# CONTRIBUTED TALK ABSTRACTS[1]

▸ M.A. NAIT ABDALLAH, *On an application of Curry-Howard correspondence to quantum mechanics.*
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We present an application of Curry-Howard correspondence to the formalization of physical processes in quantum mechanics. Quantum mechanics' assignment of complex amplitudes to events and its use of superposition rules are puzzling from a logical point of view.

We provide a Curry-Howard isomorphism based logical analysis of the photon interference problem [1], and develop an account in that framework. This analysis uses the logic of partial information [2] and requires, in addition to the resolution of systems of algebraic constraints over sets of $\lambda$-terms, the introduction of a new type of $\lambda$-terms, called *phase $\lambda$-terms.* The numerical interpretation of the $\lambda$-terms thus calculated matches the expected results from a quantum mechanics point of view, illustrating the adequacy of our account, and thus contributing to bridging the gap between constructive logic and quantum mechanics.

The application of this approach to a photon traversing a Mach-Zehnder interferometer [1], which is formalized by context $\Gamma = \{x : s,\ \langle P, \pi\rangle : s \to a^\star \to a,\ Q : s \to b^\star \to b,\ \langle J, \pi\rangle : a \to a',\ \langle J', \pi\rangle : b \to b',\ \langle P', \pi\rangle : b' \to c^\star \to c,\ Q' : b' \to d^\star \to d,\ P'' : a' \to d'^\star \to d',\ Q'' : a' \to c'^\star \to c',\ R : c \vee c' \to C,\ S : d \vee d' \to D,\ \xi_1 : a^\star,\ \xi_2 : b^\star,\ \xi_1' : c^\star,\ \xi_2' : d^\star,\ \xi_1'' : d'^\star,\ \xi_2'' : c'^\star\}$, yields inhabitation claims:

$$R(\mathrm{in}_1(Q''(J(Px\xi_1))\xi_2'')) + R(\mathrm{in}_2(P'(J'(Qx\xi_2))\xi_1')) : C$$

$$S(\mathrm{in}_1(P''(J(Px\xi_1))\xi_1'')) + \langle S(\mathrm{in}_2(Q'(J'(Qx\xi_2))\xi_2')), \pi\rangle : D$$

which are the symbolic counterpart of the probability amplitudes used in the standard quantum mechanics formalization of the interferometer, with constructive (resp. destructive) interference at $C$ (resp. $D$).

[1] P. Grangier, G. Roger, and A. Aspect, *Experimental evidence for a photon anticorrelation effect on a beam splitter: a new light on single-photon interference.*, ***Europhysics Letters***, vol. 1 (1986), pp. 173–179.

[2] M.A. Nait Abdallah, ***The logic of partial information***, EATCS Research Monographs in Theoretical Computer Science, Springer, 1995.

▸ RYOTA AKIYOSHI, *An extension of the $\Omega_{\mu+1}$-Rule.*
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By the author and G.Mints, Buchholz's $\Omega$-rule [3] was extended so that the complete cut-elimination theorem was proved in [2]. That is, any derivation of *arbitrary sequent* can be transformed into its cut-free derivation by the standard rules (with induction replaced by $\omega$-rule). The main idea is to reformulate $\Omega$-rule using Takeuti's distinction of explicit/implicit inference (cf. [5]).

In this talk we extend this approach to full $\Pi^1_1$-CA+$BI$ [1]. Basic ideas are as follows. First, we formulate Buchholz's iterated $\Omega$-rule using Takeuti's distinction of explicit/implicit inference. The formal system introduced is a ramified system based on this distinction. The idea of ramification is due to Buchholz and Schütte [4]. Second, we translate *only* implicit logical inferences into the extended iterated $\Omega$-rules while other explicit rules (especially explicit $\Pi^1_1$-CA+$BI$) are preserved. For example, a rule for implicit second-order universal quantifier is translated into the corresponding "ramified" rule. On the other hand, a rule for explicit second-order universal quantifier is just preserved. Then we have the complete cut-elimination theorem for $\Pi^1_1$-CA+$BI$.

If time is permitting, we explain what kind of reduction steps for $\Pi^1_1$-CA+$BI$ are "recovered" from the cut-elimination steps for the extended iterated $\Omega$-rule.

[1] Ryota Akiyoshi, *The complete cut-elimination theorem for $\Omega_{n+1}$-rule*, preprint, 2011.

[2] Ryota Akiyoshi and Grigori Mints, *Analysis and Extension of Omega-rule*, submitted, 2011.

[3] Wilfried Buchholz, *Explaining the Gentzen-Takeuti reduction steps*, ***Archive for Mathematical Logic***, vol. 40, pp. 255–272, 2001.

[4] Wilfried Buchholz and Kurt Schütte, *Proof Theory of Impredicative Subsystems of Analysis*, Bibliopolis, 1988.

[5] Gaisi Takeuti, *Proof Thory*, 2nd. edition, Springer, 1987.

▸ TOSHIYASU ARAI, *Proof theoretic bounds of set theories.*
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I will explain how to describe bounds on provability in set theories.

[1] Asterisked contributions are by title only.

▶ SERIKZHAN BADAEV, MANAT MUSTAFA, ANDREA SORBI, *A note on computable Friedberg numberings in the Ershov hierarchy.*
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Minimal numberings became a fashionable research topic in the classical theory of numberings at the end of the sixties. One of the main questions on minimal numberings, that is the problem of finding, up to equivalence of numberings, the possible number of minimal numberings, was settled by Yu.L. Ershov. A Friedberg numbering is a special but very important case of minimal numbering. The theory of minimal numberings, and in particular Friedberg numberings, has many successful applications in classical recursion theory, recursive model theory ([2], [5]), and theoretical computer science ([6]).The main powerful methods for constructing families of c.e. sets with a finite number of Friedberg numberings, due to Goncharov [2],use this he show that numbers of spectrum of the nonautoequivalent constructivizations of recursive models is equal to $\{\omega, 0, 1, 2 \dots\}$ ([4]). It was the starting point of some of the most important researches on algorithmic dimension of recursive models. Another application of this results was found by Kummer ([6]).

In [3], S.S. Goncharov showed that there exist classes of recursively enumerable sets admitting up to equivalence exactly one Friedberg numbering which does not induce the least element in the corresponding Rogers semilattice. Later, a simple example of a such a class was found by M. Kummer: This example appears in the paper of S.A. Badaev and S.S. Goncharov ([1]). We generalize this result to all finite levels of the Ershov hierarchy, by showing that for every $n \geq 1$, there exists a $\Sigma^{-1}_n$-computable family of sets, with only one Friedberg numbering up to equivalence, that does not induce the least element in the Rogers semilattice of the family.

[1] S.A. Badaev, S.S. Goncharov *On computable minimal enumerations,*.***In Algebra. Proceedings of the Third Internation-al Conference on Algebra, Dedicated to the Memory of M.I. Kargopolov***. Krasnoyarsk, Au-gust 23-28, 1993.- Walter de Gruyter, Berlin- New York, 1995, pp 21-32

[2] S.S. Goncharov *Computable single-valued numerations.* ***Algebra and Logic,*** 1980, v.19, n.5, pp,325–356.

[3] S.S. Goncharov, *The family with unique univalent but not the smallest enumeration.*, ***Trudy Inst. Matem. SO AN SSSR***, v.8, pp.42-48, Nauka, Novosibirsk, 1988 (Russian).

[4] Goncharov, S. S. *Problem of the number of non-self-equivalent constructivizations.* ***Algebra and Logic,*** v.19, n.6(1980), 401-414.

[5] Yu.L. Ershov,Goncharov, S. S.,*Constructive models,****Transl. from the Russian. (English) (Siberian School of Algebra and Logic) Siberian School of Algebra and Logic.***. New York, NY: Consultants Bureau. xii, 293 p. (2000) 1980.

[6] M. Kummer, *Some applications of computable one-one numberings*, ***Arch. Math. Log.***, v.30, n.4 (1990), 219-230.

▶ YERZHAN BAISSALOV, *On linearly minimal Lie and Jordan algebras.*
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Let $\mathfrak{A} = \langle A; +, \cdot \rangle$ be an infinite algebra over field $\Phi$ and $\mathfrak{T}(A)$ be its multiplication algebra [1].

The algebra $\mathfrak{A}$ is called *linearly minimal*, if each non-zero element $\mathfrak{t}$ of $\mathfrak{T}(A)$ is a surjective linear transformation of $\mathfrak{A}$ (i.e. $\mathfrak{t} : A \to A$ is an onto map) with finite kernel. The property of linear minimality is much weaker than one of definable minimality when we demand each definable (by a formula of first-order logic) set of the algebra to be finite or co-finite.

The notion of linear minimality can be also defined for rings. It is shown in [2] that the classes of linearly minimal rings and algebras coincide, and there are two main possibilities for a (non-trivial) linearly minimal algebra $\mathfrak{A}$: either $\mathfrak{A}$ is field or $\mathfrak{A}$ is a infinite-dimensional central algebra over finite field $\Phi$. Note also that when the linearly minimal algebra $\mathfrak{A}$ has trivial multiplication (i.e. $a \cdot b = 0$ for all $a, b \in A$) the study of the algebra is reduced to the study of the definably minimal Abelian group $\langle A; + \rangle$ which is done, for example, in [3].

**Theorem 1.** *Any non-trivial linearly minimal Lie algebra has an infinite locally finite subalgebra, which is linearly minimal too.*

**Theorem 2.** (1) *Any non-trivial linearly minimal Jordan algebra over field $\Phi$ with $char\Phi \neq 2$ is unital.*

(2) *Any linearly minimal unital Jordan algebra is a division algebra. If it is not associative then any element generates a finite subalgebra.*

**Remark.** When $char\Phi = 2$ the notion of linear minimality can be easily adapted for quadratic Jordan algebras [4].

**Conjecture.** *Any non-associative linearly minimal Jordan algebra has an infinite locally finite subalgebra, which is linearly minimal too.*

The following results (obtained jointly with R. Bibazarov, B. Duzban, A. Syzdykova, B. Tuktybaeva) are the first steps towards the classification of linearly minimal algebras: any non-trivial linearly minimal alternative algebra is a field, and so is any non-trivial linearly minimal Novikov algebra.

I think that the situation is not so easy and simple in the case of linearly minimal Lie or Jordan algebras.

[1] Nathan Jacobson, ***Lie algebras***, Dover books on Mathematics, Dover, 1979.

[2] Yerzhan Baisalov, *On linearly minimal rings and algebras* ***Abstracts of Mal'tsev Meeting 2011***, Novosibirsk, October 11-14, p. 73 (in Russian).

[3] Bruno Poizat, ***Groups stables***, Nur Al-Mantiq Wal-Ma'rifah, Launey, 1987.

[4] Kevin McCrimmon, ***A taste of Jordan algebras***, Universitext, Springer, 2004.

▸ LIBOR BĚHOUNEK, *Infinitesimal calculus over semilinear contraction-free logics.*
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The original notion of infinitesimal, though inconsistent over classical logic, can be formalized as consistent over suitable weaker logics. In particular, I propose a reconstruction of infinitesimals over semilinear contraction-free substructural logics (or logics of linear residuated lattices), which is naturally motivated and turns out to be strong enough for a smooth development of basic notions of classical infinitesimal calculus (including limits, derivatives, etc.). In this formalization, the truth values in linear residuated lattices are regarded as the degrees of infinitesimality, with small real numbers approximating infinitesimals to larger or smaller degrees (though none to the full degree, as there are no true infinitesimals among reals). With a few technical adjustments to this basic idea, the notions of infinitesimal calculus can formally be defined, and their properties derived, in first-order (or more conveniently, Henkin-style higher-order) logic MTL of linear bounded integral commutative residuated lattices. Since it is only the set of infinitesimals that behaves non-classically in this rendering of the infinitesimal calculus, all algebraic properties of infinitesimals are obtained for free (as infinitesimals are ordinary real numbers). The basic theorems of the calculus (such as the uniqueness of limits, the arithmetic of limits and derivatives, etc.) turn out to be easily derivable in Henkin-style higher-order MTL. If time permits, the connection between this formalization of infinitesimal calculus and non-classical topology over semilinear contraction-free logics will also be shown in the talk.

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▸ ANATOLY P. BELTIUKOV *, *Deductive synthesis of polynomial algorithms on finite partially ordered models of second-order logic.*
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A method of constructing formal intuitionistic theories, destined for deductive synthesis of polynomial algorithms, working on finite realizational models of the second-order predicate logic is proposed. Built theories suppose some relation of partial order defined on a model.

Formal theories are built in the form of sequent calculi, focused on inverse search of inference. Special rules are constructed to deal with partial orderings. Cyclic and recursive algorithms are extracted from the applications of these rules.

Estimating computational complexity of algorithms we consider second-order algorithms, that may have at entry also algorithms, but working only with data. In addition, if entrance algorithms are time polynomial then the resulting algorithm is also time polynomial. Degree of this polynom is also limited with a polynom from degrees of the initial polynoms.

The theories can be applied to programming of information systems. Proposed methods of extracting algorithms are suitable for direct construction of programs on such actual programming languages as JavaScript.

This work is continuing the works [1, 2, 3].

[1] A. P. Beltiukov, *Intuitionistic formal theories with realizability in subrecursive classes*, ***Annals of Pure and Applied Logic***, vol. 89 (1997), pp. 3–15.

[2] ——— *A strong induction scheme that leads to polynomially computable realizations*, ***Theoretical Computer Science***, vol. 322 (2004), pp. 17–39.

[3] ——— *A Polinomial Programming Language*, ***Transactions of the Institute for Informatics and Automation Problems of the National Academy of Sciences of Armenia, Mathematical Problems of Computer Science***, vol. 27 (2006), pp. 11–19.

▸ CAROLINA BLASIO AND JOÃO MARCOS,*Logics for discussion, and for agreement.*
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For a given society of reasoning agents, we will entertain situations in which they are consulted upon their opinion about the informational content of logical expressions. In the simplest case, the agents are mere sources of unstructured sentences that may be used to *assert* or to *deny* certain facts. However, invoking a judgmental attitude on the part of the agent, we will assume instead that such sentences represent information that is either *accepted* or *rejected* by the agent. For a source immersed in a classic-like environment, for instance, acceptance may be taken as dual to rejection and these may be reduced to checking satisfiability of an atom by a given assignment.

When collecting and processing the opinions of given agents, one may adopt several different strategies in defining the underlying logic of their society. A *cautious* strategy, for instance, would be one in which a given sentence is accepted by the society when at least one of the involved agents sees reason to accept it. The idea of processing the information received from agents involved in a discussion appears, e.g., in some of the oldest papers on paraconsistent logic: inconsistent opinions should be somehow accommodated when cautiously collected. In the present contribution we shall show that the correct way of dualizing the latter approach in terms of a *bold* collecting strategy would be one in which a given sentence is accepted when none of the involved agents sees reason to reject it. This will allow us to smoothly accommodate undeterminedness phenomena, typical of paracomplete logics. For some interesting illustrations we will concentrate on cases in which agents are classic-like and sentences are structured. As we shall prove, the natural broadly truth-functional semantics behind such approaches have non-deterministic features, yet is computationally well-behaved.

