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Kreisel differentiated between two forms of computability: machine-effective computability and human-effective computability (Kreisel, 1972). In his view, it has been established that machine-effective computability can be analysed in terms of Turing machines, so that the Church-Turing thesis holds for machine-effective computability. Kreisel believes that it is difficult to give a precise characterisation of the notion of human-effective computability because it is unclear what the right idealisations are for this notion. However, he does say that the notion of human-effective calculability is "analogous" to the notion of informal provability.

In this talk, I propose that something like Kreisel's notion of human-effective computability can be and has been formally investigated in the framework of Epistemic Arithmetic (Shapiro 1985). In this framework, an informal provability operator ($\Box$) is added to the first-order language of arithmetic. The operator $\Box$ is intended to express the notion of a priori knowability, and is governed by the laws of $S4$ modal logic. Then the proposed formalisation of the human-effective computability of a function expressed by a formula $\phi(x, y)$ in the language of Epistemic arithmetic is:

$$\Box\forall x\exists y\Box\phi(x, y).$$

This then gives rise to Church's Thesis for human-effective computability, which is in the literature known as ECT:

$$\Box\forall x\exists y\Box\phi(x, y) \rightarrow \text{"}\phi \text{ is Turing-computable "}.$$

The principle ECT has certain interesting consequences. For instance, it entails that there are absolutely undecidable propositions of low arithmetical complexity. Thus we obtain an analogue of Gödel's disjunctive thesis (either the human mathematical mind is not a Turing machine or there are absolutely undecidable propositions):

Either ECT is false, or there are absolutely undecidable propositions.

Thus it would be of philosophical interest to know whether ECT is true. I will argue that, unfortunately, we have at present no convincing evidence for or against the truth of ECT, so in this respect we are at present no better off than with Gödel's disjunction.

[1] Kreisel G., *Which number theoretic problems can be solved in recursive proggressions on Π^1_1-paths through* $\mathbf{O}$*?*, ***Journal of Symbolic Logic***, vol.37 (1972), pp. 311–334.

[2] Shapiro, S., *Epistemic and intuitionistic arithmetic*, ***Intensional mathematics*** (S. Shapiro, editor), North-Holland Oxford, 1985, pp. 11–46.