► KLAUS AMBOS-SPIES, *On the strongly bounded Turing degrees of simple sets.*
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We study the r-degrees of simple sets under the strongly bounded Turing reducibilities $r = \mathrm{cl}$ (computable Lipschitz reducibility) and $r = \mathrm{ibT}$ (identity bounded Turing reducibility) which are defined in terms of Turing functionals where the use function is bounded by the identity function up to an additive constant and the identity function, respectively. We call a c.e. r-degree $\mathbf{a}$ simple if it contains a simple set and we call $\mathbf{a}$ nonsimple otherwise. As we show, the ibT-degree of a c.e. set A is simple if and only if the cl-degree of A is simple, and there are nonsimple c.e. r-degrees $> \mathbf{0}$.

Moreover, we analyze the distribution of the simple and nonsimple r-degrees in the partial ordering of the c.e. r-degrees. Among the results we obtain are the following. (i) For any c.e. r-degree $\mathbf{a}$, there is a simple r-degree $\mathbf{b}$ above $\mathbf{a}$ and, for any c.e. r-degree $\mathbf{a} > \mathbf{0}$, there is a simple r-degree $\mathbf{b}$ and a nonsimple r-degree $\hat{\mathbf{b}} > \mathbf{0}$ below $\mathbf{a}$. (ii) For any wtt-complete set A, $deg_r(A)$ is simple. So, in particular, there is a c.e. r-degree $\mathbf{a}$ with no nonsimple r-degree above it (which in turn implies that the nonsimple r-degrees are not dense in the p.o. of the c.e. r-degrees). (iii) The simple r-degrees are not dense in the p.o. of the c.e. r-degrees. (iv) For any c.e. set A which is not wtt-complete there is a nonsimple r-degree above $deg_r(A)$. So, in particular, there is a Turing-complete set A such that $deg_r(A)$ is nonsimple. (v) For any c.e. r-degree $\mathbf{a} > \mathbf{0}$ there are simple and nonsimple r-degrees which are incomparable with $\mathbf{a}$. (vi) Any c.e. r-degree is the join of two nonsimple c.e. r-degrees whereas the class of the nonzero c.e. r-degrees is not generated by the simple r-degrees under join.