▸ MARIJA BORIČIĆ, *Hypothetical syllogism rule probabilized.*
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Let $A, B$ and $C$ be propositional formulae with the following probabilities of their truthfulness $P(A) = a$, $P(B) = b$ and $P(C) = c$. Then, the probabilistic versions of the hypothetical syllogism inference rule can be given as follows:

$$\frac{P(A \to B) = r \quad P(B \to C) = s}{\max\{r - a, r + s - 1\} \leq P(A \to C) \leq \min\{s + 1 - a, r + c\}}$$

in Hailperin–style, and

$$\frac{P(A \to B) \geq 1 - \varepsilon \quad P(B \to C) \geq 1 - \varepsilon}{P(A \to C) \geq 1 - 2\varepsilon}$$

for each $0 \leq \varepsilon \leq \frac{1}{2}$, in Suppes–style. These rules contain the probabilistic versions of both *modus ponens* (see [1] and [2]), for $a = 1$, and *modus tollens* rule (see [3]), for $c = 0$. We can show that a complete proof–theoretical treatment of probability operators, considered as a part of a polymodal language containing formulae of the form $A^\alpha$, with the intended meaning that $P(A) \in \alpha$, where $\alpha$ is an element of a finite algebra of subsets of $[0, 1]$, can be based on this approach. On the other side, in case when implication $A \to B$ is interpreted as conditional probability $P(B|A)$, although the probabilistic versions of *modus ponens* and *modus tollens* are quite natural (see [1], [2] or [3]), there are arguments that probabilistic versions of the hypothetical syllogism inference rule lose their usual logical sense.

[1] T. Hailperin, *Probability logic*, ***Notre Dame Journal of Formal Logic***, vol. 25 (1984), pp. 198–212.

[2] P. Suppes, *Probabilistic inference and the concept of total evidence*, ***Aspects of Inductive Inference***, (J. Hintikka and P. Suppes, editors), North–Holland, Amsterdam, 1966, pp. 49–55.

[3] C. G. Wagner, *Modus tollens probabilized*, ***British Journal for the Philosophy of Science***, vol. 54(4) (2004), pp. 747-753.

▸ QUENTIN BROUETTE, *A nullstellensatz and a positivstellensatz for ordered differential fields.*
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We consider ordered differential fields endowed with $m$ commuting derivations $\delta_1, \dots,$
$\delta_m$. Their theory has a model completion called $m$-CODF. An axiomatisation of $m$-CODF was given by M. Singer [3] (in case $m = 1$) and later by M. Tressl [4] and C. Rivière [2] (in the general case).

We define the differential real radical of a differential ideal $I$ (denoted by $\mathcal{R}^\omega(I)$ below) and note that it is the smallest differential real ideal containing $I$.

Throughout $K$ is a model of $m$-CODF.

We first obtain the analogue in this context of Dubois' nullstellensatz for real closed fields (see [1]).

Theorem 1 (Nullstellensatz). *Let $I$ be a differential ideal of $K\{X_1, \dots, X_n\}$,*

$$\mathcal{I}(\mathcal{V}(I)) = \mathcal{R}^\omega(I).$$

For any differential polynomial $f$, let $f^*$ be the ordinary polynomial obtained by substituting for each $\delta_1^{e_1} \dots \delta_m^{e_m} X_i$ a new variable $Y_k$.

Let $g_1, \dots, g_s \in K\{X_1, \dots X_n\}$ and $W := \{\bar{x} \in K^n : g_1(\bar{x}) \geq 0, \dots, g_s(\bar{x}) \geq 0\}$ and $W^* := \{\bar{x} \in K^d : g_1^*(\bar{x}) \geq 0, \dots, g_s^*(\bar{x}) \geq 0\}$ and $d$ is such that for all $i = 1, \dots, s$, $g_i \in K[Y_1, \dots, Y_d]$.

Using a result of density of differential tuples, we obtain a differential version of Stengle's positivstellensatz (see [1]).

Theorem 2 (Positivstellensatz). *Let $f \in K\{X_1, \ldots, X_n\}$ and $P$ be the cone of $K\{X_1, \ldots, X_n\}$ generated by $g_1, \ldots, g_s$.*

*Suppose that there exists an open set $O$ such that $O \subset W^* \subset \mathrm{cl}(O)$.*

$$\forall \bar{x} \in W, f(\bar{x}) \geq 0 \Leftrightarrow \exists m \in \mathbb{N}, g, h \in P, \forall \bar{x} \in W : f(\bar{x}).g(\bar{x}) = f^{2m}(\bar{x}) + h(\bar{x}).$$

[1] J. Bochnak, M. Coste, M.-F. Roy, ***Géométrie algébrique réelle***, Springer-Verlag, 1987.

[2] C. Rivière, *The theory of closed ordered differential fields with m commuting derivations*, ***Comptes rendus de l'académie des sciences Paris***, ser. I 343 (2006), pp. 151–154.

[3] M. Singer, *The model theory of ordered differential fields*, ***Journal of Symbolic Logic***, vol. 43 (1978), no. 1, pp. 82–91.

[4] M. Tressl, *The uniform compagnion for large differential fields of characteristic zero*, ***Transactions of the american mathematical society***, vol. 357 (2005), pp. 3933–3951.

▸ MAURICE CHIODO, *The computational complexity of recognising embeddings, and a universal finitely presented torsion-free group* *.
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We extend a result by Lempp [1] that recognising torsion-freeness for finitely presented groups is $\Pi_2^0$-complete; we show that the problem of recognising embeddings of finitely presented groups is at least $\Pi_2^0$-hard, $\Sigma_2^0$-hard, and lies in $\Sigma_3^0$. We conjecture that this problem is indeed $\Sigma_3^0$-complete. We give a uniform construction that, on input of a recursive presentation of a group $P$, outputs a recursive presentation $P^{\text{tor-free}}$ of a torsion-free group, which is isomorphic to $P$ whenever $P$ is itself torsion-free. We apply our constructions to form a universal finitely presented torsion-free group.

[1] S. Lempp, *The computational complexity of recognising torsion-freeness of finitely presented groups*, ***Bulletin of the Australian Mathematical Society***, vol. 56 (1997), pp. 273–277.

▸ JOHN CORCORAN, *Tarski's extensional functions* *.
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"Extensional functions" appear twice in [1]—in different senses. In his 1923 dissertation [1, pp. 1–23], "functions" are interpretable as type-theoretic mathematical "mappings": not linguistic expressions [1, pp. 5, 23]. In Tarski's 1935 truth-definition paper [1, pp. 152–278], "functions" are expressions containing free variables. Below *mapping* and *function* have contrasting senses: *mapping* always has the mathematical sense, *function* always the linguistic sense. Mappings never contain variables, functions always contain variables.

Arithmetic languages have the three-character absolute-value *function* '$|x|$' associated with the absolute-value mapping carrying numbers to their absolute values. Set-theoretic languages have the three-character singleton function '$\{x\}$' associated with the singleton mapping carrying sets to their singletons. Functions, which are not names, convert into names by replacing variables with constants: '$|-2|$' names two; '$\{\omega\}$' names $\omega$'s singleton.

The respective metalanguages have the three-character *quotation function* ' '$x$' ' convertible to expression names: ' 'a' ' *names* the first letter—the one-character expression 'a'. As Tarski knew, the *expression* ' '$x$' ' is ambiguous: in one sense it is a *function*; in another sense it is a name of the 24$^{\text{th}}$ letter, ecks. The three-character expression ' '$x$' ' is named with a *five-character* expression ' ' '$x$' ' '—applying two-character single-quotation twice.

The singleton function '$\{x\}$' and the absolute-value function '$|x|$' are both *extensional* in Tarski's implicit sense [1, p. 161]—substituting coextensive names for the variable produces coextensive names: $|2| = |1+1|$ and $\{\omega\} = \{\omega - 1\}$. However, as Tarski implied, the quotation function ' '$x$' ' is nonextensional: '2' $\neq$ '$1+1$' and '$\omega$' $\neq$ '$\omega - 1$'—no one-character expression is a three-character expression.

[1] Alfred Tarski, ***Logic, semantics, metamathematics***, Hackett, 1983.

▸ JOHN CORCORAN AND HASSAN MASOUD, *Existential-import sentence schemas: classical and relativized* *.
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The variable-enhanced-English sentence schema 'for every integer $x$ $P(x)$' translates the first-order schema '$\forall x\, \mathrm{P}(x)$' interpreted in the integers: 'for every integer $x$' translates '$\forall x$' applied to integers.

The *Classical Existential-Import Schema*, *CEIS*, has as instances every conditional whose antecedent is a universal sentence and whose consequent is *the* corresponding existential—replacing the initial universal quantifier by the existential.

If for *every* integer $x$ $\mathrm{P}(x)$, then for *some* integer $x$ $\mathrm{P}(x)$.

$$[\forall x\, \mathrm{P}(x) \rightarrow \exists x\, \mathrm{P}(x)]$$

Obviously every CEIS instance is tautological [logically true].

The *Relativized Existential-Import Schema*, *REIS*, has as instances every conditional whose antecedent is a universalized conditional and whose consequent is *the* corresponding existentialized conjunction—replacing the initial universal quantifier by the existential *and* replacing the conditional connective by conjunction.

If for every integer $x$ [if A($x$), *then* C($x$)], then for *some* integer $x$ [A($x$) *and* C($x$)].

$$[\forall x\,(\mathrm{A}(x) \to \mathrm{C}(x)) \to \exists x\,(\mathrm{A}(x)\ \ \&\ \ \mathrm{C}(x))]$$

Non-tautological REIS instances are familiar. But, contrary to textbook impressions, *certain* instances of REIS *are* tautological. A necessary and sufficient condition for REIS instances to be tautological follows.

THEOREM. *A REIS instance is tautological iff the existentialization* $\exists x\,\mathrm{A}(x)$ *of the antecedent condition* $\mathrm{A}(x)$ *is tautological.*

$$[\forall x\,(\mathrm{A}(x) \to \mathrm{C}(x)) \to \exists x\,(\mathrm{A}(x)\ \ \&\ \ \mathrm{C}(x))] \text{ is tautological}$$
$$\text{if and only if}$$
$$\exists x\,\mathrm{A}(x) \text{ is tautological.}$$

*"If" is obvious. The four key ideas in our "only-if" proof are:*

(1) $\exists x\,\mathrm{A}(x)$ *is tautological if* $\sim\exists x\,\mathrm{A}(x)$ *implies* $\exists x\,\mathrm{A}(x)$.
(2) $\sim\exists x\,\mathrm{A}(x)$ *implies* $\forall x\,(\mathrm{A}(x) \to \mathrm{C}(x))$.
(3) $\forall x\,(\mathrm{A}(x) \to \mathrm{C}(x))$ *implies* $\exists x\,(\mathrm{A}(x)\ \ \&\ \ \mathrm{C}(x))$, *by the hypothesis.*
(4) $\exists x\,(\mathrm{A}(x)\ \ \&\ \ \mathrm{C}(x))$ *implies* $\exists x\,\mathrm{A}(x)$.

This lecture complements this BULLETIN vol. 11 (2005), p. 460; vol. 11 (2005), pp. 554-555; vol. 12 (2006) pp. 219–240 and vol. 13 (2007) pp. 143–144; and vol. 17 (2011), pp. 324–325.

▸ JOHN CORCORAN AND JOAQUIN MILLER, *Meanings of show* *.
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We study uses of *show* in logic: literal, metaphorical, elliptical, etc. Some are confused—through not distinguishing literal from figurative uses.

A teacher *shows* students that not every integer is either positive or negative.
An easy proof *shows* that not every integer is either positive or negative.
Euclid's construction *shows* how to construct equilateral triangles.

Zero *shows* that not every integer is either positive or negative.

Zero is a counterexample for "every integer is either positive or negative". A counterexample for a given proposition *shows* the proposition false. Thus, zero *shows* "every integer is either positive or negative" to be false. Compare [1].

The fact that zero is neither positive nor negative *shows* that not every integer is either positive or negative.

The theorem that zero is an integer which is neither positive nor negative *shows* that not every integer is either positive or negative.

The proposition "zero is zero" does not say that it—the proposition "zero is zero"—is tautological; however, according to some, it does *show* that it is. In fact, some logicians say *every tautology itself shows that it is a tautology* ([2], 6.127).

The grammatical categories of the verb *show* are diverse. It occurs as a two-place verb completed by both subject and direct object, but it also occurs as a three-place verb requiring for its completion an indirect object as well. Moreover, sometimes it requires a human subject; it is an action verb—like *teach* and *infer*. Other times it requires an inert non-human subject; it is a timeless relation verb—like *equal* and *imply*.

[1] JOHN CORCORAN, *Counterexamples and proexamples*, this BULLETIN, vol. 11 (2005), p. 460.
[2] LUDWIG WITTGENSTEIN, ***Tractatus Logico-Philosophicus***, Kegan Paul, London, 1921.

▸ JOHN CORCORAN AND SRIRAM NAMBIAR, *Conversely: extrapropositional and prosentential* *.
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This self-contained lecture examines uses and misuses of the adverb *conversely* with special attention to logic and logic-related fields. Sometimes adding *conversely* after a conjunction such as *and* signals redundantly that a converse of what preceded will follow.

(1) Tarski read Church and, conversely, Church read Tarski.

In such cases, *conversely* serves as an *extrapropositional* constituent of the sentence in which it occurs: deleting *conversely* doesn't change the proposition expressed. Nevertheless it does introduce new implicatures:

a speaker would implicate belief that the second sentence expresses a converse of what the first expresses.

Perhaps because such usage is familiar, the word *conversely* can be used as "sentential pronoun"—or *prosentence*—representing a sentence expressing a converse of what the preceding sentence expresses.

(2) Tarski read Church and conversely.

This would be understood as expressing the proposition expressed by 1.

*Prosentential* usage introduces ambiguity when the initial proposition has more than one converse. Confusion can occur if the initial proposition has non-equivalent converses.

Every proposition that is the negation of a false proposition is true and conversely.

One sense implies that every proposition that is the negation of a true proposition is false, which is true of course. But another sense, probably more likely, implies that every proposition that is true is the negation of a false proposition, which is false: the proposition that one precedes two is not a negation and thus is not the negation of a false proposition.

The above also applies to synonyms of *conversely* such as *vice versa*. Although *prosentence* has no synonym, extrapropositional constituents are sometimes called *redundant rhetoric*, *filler*, or *expletive*.

Authors discussed include Aristotle, Boole, De Morgan, Peirce, Frege, Russell, Tarski, and Church.

▸ JOHN CORCORAN AND DANIEL NOVOTNÝ *, *Formalizing Euclid's first axiom*.
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Euclid's ***Elements*** divides its ten premises into two groups of five.

The first five (*postulates*)—applying in geometry but nowhere else—are *specifically* geometrical. The first: "to draw a line from any point to any point"; the last: the parallel postulate.

The second five (*axioms*) apply in geometry *and* elsewhere. They are non-logical principles governing *magnitude types* both geometrical (e.g., lengths, areas) and non-geometrical (e.g., durations, weights). Euclid called axioms *koinai ennoia*: *koinai* ("shared", "communal", etc.), ennoia ("designs", "thoughts", etc.). The first axiom is:

*Ta toi autoi isa kai allelois estin isa.*

Things that equal the same thing equal one another.

One first-order translation in variable-enhanced English (cf. [2, p. 121] is:

(1) Given two things $x$, $y$, *if* for something $z$, $x$ and $y$ equal $z$,
*then* $x$ equals $y$.

Translation (1) overlooks Euclid's plural construction not limited to two. Second-order translations avoid that objection.

(2) For any set $S$, *if* for something $z$, everything $x$ in $S$ equals $z$,
*then* anything $x$ in $S$ equals anything $y$ in $S$.

