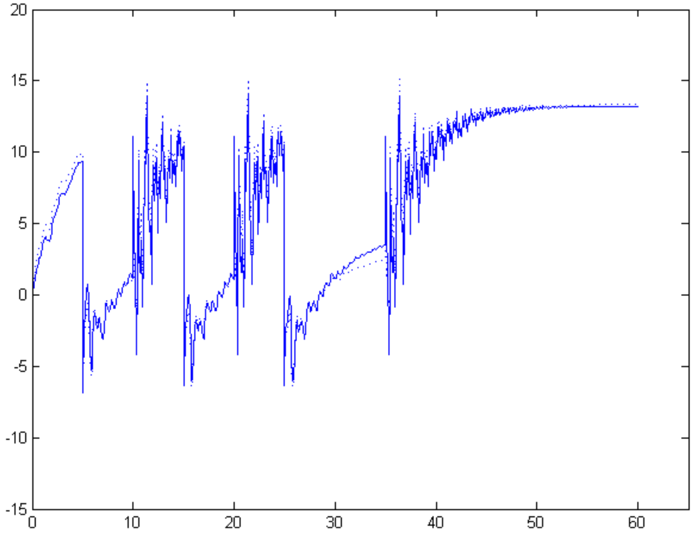
Procedure:

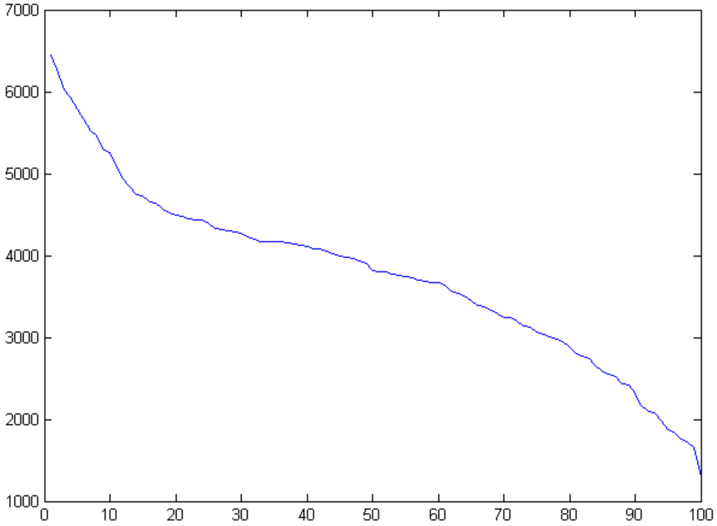
- Derive Lyapunov Inequalities of the linear reduction method
- Compute common grammian
- Build Petrov-Galerkin Projection based on them.

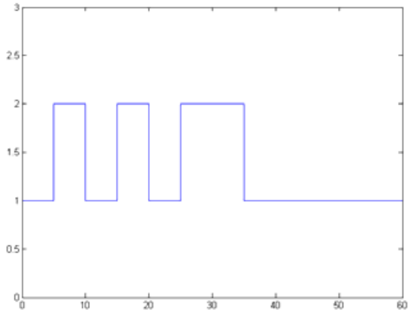
$$\begin{cases} A_\sigma \hat{P}_{cg}(\omega_1,\omega_2) + \hat{P}_{cg}(\omega_1,\omega_2) A_\sigma^{\ *} + \hat{B}_\sigma \hat{B}_\sigma^{\ *} < 0 \\ \forall \sigma \in \mathrm{K} \end{cases}$$

$$\begin{cases} A_\sigma^{\ *} \hat{Q}_{cg}(\omega_1,\omega_2) + \hat{Q}_{cg}(\omega_1,\omega_2) A_\sigma + \hat{C}_\sigma^{\ *} \hat{C}_\sigma < 0 \\ \forall \sigma \in \mathrm{K} \end{cases}$$

Reduction of Bimodal Switched linear System of order 100 to the order 87:







- Framework is a general framework for gramian based reduction methods.
- This is stability preserving reduction technique under arbitrary switches.
- On shot reduction
- It can be extended
- It is not always feasible

Future work:

- Using Lyapunov like function instead of common quadratic lyapunov function for improvement of the feasbility
- Extensions
- Other representaions
- Application of model reduction theory in control of hybrid sytems.

Thank you