▸ ALBERT VISSER, *Provability logic and the arithmetics of a theory.*
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We propose a particular way of viewing theories. We look at theories as a class of interpretations of a given weak arithmetical theory (like S^1_2). Consider a theory U. We view the interpretations of the given weak arithmetical theory in U as 'occurrences' of that given theory in U.

We will call an interpretation N of the given weak theory in U *an arithmetic of* U. The arithmetics of U have a natural ordering, the (definable) initial embedding ordering $\preceq$.

From the perspective of theories as containers of (possibly) lots of arithmetics, we study the provability logics of theories. We fully characterize the propositional modal principles for provability that hold in all arithmetics in a given theory U. The only assumption being a constraint on the complexity of the set of axioms of U. The comparatively easy success of this characterization contrasts with the remaining great open questions of provability logic concerning the provability logics of theories like S^1_2 or $\mathrm{I}\Delta_0 + \Omega_1$ (for a fixed arithmetic given by the identical embedding interpretation).

We provide an example of a theory U where the provability logic of U is not assumed at any arithmetic N in U. The idea of the example is very simple, but to verify its correctness requires some work and some theory. We need a sharpened version of a theorem independently due to Harvey Friedman and to Jan Jan Krajíček. Very roughly, this theorem says that $\preceq$-below any arithmetic N of a finitely axiomatized sequential theory, there is an arithmetic M that is Σ_1-sound. We explain the formulation of the theorem and the methods needed to prove it.