Translations (1) and (2) are "too broad": they cover all magnitude types but by amalgamating them into a hodgepodge universe containing all magnitude types—a universe violating category restrictions and not itself a magnitude type.

Translation (3) is a *second-order axiom schema* (cf. [1]) having one instance for each magnitude type. 'MAG' is placeholder for magnitude words such as *length*, *area*, etc.

(3) For any set $S$, *if* for some MAG $z$, every MAG $x$ in $S$ equals $z$,
*then* any MAG $x$ in $S$ equals any MAG $y$ in $S$.

We treat several other translations and formalizations.

[1] John Corcoran, *Schemata*, ***The Bulletin of Symbolic Logic***, vol. 12 (2006), pp. 219–240.
[2] Alfred Tarski, ***Introduction to logic***, Dover, New York, 1995.

▸ JOHN CORCORAN AND JOSÉ MIGUEL SAGÜILLO, *Euclid's weak first axiom* *.
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"Things that equal the same thing equal one another" [TES] is Euclid's first axiom in ***Elements***—written about 300 BCE, some 50 years after Aristotle's ***Analytics***. There is no trace of TES before Euclid, but afterward it was often repeated verbatim or nearly verbatim. For example, over 400 years later, Galen stated it at least three times [1, pp. 430–442]; over 700 years later Proclus stated it several times. It still influences logic. After distinguishing geometrical equality ["has-the-same-size-as"] from logical identity ["is-the-same-entity-as"], Tarski adapted TES for line-segment geometry where congruence and equality are coextensive—"Two segments congruent to the same segment are congruent to each other" [3, p. 121].

TES is the only ***Elements*** axiom governing equality alone: three others govern equality with addition, subtraction, and coincidence, respectively. A closely-related proposition (*the first axiom's twin*)—"Things that the same thing equals equal one another" [TSE]—has similar practical applications. After all, things that equal the same thing are things that the same thing equals [TES–TSE], and conversely [TSE–TES]; both are consequences of equality's symmetry.

Euclid's first axiom seems poorly chosen if characterizing equality were a goal. It is particularly weak. As a measure of its weakness, note that it doesn't imply any of the following: reflexivity, symmetry, transitivity, TSE (its twin), TES–TSE, and TSE–TES.

The independence results use the counterargument method [2, pp. 32ff] from Aristotle's ***Analytics*** and available to Euclid—after first-order translations following Tarski's examples [3, pp. 120–125].

This lecture treats the history, philosophy, and logic of Euclid's first axiom.

[1] Jonathan Barnes, ***Truth, etc.***, Oxford University Press, 2007.

[2] John Corcoran, *Argumentations and logic*, ***Argumentation***, vol. 3 (1989), pp. 17–43.

[3] Alfred Tarski, ***Introduction to logic***, Dover, 1995.

▸ JAN DOBROWOLSKI, *New examples of small Polish structures.*
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We answer some questions from [1] by giving suitable examples of small Polish structures. First, we present a class of small Polish group structures without generic elements. Next, we give a first example of a small non-zero-dimensional Polish $G$-group.

[1] Krzysztof Krupiński, *Some model theory of Polish structures*, ***Transactions of the American Mathematical Society***, vol. 362 (2010), no. 7, pp. 3499–3533.

▸ SEBASTIAN EBERHARD, *Applicative theories for logarithmical complexity classes.*
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Applicative systems are a formalisation of the lambda calculus and form the base theory of Feferman's explicit mathematics. For many linear and polynomial complexity classes corresponding applicative systems have already been developed by authors as Kahle, Oitavem, and Strahm. In contrast to the setting of bounded arithmetic, this setting allows very explicit and straightforward lower bound proofs because no coding of graphs of functions is necessary. For an overwiev, we recommend Strahm's [2].

We present natural applicative theories for the logarithmic hierarchy, alternating logarithmical time, and logarithmic space. For the first two algebras, we formalize function algebras having concatenation recursion as main principle. For logarithmical space, we formalize an algebra with safe and normal inputs and outputs. This algebra allows to shift small safe inputs to the normal side which is against the safe-normal paradigm but helps to describe logarithmical space in an elegant way.

The theories corresponding to the mentioned complexity classes all contain the predicates $\mathsf{W}$ for normal and $\mathsf{V}$ for safe words, and are similar in spirit to Cantini's theory for polytime in [1]. $t \in \mathsf{V}$ intuitively expresses that $t$ is a stored but not fully accessible content. The interplay between $\mathsf{W}$ which formalizes fully accessible content and $\mathsf{V}$ allows an easy formulation of induction principles justifying concatenation - and sharply bounded recursion.

[1] Cantini, *Polytime, combinatory logic and positive safe induction*, ***Archive for Mathematical Logic***, vol. 41 (2), pp. 169–189.

[2] Strahm, *Weak theories of operations and types*, Schindler (Ed.), ***Ways of proof theory***, pp. 441–468.

▸ PHILIP EHRLICH, *The Surreal Number Tree.*
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In his monograph *On Numbers and Games* cite1er, J. H. Conway introduced a real-closed field No, that is so remarkably inclusive that, subject to the proviso that numbers construed here as members of ordered number fields be individually definable in terms of sets of NBG, it may be said to contain 'All Numbers Great and Small'. In addition to its inclusive structure as an ordered field, No has a rich algebraico-binary tree-theoretic structure, or simplicity hierarchy, that emerges from the recursive clauses in terms of which it is defined. Among the striking simplicity-hierarchical features of No is that every surreal number can be assigned a canonical proper name called its *Conway name* (or *normal form*) that is a reflection of its characteristic simplicity-hierarchical properties. In [2], answers are provided for the following two questions that are motivated by No's structure as an ordered binary tree: (i) Given the Conway name of a surreal number, what are the Conway names of its two immediate successors? (ii) Given a chain of surreal numbers of infinite limit length, what is the Conway name of the immediate successor of the chain? The purpose of this talk is to provide an introduction to [2].

[1] J. H. Conway, ***On numbers and games***, Academic Press, 1976.

[2] Philip Ehrlich, *Conway Names, the Simplicity Hierarchy and the Surreal Number Tree*, ***The Journal of Logic and Analysis***, vol.3 (2011), no.1, pp.1–26.

▶ FREDRIK ENGSTRÖM, *Generalized quantifiers in dependence logic.*

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Partially ordered quantifier prefixes, or branching quantifiers, are needed when formalizing natural languages. The case of partially ordered existential and universal quantifiers has been studied in length, leading to systems like Hintikka and Sandu's IF-logic and Väänänen's dependence logic. For a compositional semantical analysis of these systems the framework, invented by Hodges, using sets of assignments instead of single assignements, is needed.

Branching of generalized quantifiers, however, is yet to be analyzed compositionally. We will in this talk present a compositional account of partially ordered monotone generalized quantifiers. It is based on dependence logic but with a modified dependence atom. Instead of using functional dependence we are forced to use multivalued dependence: A set of assignments $X$ satisfies the multivalued dependence $[\bar{x} \twoheadrightarrow y]$ if

$$\forall s, s' \in X \big( s(\bar{x}) = s'(\bar{x}) \to \exists s_0 \in X \big( s_0(\bar{x}, \bar{y}) = s(\bar{x}, \bar{y}) \wedge s_0(\bar{z}) = s'(\bar{z}) \big) \big),$$

where $\bar{z}$ are the variables in $X$ which are not in $\bar{x}$ or $\bar{y}$. Galliani proved in [3] that this atom if definably equivalent to the *independence atom* recently introduced by Väänänen and Grädel.

We will end by characterizing the expressive power of these extensions of dependence logic by monotone generalized quantifiers in terms of quantifier extensions of existential second-order logic.

[1] Fredrik Engström and Juha Kontinen, *Characterizing quantifier extensions of dependence logic*, ***Arxiv preprint***, arXiv:1202.5247, 2012.

[2] Fredrik Engström, *Generalized quantifiers in dependence logic*, ***Journal of Logic, Language and Information***, 10.1007/s10849-012-9162-4.

[3] Pietro Galliani, *Inclusion and exclusion dependencies in team semantics—on some logics of imperfect information*, ***Annals of Pure and Applied Logic***, vol. 163 (2012), no. 1, pp 68-84.

▶ HADI FARAHANI, HIROAKIRA ONO,
*Substructural view of Glivenko theorems and negative translations* *.

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In [3], the second author has developed a proof-theoretic approach to Glivenko theorems for substructural propositional logics. In the present talk, by using the same techniques, we will extend them for substructural predicate logics relative not only to classical predicate logic but also to an arbitrary involutive substructural predicate logic over QFLe. It will be pointed out that in this extensions, the following *double negation shift* scheme (DNS) plays an essential role.

$$(DNS): \ \forall x \neg\neg \varphi(x) \to \neg\neg \forall x \varphi(x)$$

Among others, it is shown that the Glivenko theorem holds for QFLe$^\dagger$ + (DNS) relative to classical predicate logic. Moreover, this logic is the weakest one among predicate logics over QFLe for which the Glivenko theorem holds relative to classical predicate logic.

Then we will study negative translations of substructural predicate logics by using the same approach. Our substructural analysis of Glivenko theorems will induce negative translation results of involutive substructural predicate logics over QFLe in a natural way. We introduce a negative translation, called extended Kuroda translation and the existence of the weakest logic is proved among such logics for which the extended Kuroda translation works. Thus we give a clearer unified understanding of negative translations by substructural point of view.

[1] J. Avigad, *A variant of the double-negation translation*, ***Carnegie Mellon Technical Report CMU-PHIL***, vol. 179(2006).

[2] G. Ferreira, P. Oliva, *On various negative translations*, ***Third International Workshop on Classical Logic and Computation*** (Brno, Czech Republic), (Steffen van Bakel Stefano Berardi and Ulrich Berger), vol. 47, Electronic Proceedings in Theoretical Computer Science, 2011, pp. 21–33.

[3] H. Ono, *Glivenko Theorems Revisited*, ***Annals of Pure and Applied Logic***, vol. 161(2009), pp. 246–250.

▸ THOMAS MACAULAY FERGUSON, *Ramsey's footnote and Priest's connexive logics.*
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The family of logics known as *connexive logics* are characterized by two theses. Using the notation of [1], these are

**1:** *Aristotle's Thesis* (AT) $\sim (\varphi > \sim \varphi)$, and
**2:** *Boethius' Thesis* (BT) $\sim ((\varphi > \psi) \wedge (\varphi > \sim \psi))$.

Connexive logics are distinct in that, in verifying AT and BT, they have properly superclassical theorems. Another feature of these theses is that they have historically been appealed to often in philosophical literature, albeit subtly.

One such instance is in Frank Ramsey's footnote in [3]:

> If two people are arguing "If $p$ will $q$?" and are both in doubt as to $p$, they are adding $p$ hypothetically to their stock of knowledge and arguing on that basis about $q$; so that in a sense "If $p$, $q$" and "If $p$, $\bar{q}$" are contradictories.

The assertion that $p > q$ and $p > \sim q$ are *contradictories* yields two consequences, BT itself, and

**3:** *Conditional Excluded Middle* (CEM) $(\varphi > \psi) \vee (\varphi > \sim \psi)$

Logics have been generated accounting for CEM in the spirit of Ramsey such as Stalnaker's C2 (see [1]); what has *not* been noted is that BT is a consequence of the footnote. Relaxing the term "contradictories" to "contraries," Ramsey's assertion is equivalent to BT.

By following this weakening along the lines of [1], one arrives at a family of connexive logics, including Graham Priest's connexive logics described in [2]. As this semantics is developed, an alternative semantics in the spirit of Ramsey's footnote for connexive logics emerges.

[1] Donald Nute, ***Topics in Conditional Logic***, Philosophical Studies Series in Philosophy, D. Reidel Publishing Company, 1980.

[2] Graham Priest, *Negation as cancellation, and connexive logic*, ***Topoi***, vol.18, no.2, pp.141-148.

[3] Frank Ramsey, *General propositions and causality*, ***Philosophical Papers*** (Mellor, D.H., editor), Cambridge University Press, New York, 1990, pp.145-163

▸ MICHELE FRIEND, *Genetic Proofs, Reductions and Rational Reconstruction Proofs.*
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Some theorems in mathematics have several proofs. Why? It is clear that proofs are not meant to only convince us of the truth of the theorem being proved. Rather, they give us explanations. I explore the philosophical implications of three conceptions of proof: the genetic conception, reductions and the rational reconstruction conception.

The genetic conception traces a theorem back to the setting in which the mathematician thought of the proof and the theorem. We learn the historical origin of the theorem. The reduction traces the theorem back to some more primitive conceptions. The primitive conceptions are associated with a foundational project, such as constructivism, realism or logicism. With such a proof, we learn the conceptual justification for the theorem. The 'rational reconstruction' conception of proof is one where we demonstrate that it is possible to understand a theorem in a novel setting, for example, we might give a proof in Topos theory of a theorem in geometry. We learn the spread of the theorem: in what other theories it is provable. The lessons become more interesting when the novel setting is inconsistent with the original setting.

As an example, I shall focus on Lobachevsky's solution to the problem of indefinite integrals. I shall compare Lobachevsky, Beltrami and Rodin's constructions and re-constructions, and offer one of my own, pointing out the lessons in each case. Lobachevsky give the genetic proof, Beltrami reduces the proof to Euclidean geometry, which was thought to be more obvious, or primitive. Rodin reconstructs the proof in topos theory to give a neutral proof, which is closer to Lobachevsky's purpose. I give a reconstruction using techniques developed in the paraconsistency literature, in order to justify using what might look like contradictory methods. Each type of proof teaches us different lessons.

▸ JEROEN GOUDSMIT, *On the admissible rules of Gabbay–de Jongh logics.*
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The admissible rules of a logic are those rules that can be added without affecting provability. Not all admissible rules of propositional intuitionistic logic (IPC) are derivable, a property shared with numerous modal and intermediate logics. Dick de Jongh and Albert Visser conjectured that the now well-known *Visser rules* characterize admissibility of IPC, in that these rules are sufficient and necessary to derive all admissible rules of IPC. Rozière [3] and Iemhoff [2] independently proved this.

We posit the de Jongh rules, an ostensible generalization of the Visser rules. Much like the Visser rules, they can be stratified along the natural numbers. We study the strata separately, and for logics that admit the disjunction property, we can tie admissibility of these rules to a stratified version of the extension property. Furthermore, these rules characterize admissibility for the Gabbay–de Jongh logics [1].

Many of our results hold in arbitrary intermediate logics. Internally, we mostly work with sets of formulae, shying away from semantics whenever sensible. This work might smoothen some other proofs in the literature. We employ these techniques to prove that a rule is admissible for the $n^{\text{th}}$ Gabbay–de Jongh logic if and only if it can be derived in the same logic enriched with the de Jongh rules up to the $(n+1)^{\text{th}}$ stratum. We also prove that the Gabbay–de Jongh logics have finitary unification type. This is joint work with Rosalie Iemhoff.


[1] Gabbay, Dov M. and de Jongh, Dick H.J., *A Sequence of Decidable Finitely Axiomatizable Intermediate Logics with the Disjunction Property*, ***The Journal of Symbolic Logic***, vol. 39 (1974), no. 1, pp. 67–78

[2] Iemhoff, Rosalie, *On the Admissible Rules of Intuitionistic Propositional Logic*, ***The Journal of Symbolic Logic*** vol. 66 (2001), no. 1, pp. 281–294

[3] Rozière, Paul, ***Règles admissibles en calcul propositionnel intuitionniste***, Université de Paris, 1992


▸ VOLKER HALBACH, *Self-reference* *.


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A Gödel sentence is a sentence that 'says about itself' that it's not provable; a Henkin sentence is a sentence that 'says about itself' that it's provable; a $\Sigma_1$-truth teller is a sentence that 'says about itself' that it is $\Sigma_1$-true.

I'll will try to look more closely at the way such self-referential statements are constructed. It is well known that the properties of self-referential sentences may depend on the chosen Gödel coding and on the formula representing the property in question. It less understood that the properties of self-referential statements depend also on the way, self-reference is obtained once the representing formula and the Gödel coding have been fixed.

Kreisel [1] constructed a refutable Henkin sentence. To this end he employed a non-canonical provability predicate but also a non-canonical construction to obtain self-reference. I will discuss some applications of Kreisel's basic technique and related observations by Albert Visser.


[1] Georg Kreisel, *On a problem of Henkin's*, ***Indagationes Mathematicae***, vol. 15 (1953), pp. 405–406.


▸ CHRISTOPHER HAMPSON, AGI KURUCZ, *The modal logic of 'elsewhere' as a component in product logics.*


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The finitely axiomatisable and decidable modal logic **Diff** of 'elsewhere' (or 'difference operator') is known to be Kripke complete with respect to the class of symmetric, pseudo-transitive frames. These frames closely resemble **S5**-relations (i.e. equivalence relations) and it is little surprise that the validity problems for **Diff** and **S5** have the same co-NP complexity, and both logics enjoy the finite model property.

Here we turn our attention to decision and axiomatisation problems of two-dimensional product logics $L_1 \times L_2$, by which we mean the multimodal logic of all product frames where the first component is a frame for $L_1$ and the second a frame for $L_2$. It is well-known that product logics of the form $L \times \mathbf{S5}$ are usually decidable, whenever $L$ is a decidable (multi)modal logic. We even have that $\mathbf{S5} \times \mathbf{S5}$ enjoys the exponential finite model property. Here we present some cases where the transition from $L \times \mathbf{S5}$ to $L \times \mathbf{Diff}$ not only increases the complexity of the validity problem, but in fact introduces undecidability and the lack of finite model property. We also show that no modal product logic of the form $L \times \mathbf{Diff}$ is finitely axiomatisable, whenever $L$ is between **K** and **S5**.


[1] D.M. Gabbay, A. Kurucz, F. Wolter and M. Zakharyaschev, ***Many-Dimensional Modal Logics: Theory and Applications***, Studies in Logic, Elsevier, 2003.


▸ ZUZANA HANIKOVÁ, *On logics of continuous t-norms and their residua.*


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In his monograph [6], P. Hájek proposed the (propositional) logic BL (Basic Logic) for the semantics given by continuous t-norms and their residua on the real unit interval $[0,1]$; such structures were called standard BL-algebras. In [3], it was shown that BL indeed was complete w.r.t. the class of standard BL-algebras, a subclass of its general algebraic semantics, the variety **BL** of BL-algebras. A thorough study carried out in [1] revealed that **BL** had uncountably many subvarieties. However, it can be shown ([7]) that individual standard BL-algebras generate only countably many subvarieties of **BL**. This remains true even if one considers subvarieties generated by *classes* of standard BL-algebras. Some nice properties for the logics of standard BL-algebras follow; among these are finite axiomatizability (cf. [4, 5]) and bounds on computational complexity (cf. [2, 7]). Recently, the logical landscape of which BL is a prominent element has been investigated in the book [8].

[1] Paolo Aglianò, Franco Montagna, *Varieties of BL-algebras I: General properties*, ***Journal of Pure and Applied Algebra***, vol. 181 (2003), no. 2–3, pp. 105–129.

[2] Matthias Baaz, Petr Hájek, Franco Montagna, Helmut Veith, *Complexity of t-tautologies*, ***Annals of Pure and Applied Logic***, vol. 113 (2002), no. 1–3, pp. 3–11.

[3] Roberto Cignoli, Francesc Esteva, Lluís Godo, Antoni Torrens, *Basic fuzzy logic is the logic of continuous t-norms and their residua*, ***Soft Computing***, vol. 4 (2000), no. 2, pp. 106–112.

[4] Francesc Esteva, Lluís Godo, Franco Montagna, *Equational characterization of subvarieties of BL-algebras generated by t-norm algebras*, ***Studia Logica***, vol. 76 (2004), no. 2, pp. 161–200.

[5] Nikolaos Galatos, *Equational bases for joins of residuated-lattice varieties*, ***Studia Logica***, vol. 76 (2004), no. 2, pp. 227–240.

[6] Petr Hájek, ***Metamathematics of Fuzzy Logic***, Trends in Logic, Kluwer, Dordrecht, 1998.

[7] Zuzana Haniková, *A note on the complexity of propositional tautologies of individual t-algebras*, ***Neural Network World***, vol. 12 (2002), pp. 453–460.

[8] Petr Cintula, Petr Hájek, Carles Noguera, editors. ***Handbook of Mathematical Fuzzy Logic***, College Publications, London, 2011.

▸ DANIEL HOFFMANN, *Multiplicatively iterative higher derivations.*
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For fields of characteristic $p > 0$, we will show that every derivation $D$ such that $D^{(p)} = D$ expands to a multiplicatively iterative Hasse-Schmidt derivation. It is known in the case of the standard (i.e. additive) iterativity condition: see Theorem 27.4. in [1], where the author expands a derivation $D$ such that $D^{(p)} = 0$ to an iterative Hasse-Schmidt derivation. Afterwards we will focus on a geometric axiomatisation of existentially closed Hasse-Schmidt fields with one multiplicatively iterative derivation as was done in [2] for the standard iterativity condition.

[1] Hideyuki Matsumura, ***Commutative ring theory***, Cambridge Studies in Advanced Mathematics, Cambridge University Press, 1989.

[2] Piotr Kowalski, *Geometric axioms for existentially closed Hasse fields*, ***Annals of Pure and Applied Logic***, vol.135 (2005), no.1-3, pp.286-302.

▸ TOMÁŠ HOLEČEK, *On a rule of propositional function instantiation.*
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The proofs in the first edition of Principia Mathematica [1] include asserted instances of previously asserted propositional functions (pfs), a process somewhat similar to the use of axiom-schemata. However, it was not possible to explicitly state a general rule for this essential step, i.e. something like "If a pf $\varphi$ is asserted, we can assert any pf which is an instance of $\varphi$." Two reasons made it impossible: inadmissibility of inductive definitions based on syntax of pfs and reluctance to express by general rule what we always need to apply as a particular. In reconstruction of this classical account on type theory [2, 3], we can easily define the instantiation by induction and explicate the rule in meta-language. In this talk, we will discuss the significance that the original reluctance has.

[1] Alfred North Whitehead and Bertrand Russell, ***Principia Mathematica***, vol. I, II, III, Cambridge University Press, 1910, 1912, 1913.

[2] Fairouz Kamareddine, Twan Laan, Rob Nederpelt, *Types in Logic and Mathematics before 1940*, ***The Bulletin of Symbolic Logic***, Vol. 8, No. 2 (2002), pp. 185-245.

[3] Twan Laan, *A Formalization of the Ramified Type Theory*, ***Computing Science Report***, 94/33, Eindhoven University of Technology, Eindhoven, 1994.

▸ SIMON IOSTI, *Imaginaries in Tannakian categories.*
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A Tannakian category is a category mimicking the behavior of the category of finite-dimensional representations of an algebraic group defined over some field. The Tannakian duality states an equivalence between Tannakian categories and such categories of representations. Recently, Moshe Kamensky [2] used the model-theoretic tools of internality and binding groups to prove this duality in some cases (algebraically and differentially closed fields).

Kamensky's proof involves the description of the imaginaries of a Tannakian category. In general, it seems impossible to achieve a complete description of these imaginaries. Nevertheless, such an exhaustive description is not needed to prove a Tannakian theorem. In my talk, I will present how one can expand Kamensky's framework, explain which imaginaries we need to understand to be able to prove Tannakian results in a more general context. This general context will allow to compare different kinds of Galois groups associated to differential and difference equations, in the spirit of the work of Chatzidakis, Hardouin, and Singer [1].

[1] Zoé Chatzidakis, Charlotte Hardouin, and Michael Singer, *On the definition of difference Galois groups*, ***Model theory with applications to algebra and analysis***(Zoé Chatzidakis, Dugald Macpherson, Anand Pillay, and Alex Wilkie, editors), Cambridge University Press,Cambridge,2008.pp. 73–109.

[2] Moshe Kamensky, *Model theory and the Tannakian formalism*,
*URL Address*: h.ttp://arxiv.org/abs/0908.0604

▸ KRZYSZTOF KAPULKIN, *Fibration categories and type theory.*
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The connections between Martin–Löf Type Theory and homotopy theory are now very intensively studied (see for example [1, 3]), especially in the context of Vladimir Voevodsky's 'Univalence Foundations' program. We propose the framework of fibration categories (cf. [2]) for a systematic development of these connections.

We start by verifying that the classifying category of MLTT has a natural structure of a fibration category. Further, we formalize within type theory several notions and theorems about fibrations categories such as right properness and the factorization lemma. Our special interests are in the study of the loop functor $\Omega$, spectra of types, and the homotopy limits. In particular, we internalize in type theory the construction of homotopy limits for model categories. All the formalization is done in the Coq proof assistant.

This is joint work with Jeremy Avigad (Carnegie Mellon University).

[1] Steve Awodey and Michael A. Warren, *Homotopy Theoretic Models of Identity Types*, ***Mathematical Proceedings of the Cambridge Philosophical Society***, vol. 45 (2009), no. 146, pp. 45–55.

[2] Kenneth Brown, *Abstract Homotopy Theory and Generalized Sheaf Cohomology*, ***Transactions of the American Mathematical Society***, vol. 186 (1973), pp. 419–458.

[3] Nicola Gambino and Richard Garner, *The Identity Type Weak Factorisation System*, ***Theoretical Computer Science***, vol. 409 (2008), no. 1, pp. 94–109.

▸ LAURENCE KIRBY, *Ordinal exponentiations of sets.*
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In the 1950s Tarski generalized to all sets the addition operation on the von Neumann ordinals ([2]; see also [1]). Scott followed with definitions of multiplication and exponentiation. The "high school algebra" laws of exponentiation fail in the generalized von Neumann arithmetic. The situation can be remedied by replacing the usual ordinal arithmetic of sets with one based on the finite Zermelo ordinals, in which the successor of $n$ is $\{n\}$. In the Zermelo arithmetic the "high school algebra" exponentiation laws hold.

Each of the two arithmetics of sets has advantages. The von Neumann arithmetic's elegance, flexibility and straightforward extension to the infinite arise largely from the fact that the order on the ordinals is the restriction of the membership relation. The Zermelo arithmetic, as well as the advantage mentioned above, is more economical.

[1] Laurence Kirby, *Addition and multiplication of sets*, ***Mathematical Logic Quarterly***, vol. 53 (2007), no. 1, pp. 52–65.

[2] Alfred Tarski, *The notion of rank in axiomatic set theory and some of its applications*, ***Bulletin of the American Mathematical Society***, vol. 61 (1955), p. 443. Reprinted in Alfred Tarski, ***Collected Papers***, ed. Steven R. Givant and Ralph N. McKenzie, Birkhäuser, Basel and Boston, 1986, vol. 3, p. 622.

► ALEXANDER P. KREUZER, *Non-principal ultrafilters, program extraction and higher order reverse mathematics.*
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We investigate the strength of the existence of a non-principal ultrafilter over fragments of higher order arithmetic. Let $(\mathcal{U})$ be the statement that a non-principal ultrafilter on $\mathbb{N}$ exists and let $\mathsf{ACA}_0^\omega$ be the higher order extension of $\mathsf{ACA}_0$. We show that $\mathsf{ACA}_0^\omega + (\mathcal{U})$ is $\Pi^1_2$-conservative over $\mathsf{ACA}_0^\omega$ and thus that $\mathsf{ACA}_0^\omega + (\mathcal{U})$ is conservative over $\mathsf{PA}$.

Moreover, we provide a program extraction method and show that from a proof of a strictly $\Pi^1_2$ statement $\forall f\, \exists g\, \mathsf{A}_{qf}(f,g)$ in $\mathsf{ACA}_0^\omega + (\mathcal{U})$ a realizing term in Gödel's system $T$ can be extracted. This means that one can extract a term $t \in T$, such that $\forall f\, \mathsf{A}_{qf}(f, t(f))$.

[1] Alexander P. Kreuzer, *Non-principal ultrafilters, program extraction and higher order reverse mathematics*, to appear in ***Journal of Mathematical Logic***.

► KANAT KUDAIBERGENOV, *Generalizations of o-minimality to partial orders.*
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For linearly ordered structures one has an important notion of o-minimality and several its generalizations. In this talk I will discuss some generalizations of o-minimality to partial orders.

► BEIBUT KULPESHOV, *On self-definable subsets in weakly o-minimal structures* *.
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We continue studying the notion of *weak o-minimality* originally studied by D. Macpherson, D. Marker and C. Steinhorn in [1]. A subset $A$ of a linearly ordered structure $M = \langle M, =, <, \dots \rangle$ is *convex* if for any $a, b \in A$ and $c \in M$ whenever $a < c < b$ we have $c \in A$. *A weakly o-minimal structure* is a linearly ordered structure $M$ such that any definable (with parameters) subset of $M$ is a finite union of convex sets in $M$. Real closed fields with a proper convex valuation ring provide an important example of weakly o-minimal structures.

If $M$ is a structure and $A \subset M$, we say that $A$ is *self-definable* if $A$ is definable in $M$ with parameters which are elements of $A$. A self-definable subset $A$ of an $\aleph_0$–categorical structure $M$ is *good* if for all $n < \omega$ every $n$-type over $A$ realized in $M$ is isolated. Self-definable sets were considered in [2] concerning the notion of *Jordan set*.

Here we discuss some equivalent conditions for goodness of self-definable subsets in an $\aleph_0$–categorical weakly o-minimal theory, and for this we use some results obtained in [3].

[1] H.D. Macpherson, D. Marker, Ch. Steinhorn, *Weakly o-minimal structures and real closed fields*, **Transactions of the American Mathematical Society**, 352 (2000), pp. 5435–5483.

[2] H.D. Macpherson, Ch. Steinhorn, *On variants of o-minimality*, **Annals of Pure and Applied Logic**, 79 (1996), pp. 165-209.

[3] B.Sh. Kulpeshov, $\aleph_0$-*categorical quite o-minimal theories*, **The Bulletin of Novosibirsk State University**, series: mathematics, mechanics and informatics, 11 (2011), pp. 45-57.

► SORI LEE, *Sight realizability: the arithmetic in subtoposes of the effective topos.*
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The internal (first-order) arithmetic of the effective topos is Kleene's realizability. This talk presents a realizability-like description for the arithmetic in *subtoposes* of the effective topos.

The effective topos has as its least (non-degenerate) subtopos the category of sets, whose internal arithmetic is of course the true arithmetic. Also well-known is the fact that the (opposite) semi-lattice of Turing degrees embed into the lattice of subtoposes of the effective topos, manifesting the vast size of the latter structure. The work [1] establishes an infinite family of new examples, with the technique behind being to understand entities that represent subtoposes in terms of a certain kind of well-founded trees called *sights*.

As a by-product of this, we obtain our "realizability" semantics for the arithmetic in subtoposes. If $\theta$ is a subtopos, we define a relation '$\theta$-realizes' between numbers and arithmetic sentences in the same inductive way as the original realizability, with only changes in the implication and universal quantifier clauses. For example, the implication clause has the following look.

> $n$ '*$\theta$-realizes*' $\phi \Rightarrow \psi$ if for each $\theta$-realizer $m$ of $\phi$ there is a "$(\varphi_n(m), \theta)$-dedicated" sight $S$ such that each "$\varphi_n(m)$-value of" $S$ does $\theta$-realizes $\psi$.

The subtopos $\theta$ is secretly just a sequence of collections of natural number sets, and each notion appearing above ('sight', 'dedicated', etc) is free of topos theory. This leaves us with plenty of models of Heyting arithmetic described in elementary terms.

In the talk we introduce and discuss this 'sight realizability'.

[1] Sori Lee, *Subtoposes of the Effective Topos*, ***Master's Thesis, Utrecht University***, arXiv:1112.5325, 2011.

[2] Sori Lee, Jaap van Oosten, *Basic Subtoposes of the Effective Topos*, arXiv:1201.2571, 2012.

▸ GRAHAM E LEIGH, *Theories of truth over intuitionistic logic* $^*$.
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We investigate the role classical principles play in restricting the freedom to add semantic concepts such as truth to the language of arithmetic. In particular we consider two collections of natural principles of truth both of which are consistent over Heyting arithmetic, but inconsistent over classical Peano arithmetic. We show that the two intuitionistic theories of truth have the same $\Pi^0_2$ consequences as their consistent classical counterparts and argue that in the analysis of formal theories of truth, intuitionistic logic can play an intermediary role between full classical logic in which paradoxes abound and much weaker logics such as partial or para-consistent logics that are mathematically not well understood.

▸ LAURENŢIU LEUŞTEAN, *An application of proof mining in nonlinear analysis.*
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*Proof mining* is an area of applied proof theory concerned with the extraction of hidden finitary and combinatorial content from proofs that make use of highly infinitary principles. This line of research, developed by Ulrich Kohlenbach in the 90's, has its roots in Georg Kreisel's program on *unwinding of proofs*, initiated in the 50's. A comprehensive reference for proof mining is Kohlenbach's book [3].

We present an application of proof mining to the asymptotic behaviour of firmly nonexpansive mappings, a class of functions which play an important role in metric fixed point theory and optimization due to their correspondence with maximal monotone operators.

We obtain effective and highly uniform rates of asymptotic regularity for the Picard iterations of firmly nonexpansive mappings in uniformly convex $W$-hyperbolic spaces, a class of geodesic spaces that generalize both CAT(0) spaces and uniformly convex Banach spaces. In the case of CAT(0) spaces, the rate of asymptotic regularity is quadratic. These results, contained in a joint paper with D. Ariza-Ruiz and G. Lopez-Acedo [1], are new even for uniformly convex Banach spaces. Furthermore, they are guaranteed by general logical metatheorems proved by P. Gerhardy and U. Kohlenbach [2] for different classes of metric and normed spaces and adapted in [4] to uniformly convex $W$-hyperbolic spaces.

[1] D. Ariza-Ruiz, L. Leuştean, G. Lopez-Acedo,*Firmly nonexpansive mappings in classes of geodesic spaces*, ***arXiv:1203.1432v1 [math.FA]***, 2012.

[2] P. Gerhardy, U. Kohlenbach, *General logical metatheorems for functional analysis*, ***Transactions of the American Mathematical Society***, vol. 360 (2008), pp. 2615–2660.

[3] U. Kohlenbach, ***Applied proof theory: Proof interpretations and their use in mathematics***, Springer Monographs in Mathematics, Springer-Verlag, 2008.

[4] L. Leuştean, *Proof mining in $\mathbb{R}$-trees and hyperbolic spaces*, ***Electronic Notes in Theoretical Computer Science***, vol. 165 (2006), pp. 95–106.

▸ SONIA L'INNOCENTE, *Diophantine Sets of Representations.*
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This is a joint work with Ivo Herzog [1].

The aim of this report is that of proving some recursive results on sets of certain representations of the Lie algebra sl$(2,k)$, denoted by $L$, of $2\times 2$ traceless matrices with entries in a field $k$ of characteristic 0. It is known that the Lie algebra $L$ acts by derivations on the ring $k[x,y]$ of polynomials in two variables. This representation of $L$ admits a decomposition

$$k[x,y]=\bigoplus_{n\geq 0}k[x,y]_n,$$

where $k[x,y]_n$ is the $(n+1)$-dimensional $k$-vector subspace of homogeneous polynomials of total degree $n$. $\varphi(v)$ is a positive-primitive formula in the free variable $v$ in the language $\mathcal{L}$ of representations of $L$, then the $k$-vector subspace of $k[x,y]$ defined by $\varphi(v)$ respects this decomposition,

$$\varphi(k[x,y])=\bigoplus_{n\geq 0}\varphi(k[x,y]).$$

To understand the pp-definable $k$-subspace $\varphi(k[x,y])$, it is suffices to know the pp-definable $k$-subspaces

$\varphi(k[x,y]_n)$. The main task of this work is to prove

THEOREM 1. *If the field $k$ is recursively presented, then the function $n \mapsto \dim_k\ \varphi(k[x,y]_n)$ is recursive.*

This theorem is part of the program, enunciated by the author and Macintyre [2], to extend the recursive presentation of $k$ to one of the von Neumann $k$-algebra $U'(L)$ of definable scalars of the representation $k[x,y]$, and to prove the decidability of the theory of $U'(L)$-modules. One goal of this program is to provide a procedure that decides for every true implication of pp-formulas $\vdash \tau(v) \rightarrow \sigma(v)$, and natural number $n$, whether the $k$-space $\sigma(k[x,y]_n)/\tau(k[x,y]_n)$ is nonzero. It follows from the theorem that for every $n$, the function

$$n \mapsto \dim_k \sigma(k[x,y]_n)/\tau(k[x,y]_n) = \dim_k \tau(k[x,y]_n) - \dim_k \sigma(k[x,y]_n)$$

is recursive. Funrthermore, Theorem 1 yields important information regarding the topology of the Ziegler spectrum of $U'(L)$. We prove that certain clopen subsets of this topological space are diophantine.

[1] IVO HERZOG, S. L'INNOCENTE, *Diophantine Sets of Representations*, In progress.

[2] S. L'INNOCENTE, A. MACINTYRE, *Towards decidability of the theory of pseudo-finite dimensional representations of sl$(2,k)$*, ***Andrzej Mostowski and Foundational Studies*** (A. Ehrenfeucht, V.W. Marek and M. Srebrny, editors), IOS Press, 2007, pp. 235–260.

▶ ZACHIRI MCKENZIE, *Automorphisms of models of set theory and* NFU.
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NFU is the subsystem of Quine's 'New Foundations' (NF) obtained by weakening the extensionality axiom to allow urelements. It was first introduced by Ronald Jensen in [1]. Jensen demonstrated that a model of NFU can built from a model of a subsystem of ZFC that admits an automorphism. We will recall Jensen's construction and demonstrate that models of natural strong extensions of NFU can be built from models of subsystems of ZFC that admit automorphisms with certain 'nice' properties. Our particular focus will be on an extension of NFU obtained by adding $\mathrm{AxCount}_{\leq}$ which is a natural weakening of Rosser's Axiom of Counting [2]. A model of $\mathrm{NFU} + \mathrm{AxCount}_{\leq}$ can be obtained from a model of a strong enough subsystem of ZFC that admits an automorphism which does not move any natural number down. We will indicate how a model admitting such an automorphism can be produced from a non-standard $\omega$-model of a subsystem of ZFC. This result allows us to show that NFU plus Rosser's Axiom of Counting proves the consistency of $\mathrm{NFU} + \mathrm{AxCount}_{\leq}$.

[1] JENSEN, R., *On the Consistency of a Slight(?) Modification of Quine's New Foundations*, ***Synthese***, vol. 19 (1969), pp. 250–263.

[2] ROSSER, J. B., ***Logic for Mathematicians***, McGraw-Hill, New York, 1953.

▶ DAVID MILLER, *Probabilistic generalizations of deducibility.*
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The guiding idea of the *logical interpretation of probability* (von Kries, Waismann, Popper, Kneale, and others) is that the upper limit of probability is certainty or logical necessity. The relation $p(c \mid a) = 1$ (or, more accurately, $\forall b\, p(c \mid ab) = 1$) represents a *logically necessary connexion* between the sentences $a$ and $c$, that is, the *deducibility* of $c$ from $a$, while lesser values of $p$ indicate that, whatever connexion there may be between $a$ and $c$, it falls short of necessity. It is generally understood, in addition, that $p$ is a measure on the ranges of its arguments, the sets of possibilities that they admit; whence the conditional probability $p(c \mid a)$ measures the proportion (in an appropriately generalized sense) of those possibilities admitted by $a$ that are admitted also by $c$, and takes its maximum value, namely 1, when they all are. As anticipated, $p(c \mid a) = 1$ when $c$ is deducible from $a$. At the other extreme $p(c \mid a) = 0$ if the range of $a$ excludes every possibility that is admitted by $c$, that is, if $\neg c$ is deducible from $a$. The function $p$, so construed, satisfies all the usual (finitary) axioms for probability, such as those of Kolmogorov, or those of the more general system given in [1], appendix ∗v.

Less often considered are several other functions that provide alternative, and perhaps more illuminating, generalizations of deducibility. Since $c$ is deducible from $a$ if & only if $\neg a$ is deducible from $\neg c$, for example, and also if & only if $c$ is deducible from $a \vee c$, the functions $q(c \mid a) = p(\neg a \mid \neg c)$ and $d(c \mid a) = p(c \mid a \vee c)$, which are not in general equal to $p(c \mid a)$, also suggest necessary conditions for deducibility. Deducibility itself may be defined by $\forall b\, q(b \rightarrow c \mid a) = 1$ and by $\forall b\, d(cb \mid ab) = 1$. The full range of possibilities is explored, and some philosophical consequences are drawn.

[1] KARL R. POPPER, ***The Logic of Scientific Discovery***, London: Hutchinson 1959.

▸ TAKAKO NEMOTO, *The proof theoretic strengths of determinacy between $\Sigma^0_1$ and $\Delta^0_2$.*
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[3] proved the equivalences between $\Sigma^0_1 \wedge \Pi^0_1$ determinacy and $\Pi^1_1$ comprehension, and between $\Delta^0_2$ determinacy and $\Pi^1_1$ transfinite recursion over $\mathsf{RCA}_0$. The idea of the latter proof is as follows:

1. Find a well-ordering $W$ such that a given $\Delta^0_2$ game can be represented as an union of $(\Sigma^0_1 \wedge \Pi^0_1)$ games along $W$.
2. Iterate the proof of the former equivalence.

Then, by restricting the well-ordering in 1, we can define many subclasses of $\Delta^0_2$ games. In this talk, we show the equivalences between the determinacy of such classes and schemata of restricted transfinite recursion. We also consider the proof theoretic strengths of them by constructing $\beta$-models.

[1] Takako Nemoto, *Determinacy of Wadge classes and subsystems of second order arithmetic,* Mathematical Logic Quarterly, 55 (2009) pp. 154–176.

[2] S. G. Simpson, *Subsystems of Second Order Arithmetic,* Springer (1999).

[3] K. Tanaka, *Weak axioms of determinacy and subsystems of analysis I: $\Delta^0_2$-games,* Zeitschrift für Mathematische Logik und Grundlagen der Mathematik 36 (1990), pp. 481–491.

▸ VLADISLAV NENCHEV, *Dynamic relational mereotopology: First-order and modal logics for stable and unstable relations.*
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This paper presents first-order and modal logics of stable and unstable versions of four mereotopological relations: *part-of*, *overlap*, *underlap* and *contact* (denoted $\leq$, $\mathsf{O}$, $\mathsf{U}$ and $\mathsf{C}$). These relations are formally defined with Contact algebras $(\underline{B}, \mathsf{C})$, where $\underline{B}$ is a Boolean algebra and $\mathsf{C}$ is the contact relation (see [3], [2]). Standard models for Contact algebras are the regular closed sets in a topological space and the topological contact. The other relations are defined: $\leq$ is the Boolean ordering, $\mathsf{O}$ corresponds to non-empty intersection and $\mathsf{U}$ is the dual of $\mathsf{O}$. *Mereotopological structures* are relational structures with the four relations $\leq$, $\mathsf{O}$, $\mathsf{U}$ and $\mathsf{C}$ (see [2]). Their stable and unstable counterparts are defined over Cartesian products of mereotopological structures. Let $I$ be a set of moments of time and $(W_i, \leq_i, \mathsf{O}_i, \mathsf{U}_i)$ be a mereotopological structure for every $i \in I$. Then the stable and unstable contact is defined $x \ \mathsf{C}^\forall \ y$ iff $(\forall i \in I)(x_i \ \mathsf{C}_i \ y_i)$, $x \ \mathsf{C}^\exists \ y$ iff $(\exists i \in I)(x_i \ \mathsf{C}_i \ y_i)$. $\leq^\forall$, $\leq^\exists$, $\mathsf{O}^\forall$, $\mathsf{O}^\exists$, $\mathsf{U}^\forall$ and $\mathsf{U}^\exists$ are defined similarly in [1].

The current system is an extension of the one presented in [1]. Both systems are relational variants of the dynamic mereotopology from [3]. These works are developments in the area of alternative theories of space and time, started by Alfred Whitehead. They combine relations from mereotopology with simple temporal properties like *stability* and *unstability.* Whitehead used mereotopology as a base to build a new point-free theory of space. The fact that we combine spacial and temporal properties in one, rather than using different operators for space and time, corresponds to Whitehead's idea that the theory of time cannot be separated from the theory of space (see [4]).

The paper continues with axiomatization of the new system. The completeness is proved via a generalization of the Stone-like representation techniques for distributive lattices. We prove completeness for the quantifier-free fragment of the corresponding first-order logic, its decidability and show that its satisfiability problem is in NP. The full first-order logic is hereditary undecidable. We use the relational structures as semantic base of a polymodal logic for which we provide a complete axiomatization.

[1] Vladislav Nenchev, *Logics for stable and unstable mereological relations*, ***Central European Journal of Mathematics***, vol. 9 (2011), no. 6, pp. 1354–1379.

[2] Yavor Nenov and Dimiter Vakarelov, *Modal Logics for Mereotopological Relations*, ***Advances in Modal Logic***, vol. 7, College Publications, 2008, pp. 249–272.

[3] Dimiter Vakarelov, *Dynamic mereotopology: A point-free theory of changing regions. I. Stable and unstable mereotopological relations*, ***Fundamenta Informaticae***, vol. 100 (2010), no. 1–4, pp. 159–180.

[4] Alfred N. Whitehead, ***Process and Reality***, New York: MacMillan, 1929.

▸ JOSEPH W. NORMAN, *Hot buttered conditionals, tangled up in grue: Goodman's riddles solved by parametric probability analysis.*
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Conditional statements, whether factual or counterfactual, make perfect sense as constraints on probabilities. Using probability networks, the computed results of probability queries are quotients of sums of products of input probabilities. With inputs $\mathrm{Pr}_0(H) = x$ and $\mathrm{Pr}_0(M \mid H) = y$ stating that the butter was heated with probability $x$ and that the conditional probability of melting given heating is $y$, the query $\mathrm{Pr}(M \mid H)$ yields the result $xy/x$: the product $\mathrm{Pr}_0(H) \times \mathrm{Pr}_0(M \mid H)$ divided by $\mathrm{Pr}_0(H)$.

Interpreting the sentence "If that piece of butter had been heated it would have melted" as the constraint $y = 1$ and "If that piece of butter had been heated it would *not* have melted" as $y = 0$ produces the desired semantics: these constraints are clearly inconsistent; and neither constraint on $y$ affects $\Pr(H)$, which is $x$. Note that if $x > 0$ then the output $\Pr(M \mid H)$ simplifies to the input $y$; however if $x = 0$ then $\Pr(M \mid H)$ yields the indeterminate form $0/0$ regardless of $y$. Algebra says what we mean.

Concerning 'grue' we model explicitly the ambiguous correlation between the basic green/blue proposition, the composite grue/bleen proposition, and time. Parametric probability analysis demonstrates that it is always correct to make complementary predictions about future green and future grue, regardless of how this ambiguity is resolved.

[1] Joseph W. Norman. *The Logic of Parametric Probability.* Preprint at `arXiv:1201.3142v2 [math.LO]`. January, 2012.

[2] Nelson Goodman. ***Fact, Fiction, and Forecast.*** 4th edition. Harvard, 1983.

▸ ANDREA PEDEFERRI, *Is it original only once?.*
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What does it mean for an object to be "as new", or "the same as it was when it was new"? We can say that to be ?as new?, the object must be (after a period of time $t$) identical to itself when it was new. However, when we say that, for example, an object is identical to itself after two years, it would be strange to expect to find exactly the same object we saw two years earlier (provided no big traumatic modifications happened in between). Identical doesn?t mean immutable. Moreover, identity is a notion that has some precise logical characterizations and limitations. Are the two objects in question identical in the sense of Leibniz?s law? It is not easy to say so. I propose a better notion, which seems to avoid the problem of using identity in such cases: the notion of originality. An object is "as new" (after some $t$ from its "birth") if it is close enough to its "original state". The notion of originality for objects allows a limited kind of modifications to the object, which do not alter the originality of the object itself.

In this paper I will sketch a formal description of the notion of originality (for physical, non-living objects) and of the property of being close to an original state. In order to do that I will use some tools from mereology to draw a distinction between the notions of identity, sameness and originality. On the basis of this I will then give a formal account to the notions of restoration and conservation. I will do it by restrict the mereological property of parthood to that of being a *component*, and I will show how you can formally describe restoration and conservation in terms of particular mereological sums and subtractions of components.

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In [1] Friedman introduces adjacent Ramsey theory, including a series of theorems independent of Peano Arithmetic which, though similar to other Ramsey-like theorems, avoid its language. We examine the provability and phase transitions in Peano Arithmetic and its fragments of one of those theorems.

Following the notations from [1], we call a function $C : R^k \to \mathbb{N}^r$ $f$-limited if $\max C(x) \leq \max(f(\max x), 1)$. Let $\mathrm{AR}^k_f$ be the following statement:

For every $r$ there exists $R$ such that for every $f$-limited function $C : R^k \to \mathbb{N}^r$ there are $x_1 < \cdots < x_{k+1} < R$ with $C(x_1, \ldots, x_k) \leq C(x_2, \ldots, x_{k+1})$.

Take $f^k_\alpha(i) = \sqrt[H^{-1}_\alpha(i)]{\log^k(i)}$ and $g_\alpha(i) = \log^{H^{-1}_\alpha(i)}(i)$ where the $k$ in the exponent at the log indicates number of iterations and $H_\alpha$ is the $\alpha$-th function in the Hardy hierarchy.

Theorem 1. *Take $\omega_0 = 1$, $\omega_{n+1} = \omega^{\omega_n}$, then:*
$\mathrm{I}\Sigma_k \nvdash \mathrm{AR}^k_{\mathrm{id}}$,
$\mathrm{PA} \vdash \forall k \mathrm{AR}^k_{g_\alpha}$ *if and only if* $\alpha < \varepsilon_0$,
$\mathrm{I}\Sigma_{k+1} \vdash \mathrm{AR}^{k+1}_{f^k_\alpha}$ *if and only if* $\alpha < \omega_{k+2}$.

[1] Harvey Friedman, *Adjacent Ramsey theory,*
***http://www.math.osu.edu/ ~friedman.8/pdf/PA%20incomp082910.pdf***

▸ MIKHAIL G. PERETYAT'KIN, *On model-theoretic properties that are not preserved on the pairs of mutually interpretable theories.*
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We consider theories in first-order predicate logic with equality. *Incomplete theories* are normally studied. An infinite model $\mathfrak{M}$ is said to be *minimal* (synonym: a *Jónsson model*), if for any model $\mathfrak{N}$, $\mathfrak{N} \preccurlyeq \mathfrak{M} \wedge \mathrm{Card}(\mathfrak{N}) = \mathrm{Card}(\mathfrak{M})$ implies $|\mathfrak{N}| = |\mathfrak{M}|$. An interpretation $I$ of theory $T$ in domain $U(x)$ of theory $H$ is called $\exists\cap\forall$*-presentable*, if the domain $U(x)$ of the interpretation and $I$-image of each predicate of $T$ is presentable in $H$ by an $\exists$-formula, and simultaneously, by a $\forall$-formula. Theories $T$ and $H$ are called *mutually* $\exists\cap\forall$*-definably interpretable in each other*, if there is an $\exists\cap\forall$-definable interpretation $I$ of $T$ in $H$

and an $\exists \cap \forall$-definable interpretation $J$ of $H$ in $T$ such that, for any sentence $\Phi$ of $T$ and any sentence $\Psi$ of $H$, we have $T \vdash \Phi \leftrightarrow J(I(\Phi))$ and $H \vdash \Psi \leftrightarrow I(J(\Psi))$ ensuring an isomorphism of the Tarski-Lindenbaum algebras of these theories.

**Theorem 1.** *There are complete decidable theories $T_i$ and $H_i$, $i = 0, 1$, of finite pure predicate signatures without finite models, such that $T_i$ and $H_i$ are mutually $\exists\cap\forall$-definably interpretable in each other for $i = 0, 1$; moreover, the following properties are satisfied*:

(a) $T_0$ *is model-complete, while* $H_0$ *is not model-complete*;

(b) $T_1$ *is finitely axiomatizable, while* $H_1$ *is not finitely axiomatizable*;

(c) $T_1$ *has a minimal model, while* $H_1$ *does not have such a model*;

(d) $T_1$ *has a model with first-order definable elements, while* $H_1$ *does not have such a model; furthermore,* $H_1$ *does not even have a model with almost first-order definable* (*algebraic*) *elements.*

By $\mathbb{L}^{ea}$, we denote the collection of those model-theoretic properties $\mathfrak{p}$ which are preserved on each pair of mutually $\exists\cap\forall$-definably interpretable computably axiomatizable theories of finite signatures. Theorem 1 shows that the semantic layer $\mathbb{L}^{ea}$ covers neither Cartesian, nor Cartesian-quotient, nor even model quasiexact layer of model-theoretic properties. This gives a negative answer to Question 7 posed in [1].

[1] Peretyat'kin M.G., *Finitely axiomatizable theories and similarity relations*, ***American Mathematical Society Translations***, (2) Vol. 195 (1999), pp. 309–346.

▶ PAULA QUINON, *Numerals and numbers. Problems of encodings and denotations.*
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This talk proposes a study of so-called "deviations" which are claimed to occur as consequences of accepting the formal definition of the concept of computability that assumes of human intuitions about computation that they concern operations on strings (as captured by Turing's thesis) rather than abstract knowledge of functions defined on natural numbers (Church's thesis) [2].

The study involves specification of the relationship between the syntactic (numerals) and the semantic (numbers) levels of the language of number theory, and the denotation functions acting between those two. The "deviations" – resulting in "computability" of some uncomputable functions (the halting problem is the most commonly quoted example) – have been claimed to occur on both of those levels ([1], [2], [3]). Certain constraints on the properties of the denotation functions have been also proposed. These constraints aim to single out the class of dentation functions acceptable for number-theoretical purpose ([4]).

The central claim of this talk is that the harmful aspect of these "deviations" can be avoided by detailed insight into the dichotomy between syntax and semantics. Additionally, some remarks on denotation functions are formulated. I claim that the presented results shed some light on the number-concept as investigated by cognitive scientists.

[1] Paul Benacerraf, *Recantation, or: Any Old ω-Sequence Would Do After All*, ***Philosophia Mathematica***, vol. 4 (1996), no. 3, pp. 184–189.

[2] Michael Recorla, *Church's Thesis and the Conceptual Analysis of Computability*, ***Notre Dame Journal of Formal Logic***, vol. 48 (2007), pp. 253–280.

[3] B. Jack Copeland, Diane Proudfoot, *Deviant Encodings and Turings Analysis of Computability*, ***Studies in History and Philosophy of Sciences***, vol. 41 (2010), no. 3, pp. 247–252.

[4] Stewart Shapiro, *Acceptable Notation*, ***Notre Dame Journal of Formal Logic***, vol. 23 (1982), no. 1, pp. 14–20.

▶ BENJAMIN RIN, *The computational strengths of α-length ITTMs* *.
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In [1], open questions are raised regarding the computational strengths of so-called $\infty$-$\alpha$-Turing machines, a family of models of computation resembling the infinite-time Turing machine (ITTM) model of [2], except with $\alpha$-length tape (for any $\alpha \geq \omega$). Let $T_\alpha$ refer to the model of length $\alpha$. So $T_\omega$ is just the ITTM model. Let $\succ$ stand for "is computationally stronger than". In attempting to address the open questions, I present the following results: (1) $T_{\omega_1} \succ T\omega$. (2) There exists a countable $\alpha$ such that $T_\alpha \succ T_\omega$. In fact, there is a hierarchy of countable machines of increasing strength, corresponding to the (weak) transfinite Turing-jump operator $^\nabla$. (3) There is a countable ordinal $\mu'$ such that for every countable $\mu \geq \mu'$, neither $T_\mu \succ T_{\omega_1}$ nor $T_{\omega_1} \succ T_\mu$ — that is, the machines $T_{\omega_1}$ and $T_\mu$ are computation-strength incommensurable. The same holds true for any machine of length greater than $T_{\omega_1}$.

[1] Peter Koepke *Ordinal Computibility* **Lecture Notes in Computer Science, vol. 5365 (2009), pp. 280-289**

[2] Joel Hamkins and Andy Lewis *Infinite time Turing machines* **Journal of Symbolic Logic, vol. 65 (2000), no. 2, pp. 567-604**

▸ GEMMA ROBLES, *Depth relevance and the contraction axiom* *.
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A propositional logic has the *depth relevance property* (drp) if in all its theorems of the form $A \to B$, $A$ and $B$ share a propositional variable at the same depth (see [1]). In [1], a particular logic, DR, is defined by restricting with the drp the class of logics verified by Meyer's six-valued Crystal matrix. DR is motivated by its rejection of the contraction axiom $(A \to .A \to B) \to .A \to B$ used in the derivation of Curry Paradox in naive set theory.

The aim of this paper is to generalize Brady's strategy by defining a class of general model structures built upon what we label weak relevant matrices. The contraction axiom, together with a number of related theses, is falsified in any of these model structures.

[1] R. T. Brady, *Depth Relevance of some Paraconsistent Logics*, ***Studia Logica***, vol. 43 (1984), pp. 67-73.

▸ JASON RUTE, *Martingale convergence and algorithmic randomness.*
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Recently there has been a good deal of interest in the interaction between algorithmic randomness and computable analysis, especially a.e. convergence theorems. In this talk I will show a fruitful relationship between martingale convergence and randomness.

Martingales, which are a formalization of the notion of betting strategy, have historically been studied in two contexts. On the computability side, they have become a useful tool for information theory and algorithmic randomness; while on the analysis side, martingales have also become the foundation of modern probability theory and finance, with a variety of applications to analysis. Traditionally, algorithmic randomness has been concerned with at which points a nonnegative dyadic martingale succeeds (wins arbitrarily large amounts of money), while probability theory has been concerned with whether more general classes of martingales converge pointwise a.e.

I will present a variety of martingale convergence theorems, and I will show how they relate to Schnorr, computable, Martin-Löf, and weak 2-randomness. These martingale convergence theorems imply facts about differentiability, the law of large numbers, and de Finetti's theorem. They also are closely related to the ergodic theorems.

Further, the tools used to study randomness and martingales have close connections to constructive and computable analysis, reverse mathematics, proof theory, and hard/quantitative/numerical analysis.

▸ FRANCISCO SALTO, GEMMA ROBLES, JOSÉ M. MÉNDEZ, *Strong relevant matrices* *.
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The aim of this paper is to define a general class of logical matrices called "Strong relevant matrices" (srm). Any logic S verified by a srm has the following properties.

1. (Strong variable-sharing property). If $A \to B$ is a theorem of S, then some variable occurs as an antecedent part (ap) or else as a consequent part (cp) of both $A$ and $B$.
2. (No loose pieces property). If $A$ is a theorem of S and $A$ contains no conjunctions as aps and no disjunctions as cps, every variable in $A$ occurs once as ap and once as cp.

Our result generalizes that of Anderson and Belnap for the logics E and R (see [1], §22.1.2).

[1] Anderson, A. R., Belnap, N. D. Jr., ***Entailment. The Logic of Relevance and Necessity***, vol. 1, Princeton University Press, 1975.

▸ LUCA SAN MAURO, *Aspects of the theory of computable enumerable equivalence relations.*
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*(joint work with Uri Andrews, Steffen Lempp, Joseph S. Miller, Keng Meng Ng, Andrea Sorbi)*

This talk is about computable enumerable equivalence relations (ceers). We study them under the following reducibility: if $R, S$ are equivalence relations on $\omega$, we say that $R$ is *reducible* to $S$ ($R \leq S$) if there exists a computable function $f$ such that, for every $x, y$, $xRy \Leftrightarrow f(x)Sf(y)$.

The reducibility was introduced by Ershov [1], with respect to the theory of numberings, and later developed in Bernardi and Sorbi [2], and Lachlan [3].

Recently, new motivations occurred while considering a computable analogue of the so-called Borel reducibility, as in Gao and Gerdes [4].

In this talk, we focus on two of the main aspects of the topic. Firstly, we approach the degree structure generated by the reducibility. We show that the structure form a bounded poset which is neither a lower semilattice, nor an upper semilattice. In addition, we prove that its first order theory is undecidable.

Secondly, we turn our attention to universal ceers. We review classical definitions from the existing literature and we show that: the uniformly effectively inseparable ceers are universal, while there are effectively inseparable ceers that are not universal.


[1] Yu. L. Ershov, ***Theory of numberings***, Nauka, Moskow, 1977.

[2] C. Bernardi and A. Sorbi, *Classifying positive equivalence relations*, ***J. symbolic logic***, vol. 48 (1983), no. 3, pp. 529–538.

[3] A. H. Lachlan, *A note on positive equivalence relations*, ***Mathematical logic quaterly***, vol. 33 (1987), no. 1, pp. 43–46.

[4] S. Gao and P. Gerdes, *Computably enumerable equivalence realations*, ***Studia logica***, vol. 67 (2001), no. 1, pp. 27–59.


▶ SAM SANDERS, *Reuniting the antipodes: Bringing together Constructive and Nonstandard Analysis.*


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*Constructive Analysis* was introduced by Errett Bishop to identify the *computational meaning* of mathematics. In the spirit of intuitionistic mathematics, notions like *algorithm*, *explicit computation*, and *finite procedure* are central. The exact meaning of these vague terms was left open, to ensure the compatibility of Constructive Analysis with several traditions (classical, intuitionistic and recursive) in mathematics. Constructive Reverse Mathematics (CRM) is a spin-off of Harvey Friedman's famous *Reverse Mathematics* program, based on Constructive Analysis. Bishop famously derided Nonstandard Analysis for its lack of computational meaning. In this talk, we introduce '$\Omega$-invariance': a simple and elegant definition of *finite procedure* in (classical) Nonstandard Analysis. Using an intuitive interpretation, we obtain many results from CRM, thus showing that $\Omega$-invariance is quite close to Bishop's notion of *finite procedure* and algorithm. We briefly discuss philosophical implications and future work with regard to Per Martin-Löf's *Type Theory*, which is intended as a foundation for Constructive Analysis.


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▶ ANDREY SARIEV, *The $\omega$-Turing degrees.*


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In this paper the study of the partial ordering of the $\omega$-Turing degrees is initiated. Informally, the considered structure is derived from the structure of $\omega$-enumeration degrees described by Soskov [1] by replacing the usage of the enumeration reducibility and the enumeration jump in the definitions with Turing reducibility and Turing jump respectively. The main results include a jump invertion theorem, existence of minimal elements and minimal pairs.


[1] I.N. Soskov, *The $\omega$-enumeration degrees*, ***Journal of Logic and Computation***, to appear.

[2] I.N. Soskov, H. Ganchev, *The jump operator on the $\omega$-enumeration degrees*, ***Annals of Pure and Applied Logic***, to appear.


▶ KENTARO SATO, *Proof-theoretic Strength Results of Analogues of Small Large Cardinal Hypotheses in Second Order Systems.*


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Second-order set theory (class theory, or theory of classes and sets) attracted little attention after basic results had been obtained in the early age. Recently it has regained attention, from several perspectives (e.g., [1] from truth theory, and [2, 3] from operational set theory). Among them, to investigate the strengths of set-theoretic axioms in various situations, the speaker has been working on the comparison among several second-order or two-sorted frameworks (e.g., [4]).

The speaker has obtained several results on the proof theoretic strengths of systems extending the base theory **NBG** by those axiom schemata that can be seen as analogues of schemata considered in second order number theory (e.g., see [5]) and found that there are many dissimilarities.

This talk will focus on impacts of "large-cardinal-like" axioms on these results: For example, since König's

Lemma is provable in $\mathbf{ACA}_0$ (whose analogue is **NBG**), it seems interesting to consider what is changed if we add to the base theory the analogue of König's Lemma, while the analogue states that Ord is weakly compact, and is (proof-theoretically) far beyond Morse-Kelley set theory **MK**, the analogue of full second order number theory $\mathbf{Z}_2$.

[1] K. Fujimoto, *Classes and truths in set theory*, Published online in ***Annals of Pure and Applied Logic***.

[2] G. Jäger, *Full operational set theory with unbounded existential quantification and power set*, ***Annals of Pure and Applied Logic***, vol. 161 (2009), no. 1, pp. 33–52.

[3] G. Jäger and J. Krähenbühl, $\Sigma^1_1$ *choice in a theory of sets and classes*, ***Ways of Proof Theory***, (R. Schindler, editor), Ontos Verlag, 2010, pp. 283–314.

[4] K. Sato, *The strength of extensionality II: weak weak set theories without infinity*, ***Annals of Pure and Applied Logic***, vol. 162 (2011), no. 8, pp. 579–646.

[5] S. Simpson, ***Subsystems of Second Order Arithmetic***, Springer-Verlag, 1999.

▸ DENIS I. SAVELIEV, *On Zariski topologies on Abelian groups with operations.*
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We consider universal algebras consisting of an Abelian group endowed with operations (of arbitrary arity) satisfying the generalized distributivity law, i. e. such that the unary operations obtaining from them by fixing all but one arguments are endomorphisms of the group. Instances of such algebras include rings, modules, linear algebras, differential rings, etc. Given such an algebra $K$, a closed basis of the Zariski topology on its Cartesian product $K^n$ consists of finite unions of sets of solutions of equations $t(x_1, \dots, x_n) = 0$ for all terms $t$ of $n$ variables over $K$; it is the least $T_1$ topology in which all operations are continuous. We prove that for every such infinite $K$ and any $n$, the space $K^n$ is nowhere dense in the space $K^{n+1}$. A fortiori, all such $K$ are nondiscrete (this fact was previously established for commutative associative rings by Arnautov [1]). Our proof uses a multidimensional generalization of Hindman's Finite Sums Theorem, a strong Ramsey-theoretic result obtained via algebra of ultrafilters [2].

[1] V. I. Arnautov, *Nondiscrete topologizability of countable groups*, ***Doklady Akademii Nayk SSSR***, vol. 191 (1970), pp. 747–750.

[2] N. Hindman, D. Strauss, ***Algebra in the Stone–Čech compactification***, de Gruyter Expositions in Mathematics 27, Walter de Gruyter, 1998.

▸ ANTON SETZER, *How to reason coinductively informally.*
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This research was supported by EPSRC grant EP/G033374/1 "Theory and application of induction-recursion", and written as a visiting fellow of the Newton Institute, Cambridge University.

Whereas formally an inductively defined set is defined as a least fixed point, one rarely argues directly using this definition. Instead we use usually the induction rules derived from this principle. In fact we have developed a culture of informally arguing inductively by referring to the induction hypothesis, and often use extended induction principles such as course of value induction. Coinductively defined sets are greatest fixed points, however proofs about coinductively defined sets are usually either carried out by referring directly to the definition, or using game theoretic approaches. Therefore coinductive proofs appear to be quite complicated and are usually not taught in the early parts of a mathematics or computer science curriculum.

In the interactive theorem prover Agda, proofs by induction are given as recursive functions which pass a termination checker. The termination checker verifies that the induction hypothesis is used correctly. In a similar way coinductive proofs are given as well as recursive functions, passing the termination checker. The termination checker checks whether the recursive call, which we call the *coinduction hypothesis* is used correctly.

In this talk we will develop rules for coinduction in the same way as it is done for induction. We show how informal proofs by coinduction can be carried out by referring to the coinduction hypothesis in an appropriate way. As when referring to the induction hypothesis for inductive proofs the same care has to be applied when referring to the coinduction hypothesis in coinductive proofs.

We will illustrate this by showing how to carry out informal proofs by bisimulation.

▸ MICHAEL SHENEFELT, *Why did Symbolic Logic Emerge During the Industrial Revolution.*
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The Industrial Revolution, beginning in the late eighteenth and early nineteenth cen- turies, gave the world new conveniences, new factories, new cities, and new problems but also a new kind of logic.

Before the nineteenth century, farsighted thinkers had long toyed with the idea of a fully symbolic logic, but they had never turned any such project into reality. Only with the advent of large-scale manufacturing did symbolic logic finally take shape. The first fully symbolic systems were laid out by George Boole and Augustus De Morgan, both of whom published major books in England in 1847 just as England's Industrial Revolution was in full swing.

In fact, the correlation between industrialization on the one hand and the development of abstract algebras and symbolic logic on the other is close. Boole, De Morgan, and George Peacock (author of the influential Treatise on Algebra) all came from England during a period of intense industrialization, and, later in the century, the eminent figures of Gottlob Frege, Georg Cantor, and Richard Dedekind appeared in Germany just as Germany, too, industrialized. Giuseppe Peano perfected his axioms at the University of Turin at about the same time that the Automobile Factory of Turin (whose acronym in Italian is FIAT) built its first automobiles.

Is this correlation just coincidence or is it cause and effect?

In this paper, I shall argue that a key factor behind symbolic logic's growth was the Industrial Revolution itself. Specifically, the Industrial Revolution convinced large numbers of logicians and mathematicians of the immense power of mechanical operations. Whole generations witnessed this power, and out of these generations the logicians of the age were recruited. Boole, like other logicians, explicitly concerned himself with the mechanical.

Symbolic logic has had far-reaching effects, but behind its development was a powerful economic and social stimulus.

▸ DARIUSZ SUROWIK, *Knowledge and intuitionistic tense logic.*
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If we want to describe (from logical point of view) knowledge changing in time, we usually use for this purpose some combined logics. These combined logic usually combine some epistemic logic with some temporal logic (based on classical logic). However, it seems, that except of these systems, we can to describe knowledge changing in time in a language of intuitionistic tense logic.

In our speech we consider the intuitionistic tense logic $IT^K$. It is an extension of intuitionistic propositional logic with the temporal operators: $F, P, G, H$. Semantics for $IT^K$ is Kripke-style semantics. Basic notion of our semantics is a notion of *state of knowledge.* Intuitionistic negation and implication can be considered as a modalized classical negation and implication. So, we may to consider our intuitionistic tense logic as a logic of knowledge changing in time. However, in our language there is no explicite epistemic operator. Knowledge is not considered on syntactical level, but it is considered on semantical level, only. In our speech we give a semantics for $IT^K$. We also give sound and complete axiomatization with respect to proposed semantics.

Moreover, we prove, that Ewald's system $IK_t$ is included in the $IT^K$ ($IK_t \subseteq IT^K$.)

[1] van Benthem J., *The information in intuitionistic logic*, ***Synthese, 2009***, 167:2, pags 251-270

[2] Ewald W. B., *Intuitionistic tense and modal logic*, ***Journal of Symbolic Logic, 1986***, Volume 51, Nr 1.

[3] Surowik D., *Tense logic without the principle of the excluded middle*, ***Topics in Logic Informatics and Philosophy of Science, 1999***, Białystok

▸ ANDREW SWAN, *The failure of the existence property for CZF.*
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A theory, $T$ is said to have the *existence property* (sometimes called the *set existence property*) if for each formula $\phi(x)$ such that $T \vdash (\exists x)\phi(x)$, there is another formula, $\chi(x)$ such that $T \vdash (\exists! x)(\phi(x) \wedge \chi(x))$. The existence property is sometimes expected for constructive theories on the basis that the existence of mathematical objects should only asserted if they can be "mentally constructed." However, the existence property fails for some set theories regarded as constructive. In this talk we will show the new result that in fact the existence property fails for what is today the most widely studied constructive set theory, CZF.

The cause of this failure is the subset collection axiom schema. Subset collection can be regarded as a strengthened version of the exponentiation axiom that is validated by Peter Aczel's interpretation of set theory into Martin-Löf type theory. Because of this interpretation it can be regarded as predicative, as opposed to the much stronger power set axiom. We show that subset collection asserts the existence of a particular set of multivalued functions from Baire space to the naturals that cannot be defined from within CZF.

To prove this we define the notion of embedding one realizability interpretation into another. We will show that there are two realizability interpretations with essentially different witnesses of subset collection that can both be embedded into the standard McCarty style realizability interpretation of IZF.

▸ GIUSEPPINA TERZO, *On Shapiro's conjecture.*
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In 1958 Shapiro posed in [1] the following conjecture which comes out of complex analysis and involves solutions of system of exponential polynomials with only one iteration of exponentiation and with complex coefficients:

*If two exponential polynomials have infinitely many common roots, then they are both multiples of some third exponential polynomial.*

We give a positive answer to this conjecture over the complex field and more in general over an exponential algebraically closed field of characteristic 0, assuming Shanuel's Conjecture.

(Joint work with Paola D'Aquino and Angus Macintyre)

[1] H. S. Shapiro: *The Expansion of mean-periodic functions in series of exponentials*, ***Communications on Pure and Applied of Mathematics***, vol. 11, (1958), pp. 1-21.

▸ TINKO TINCHEV, *Modal approach to region-based theories of space: canonicity.*
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Region-based theories of space study properties of the regions—formal analog of the "bodies"—instead of abstract notions like points with respect to axiomatization, complexity of satisfiability problem etc. Usually the regions are taken to be the regular closed sets (or regular open sets) in a given topological space $\mathbb{T} = (T, \tau)$ from a class $\mathfrak{T}$ of spaces. Typical properties are contact, $n$-contact, internal (strong) contact, connectedness, $n$-connectedness, boundedness, convexity etc. For example, $n$ regions $A_1, \dots, A_n$ are in $n$-contact if they have a common point, $A_1 \cap \dots \cap A_n \neq \varnothing$. On the other hand, regular closed sets form a Boolean algebra under inclusion with bottom $0 = \varnothing$ and top $1 = T$. Normally, the first-order theories of this kind for the spaces (classes of spaces) which take attention are very complex. In contrast their universal fragments often allow formal handling and for practical spatial reasoning are good enough.

In the present talk we propose a sufficiently general condition for completeness with respect to the canonical model whenever the above mentioned universal fragments are treated as fragments of appropriate modal language.

▸ J.A. TUSSUPOV, *Categoricity and Complexity Relations over Structures With Two Equivalences.*
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We will consider the problems on algorithmic complexity of isomorphic and definable properties on models and connections with Scott families.
In paper [1] authors showed that for each computable ordinal $\alpha$ there is a structure that is $\Delta^0_\alpha$ categorical but not relatively $\Delta^0_\alpha$ categorical. This structure of the countable relational language. S.S. Goncharov [2] suggested the method of definability structure with countable computable set of predicates where arity of them bounded by finite number to the oriented graph such that categoricity is preserving. J.A. Tussupov [3] suggested the method of definability oriented graph to the bipartite graph, and to the structure with two equivalence such that categoricity is preserving for the computable successor ordinal $\alpha$. This construction is true for the limit ordinal $\alpha$.

Let $\mathcal{A}$ be a computable structure.

We say that $\mathcal{A}$ is $\Delta^0_\alpha$ *categorical* if for all computable $\mathcal{B} \cong \mathcal{A}$, there is a $\Delta^0_\alpha$ isomorphism from $\mathcal{A}$ to $\mathcal{B}$. We say that $\mathcal{A}$ is *relatively* $\Delta^0_\alpha$ categorical if for all computable $\mathcal{B} \cong \mathcal{A}$, there is a $\Delta^0_\alpha(\mathcal{B})$ isomorphism from $\mathcal{A}$ to $\mathcal{B}$.

A *Scott family* for $\mathcal{A}$ is the set $\Phi$ of formulas, with a fixed tuple of $\bar{c}$ in $\mathcal{A}$, such that 1) each tuple of parameters in $\mathcal{A}$ satisfies some formula $\varphi \in \Phi$, and 2) if both $\bar{a}$, $\bar{b}$ satisfy the same formula $\varphi \in \Phi$, then there is an automorphism of $\mathcal{A}$ mapping $\bar{a}$ to $\bar{b}$.
A *formally* $\Sigma^0_\alpha$ *Scott family* is a $\Sigma^0_\alpha$ Scott family that is made up of "computable $\Sigma_\alpha$" formulas.

Let $\mathcal{A}$ be a computable structure and $R$ be a relation on $\mathcal{A}$. We say that $R$ is *intrinsically* $\Sigma^0_\alpha$ if in all computable $\mathcal{B} \cong \mathcal{A}$ the image of $R$ in $\mathcal{B}$ is $\Sigma^0_\alpha$.

We say that $R$ is *relatively intrinsically* $\Sigma^0_\alpha$ if in all computable $\mathcal{B} \cong \mathcal{A}$, the image of $R$ is $\Sigma^0_\alpha(\mathcal{B})$.

We say that $R$ is *intrinsically* if for each automorphism $f$ of the structure $\mathcal{A}$ the image $f(R) \subseteq R$.

The structure $\mathcal{A}$ with binary predicate $P(x, y)$ is called the structure with bipartition binary predicates $P(x, y)$ if for the sets $K_1 = \{x : \mathcal{A} \models \exists y P(x, y)\}$ and $K_2 = \{x : \mathcal{A} \models \exists y P(y, x)\}$ satisfy the conditions: $K_1 \cap K_2 = \emptyset$ and $K_1 \cup K_2 \neq \emptyset$.
Let $\sigma_0 = \langle P^2(x, y)\rangle$ is signature with the bipartition binary predicates $P(x, y)$ and
$\sigma_0 = \langle E^2_0(x, y), E^2_1(x, y)\rangle$ is signature with two equivalences $E^2_0(x, y)$,$E^2_1(x, y)$.

Let $\mathcal{A}$ structure of signature $\sigma_i$, where $i = 0, 1$.

**Theorem 1.** *For each computable ordinal $\alpha$ there is a computable structure $\mathcal{A}$ of signature $\sigma_i$ that is $\Delta^0_\alpha$ categorical but not relatively $\Delta^0_\alpha$ (and without formally $\Sigma^0_\alpha$ Scott family).*

**Theorem 2.** *For each computable ordinal $\alpha$ there is a computable structure $\mathcal{A}$ of signature $\sigma_i$ with additional relation $R$ that is intrinsically $\Sigma^0_\alpha$ but not relatively intrinsically $\Sigma^0_\alpha$* on $\mathcal{A}$.

[1] J. Chisholm, E. B. Fokina, S. S. Goncharov, V. S. Harizanov, J. F. Knight, and S. Miller. *Intrinsic bounds on complexity and definability at limit levels.* ***J. of Symbolic Logic,*** Vol.74, No.3,2009, pp.1047-1060.

[2] Goncharov S. S., *Isomorphisms and definable relations on Computable Models*, ***Proceeding of the Logic Colloquium 2005, Athens***, pp..26–45

[3] J.A. Tussupov, *Isomorphisms And Algorithmic Properties Structures With Two Equivalences* ***Abstracts of Logic Colloquium 2011***, Barcelona, Spain, July 11-16, pp. 107-109.

▸ JEROEN VAN DER MEEREN, *Well partial orderings and recursively defined trees.*
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Well partial orderings play an important role in for example logic, mathematics and computer science [1]. They are the essential ingredient of famous theorems like Higman's lemma and Kruskal's theorem. The maximal order type of a well partial ordering is most of the time also the proof-theoretical ordinal of a specific theory $T$. There exists a general principle for computing the maximal order type of well partial orderings of recursively defined trees [2]. In this talk, I will introduce those recursively defined trees and discuss recent results of their maximal order types. These recursively defined trees are introduced for studying trees with a Friedman-style gap-condition [3].

[1] T. Becker, V. Weispfenning, in cooperation with H. Kredel, ***Gröbner bases: A computational approach to commutative algebra***, Graduate Texts in Mathematics (v. 141), Springer-Verlag, 1993.

[2] A. Weiermann, *A computation of the maximal order type of the term ordering on finite multisets*, ***Mathematical Theory and Computational Practice*** (5th Conference on Computability in Europe, Heidelberg, Germany, July 19-24, 2009), (K. Ambos-Spies, B. Löwe and W. Merkle, editors), vol. 5635/2009, Springer Berlin / Heidelberg, 2009, pp. 488–498.

[3] S. G. Simpson, *Nonprovability of certain combinatorial properties of finite trees*, ***Harvey Friedman's research on the foundations of mathematics***, Studies in Logic and the foundation of mathematics, (L. A. Harrington, M. D. Morley, A. Scedrov, S. G. Simpson, editors), Elsevier Science Publishers B.V., P.O. Box 1991, 1000 BZ Amsterdam, The Netherlands, 1985, pp. 87–117.

▸ ROMAN WENCEL, *Definable connectedness in the weakly o-minimal context.*
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Let $\mathcal{M} = (M, \leq, \dots)$ be a weakly o-minimal structure and denote by $\overline{M}$ the definable completion of $M$ with respect to $\mathcal{M}$, i.e. the set $M$ together with all non-rational cuts definable in $\mathcal{M}$. Clearly, $\overline{M}$ has a natural linear ordering and $M$ is a dense subset of $\overline{M}$.

A set $X \subseteq M^m$ definable in $\mathcal{M}$ is called definably connected if it is not a union of two disjoint non-empty definable open subsets of $X$. One can easily show that if $\mathcal{M}$ is o-minimal, then every set definable in $\mathcal{M}$ has finitely many definably connected components, and an image of a definably connected set by a continuous definable map is definably connected.

Unfortunately, such a notion of definable connectedness in general does not work in the weakly o-minimal context. There are easy examples of linearly ordered structures with weakly o-minimal theory whose universe is definably totally disconnected.

A partial solution to this issue was proposed by H. Tanaka in [1] for a class of weakly o-minimal structures with the strong cell decomposition property, introduced in [2]. H. Tanaka gives a definition of so called weak definable connectedness for sets definable in weakly o-minimal structures with the strong cell decomposition property. In the sense of [1] strong cells are weakly definably connected, which in particular implies that a set definable in a weakly o-minimal non-valuational expansion of an ordered group has finitely many weakly definably connected components.

In this talk I am going to discuss a variant of weak definable connectedness, which seems to be suitable for sets definable in models of arbitrary weakly o-minimal theories. Namely, one can show that sets definable in models of weakly o-minimal theories have finitely many weakly definably connected components.

Generalizing ideas concerning weakly o-minimal structures with the strong cell decomposition property, one can introduce a notion of a completion $\overline{X} \subseteq \overline{M}^m$ of a set $X \subseteq M^m$ definable in an arbitrary weakly o-minimal structure, which in case of strong cells coincides with the completion defined in [2]. We say that a function $f : X \longrightarrow M^n$ is strongly continuous if it has a (necessarily unique) continuous extension $\overline{f} : \overline{X} \longrightarrow \overline{M}^n$. Among other things, one can show that if $\mathcal{M} = (M, \leq, \dots)$ has weakly o-minimal theory, $X \subseteq M^m$ is a definable weakly definably connected set and a function $f : X \longrightarrow M^n$ is definable and strongly continuous, them $f[X]$ is weakly definably connected.

[1]H. Tanaka, *Weakly o-minimal structures and weakly definably connected*, ***Far East Journal of Mathematical Sciences***, vol. 34 (2009), no. 2, pp. 177–187.

[2]R. Wencel, *Weakly o-minimal non-valuational structures*, ***Annals of Pure and Applied Logic***, vol. 154 (2008), no. 3, pp. 139–162.

▸ DAN E. WILLARD, *On the Allowed Limited Permissible Extents in Which Various Different Self-Justifying Logics Can Formally Recognize Their Own Consistency.*
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We have published a series of articles during 2001-2009 about generalizations and boundary-case exceptions to the Second Incompleteness Theorem, including six papers in the *JSL* and*APAL*. (Citations to these articles and a formalism that both unifies and extends their techniques can be found in [2].) Our goal in this talk is to summarize the significance of the latter's unification formalism.

Our partial evasions of the Second Incompleteness can obviously elude the force of Second Incompleteness Theorem under only unusual extremal circumstances because the combined work of Pudlák, Solovay, Nelson and Wilkie-Paris implies that essentially all natural axiom systems, that merely recognize Successor as a total function, are unable to recognize their own consistency, when a Hilbert-style method of deduction is used. Our six journal articles and [2]'s unification formalism do show that boundary-case exceptions to the Second Incompleteness Theorem do exist when either:

1. The assumption that successor is a total function is dropped, in a context where Addition and Multiplication are treated as two 3-way relations (e.g. see [1])
2. Or when Addition is treated as a total function (e.g. as by "$\forall x \forall y \exists z \text{ Add}(x, y, z)$"), and the self-justifying system can recognize its consistency under a deduction method that lacks a Modus Ponens Rule, such as semantic tableaux,

It is obvious that Items (1) and (2) amount to being no more than being *Boundary-Case Exceptions* to the Second Incompleteness Theorem, in light of the aforementioned power of the Second Incompleteness Theorem. Our papers have diligently used the preceding italicized phrase, so as to avoid any possible confusion.

Our report [2] illustrates how such results are of epistemological interest because they explain how a Thinking Being can maintains at least some *instinctive* (partial) faith in its own consistency, despite the formidable barriers imposed by the Second Incompleteness Theorem.

[1] D. Willard, "A Generalization of the Second Incompleteness Theorem and Some Exceptions to It", *Annals of Pure and Applied Logic* 141 (2006) pp. 472-496.

[2] — , "A Detailed Examination of Methods for Unifying, Simplifying and Extending Several Results About Self-Justifying Logics", http://arxiv.org/abs/1108.6330.

▸ TIN LOK WONG AND RICHARD KAYE, *The model theory of generic cuts.*
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We study pairs of the form $(M, I)$ where $M$ is a nonstandard model of Peano arithmetic, and $I$ is a *cut* of $M$. Cuts have been extensively studied since the 1970s, mainly because of the relationship with independence results such as the Paris–Harrington theorem. However, surprisingly little about such pairs $(M, I)$ exists in the literature. We start filling this gap by investigating these pairs along the tracks of Robinson-style model theory. Arithmetic usually does not fit well into this theory, but it turns out that the *generic cuts*, a new family of cuts recently discovered by the first author, fit in rather nicely. Amongst other results, we showed that pairs $(M, I)$ with $I$ generic are existentially closed.

▸ MITKO YANCHEV, *Part restrictions in Description Logics: reasoning in polynomial time complexity.*
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Description Logics (DLs) are logical formalism widely used in knowledge-based systems for both explicit knowledge representation in the form of taxonomy, and inferring new knowledge out of the presented structure by means of a specialized inference engine ([1]). The representation language, called *concept language*, comprises expressions with only unary an binary predicates, called *concepts* and *roles*. In the semantics these are interpreted as subsets and binary relations respectively. With their syntax and interpretation various DLs can be viewed as syntactical variants or restricted fragments of some modal logics. Concept languages differ mainly in the constructors adopted for building complex concepts and roles, and they are compared with respect to their expressiveness, as well as with respect to the complexity of reasoning in them. The language $\mathcal{AL}$ is usually considered as a "core" one, having the basic set of constructors: $\neg A$ (atomic negation), $C \sqcap D$ (intersection), $\forall P.C$ (universal role quantification), and $\exists P.\top$ (restricted existential role quantification).

In the present talk we introduce new concept constructors, called *part restrictions*, capable to distinguish a part of a set of successors. These are $MrP.C$ and (the dual) $WrP.C$, where $r$ is an arbitrary rational number in (0,1), $P$ is an atomic role, and $C$ is a concept. The concept $MrP.C$ is interpreted by the set of all objects $x$ such that $\left|\{y \mid (x,y) \in P^{\mathcal{I}} \ \& \ y \in C^{\mathcal{I}}\}\right| > r\left|\{y \mid (x,y) \in P^{\mathcal{I}}\}\right|$. Part restrictions essentially enrich the expressive capabilities of Description Logics, and, as we show for a particular language, they do that with no extra cost of complexity.

We consider the language $\mathcal{ALP}^{\varepsilon}$ extending $\mathcal{AL}$ with limited part restrictions adopting only atomic and negated atomic concepts. We show that this language, while extending the expressive power of $\mathcal{AL}$, keeps the same P-time upper bound for the complexity of the main reasoning task in DLs—checking the subsumption between concepts. For, a completion calculus based on tableau technique is used.

[1] F. Baader, D. Calvanese, D. McGuinness, D. Nardi, and P. Patel-Schneider, editors. ***The Description Logic Handbook: Theory, Implementation, and Applications***, Cambridge University Press, New York, 2003.

▸ FAN YANG, *Implications in dependence and independence logic.*
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Dependence logic (**D**) [Väänänen, 2007] and independence logic (**Ind**) [Grädel, Vään-änen, 2011] are new logics incorporating the concept of dependence and independence into first-order logic. The compositional semantics of these logics are defined with respect to sets of assignments, called *teams*. Team semantics was originally introduced by Hodges [1997] for independence friendly logic [Hintikka, Sandu 1989]. Both dependence logic and independence logic have the same expressive power as existential second order logic [Galliani, Grädel, Kontinen,Väänänen].

The negation of neither of these two logics is classical; this fact therefore raises the question of how to define implications in these two logics. Basing on team semantics and the downwards closure property of dependence logic, Abramsky and Väänänen [2009] introduced intuitionistic implication ($\rightarrow$) and linear implication ($\multimap$) for dependence logic. In this talk, we show that on sentence level, dependence logic extended with these two implications ($\mathbf{D}^{[\rightarrow,\multimap]}$) have the same expressive power as the full second order logic [Yang 2010], while on the formula level, $\mathbf{D}^{[\rightarrow,\multimap]}$ characterizes exactly second order downwards monotone properties.

On the other hand, dependence logic is contained in independence logic [Grädel, Väänänen 2011], and independence logic can be, in certain sense, broken into two logics, namely inclusion logic (**I**) and exclusion logic (**E**) [Galliani 2011]. As independence logic does not have the downwards closure property, the intuitionistic implication or linear implication does not do the same job in independence logic as in dependence logic. In this talk, we introduce a maximal implication ($\hookrightarrow$) in the context of independence logic and show that on the sentence level, independence logic extended with maximal implication ($\mathbf{Ind}^{\hookrightarrow}$) has the same expressive power as the full second order logic (thus on the sentence level, $\mathbf{Ind}^{\hookrightarrow} = \mathbf{D}^{[\rightarrow,\multimap]}$). The same hold for **I** extended with $\hookrightarrow$ ($\mathbf{I}^{\hookrightarrow}$) and **E** extended with $\hookrightarrow$ ($\mathbf{E}^{\hookrightarrow}$) as well, namely on the sentence level

$$\mathbf{D}^{[\rightarrow,\multimap]} = \mathbf{Ind}^{\hookrightarrow} = \mathbf{I}^{\hookrightarrow} = \mathbf{E}^{\hookrightarrow}.$$

In addition, on the formula level, both $\mathbf{Ind}^{\hookrightarrow}$ and $\mathbf{I}^{\hookrightarrow}$ characterize exactly second order properties.

▸ XUNWEI ZHOU, *Meaningfulness—meaninglessness duality for distinguished sets* *.
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An empty set has nothing in it. A universal set represents the universe, it is ubiquitous. In mutually-inversistic set theory, an empty set and a universal set are called by a joint name distinguished sets. But in abstract set operations, distinguished sets are bound to yield, to be operated on. For example, the intersection of P and ∼P is the empty set ∅. In mutually-inversistic set theory, P∩∼P=∅ is called a quasi-set connection proposition. In mutually-inversistic set theory, there is the meaningfulness—meaninglessness duality for distinguished sets: distinguished sets occurring in quasi-set connection propositions and power sets are meaningful, occurring elsewhere are meaningless. Meaningless distinguished sets correspond to proper classes in axiomatic set theory. Mutuall-inversistic set theory is logical-mathematical paradox-free. It is free from Russell's paradox, because x∉x is a universal set, a meaningless distinguished set. It is free from the greatest ordinal number paradox, because the set of all ordinal numbers is a universal set, a meaningless distinguished set. It is free from the greatest cardinal number paradox, because the set of all sets is a universal set, a meaningless distinguished set. There are logical-mathematical paradoxes in naïve set theory. Axiomatic set theory is logical-mathematical paradox-free, but is complex. It has to construct sets and classes in parallel. Mutually-inversistic set theory is logical-mathematical paradox-free, and is simpler than axiomatic set theory. It need not introduce classes.