Statistical Physics of Communicating Processes

Vincent Danos

U of Edinburgh, CNRS

SynThsys Centre

introduction

what?

What: we want to write Newton's equations of motion (or the Hamiltonian) of a set of communicating processes:

$$\begin{pmatrix} x \\ v \end{pmatrix} \mapsto \begin{pmatrix} x + v\,dt \\ v + F/m\,dt \end{pmatrix} = \begin{pmatrix} x + v\,dt \\ v - 1/m\,\partial_x V\,dt \end{pmatrix}$$

where V is the potential

Actually, the state space of CCS processes is not continuous, so velocity undefined[1]; so we use a Metropolis form.

Metropolis?

Suppose given: a potential on X, that is to say a function $V : X \to \mathbb{R}$; and a symmetric graph G on X with finite out-degree. If X is finite, one can always find a rate function q with support G for which $\pi_V(x) := \exp(-V(x))$ is an equilibrium.

For instance, for any transition (x, y) in G, we set $q(x, y) := 1$ if $V(x) \geq V(y)$, $q(x, y) := \exp(V(x) - V(y))$ else.

The first clause says that one is always willing to travel 'downhill', while the second says that one is increasingly reluctant to travel 'uphill'. This $|q|$ is clearly symmetric, and one can readily see that Detailed Balance holds:

$$q(y, x)/q(x, y) = e^{V(y)-V(x)} = \pi_V(x)/\pi_V(y) \tag{1}$$

what? (2)

borrow from stat phys
    distributed CT Metropolis

build a potential energy function to drive kinetics

NB: lower energy/higher probability

why?

Why: 1) convenient: generates implicitely a quantitative dynamics (so we do not have to describe the actual behaviour, it follows), 2) conceptual: decentralized computation as physical dynamics, 3) analytic tool: energy constraint is a structuring constraint, one can use that for analysis[1] 4) opportunity: quantize (make quantitative) rCCS so that we can start really computing and perhaps talk about efficiency and learning[2] 5) *post hoc* reason: nice result.

outline

- processes CCS/reversible processes rCCS
- quantizing rCCS
- concurrent and convergent rCCS potentials
- a solution/sufficient condition

1

processes CCS/reversible processes rCCS

CCS the idea

minimal model

processes can fork
and synch on multiset of channels
(predefined)

Robin Milner circa 1980

rCCS the idea

two aspects in solving a distributed problem:
- local steps towards a solution
- backtracking (deadlock escape)

centralized case: can try to always make progress to solution, but NP!

decentralized case: one has to!

NB: decentralization = for efficiency or given granularity

rCCS the idea (2)

set backtrack in the infrastructure
code easier to prove and understand

universal backtrack strategy

$$p \quad \to \quad \Gamma \cdot p$$

i.e., add history to a process

rCCS results

universal cover property

distributed history characterizes traces up to concurrent moves

syntax-independent history construction

(eg works for Petri nets, pi-calculus)

weak-bisimulation

rev(p) + irreversible actions / causal transition
system(p) - only irreversible actions observable

Jean Krivine

Pawel Sobocinski

2004-2006

Ulidowski et al, Stefani et al.

beyond weak-bisimulation

∞-hesitation, no efficiency measure

need to probabilize rev(p)

exhaustivity of backtrack as probabilistic equilibrium

which probabilistic structure?

borrow from stat phys
distributed CT Metropolis

build a potential energy function to
drive kinetics

concurrent & convergent

2

the reversible CCS transition system

csq on possible potentials

reversible communicating processes

memory

fork

$$\Gamma \cdot (p_1, \ldots, p_n) \rightarrow^f \Gamma 1 \cdot p_1, \ldots, \Gamma n \cdot p_n$$

synch on a1, ..., am

$$\Gamma_1 \cdot (a_1 p_1 + q_1), \ldots, \Gamma_m \cdot (a_m p_m + q_m) \rightarrow^s_{\vec{a}}$$
$$\Gamma_1(\vec{\Gamma}, a_1, q_1) \cdot p_1, \ldots, \Gamma_m(\vec{\Gamma}, a_m, q_m) \cdot p_m$$

with a unique naming scheme and enough info to reverse uniquely

what can we say about the generated TS?

symmetric TS (so strongly connected)

"simplicity" of TS: at most one jump

(slight pb with sums)

acyclic up to concurrent moves

countable state space (recursion)

from potential to dynamics (CTMC)

potential/rate ratio constraint

$$q(y,x)/q(x,y) = e^{V(y)-V(x)} = \pi_V(x)/\pi_V(y)$$

$$\sum_X e^{-V(x)} < \infty$$

definition of convergence

explosive growths

		event horizon	nb
$q \to^{f}$	$0 \cdot p(a), 1 \cdot p(\bar{a})$	$1, 1$	1
$\to^{fs}$	$0a0 \cdot p(a), 0a1 \cdot p(a), 1\bar{a}0 \cdot p(\bar{a}), 1\bar{a}1 \cdot p(\bar{a})$	$2, 2$	2
$=$	$0a0 \cdot a(p(a), p(a)), 0a1 \cdot a(p(a), p(a)),$		
	$1\bar{a}0 \cdot \bar{a}(p(\bar{a}), p(\bar{a})), 1\bar{a}1 \cdot \bar{a}(p(\bar{a}), p(\bar{a}))$		
$\to^{fs}$	$0a0a0 \cdot p(a), 0a0a1 \cdot p(a), 0a1a0 \cdot p(a), 0a1a1 \cdot p(a),$	$4, 4$	$4!$
	$1\bar{a}0\bar{a}0 \cdot p(\bar{a}), 1\bar{a}0\bar{a}1 \cdot p(\bar{a}), 1\bar{a}1\bar{a}0 \cdot p(\bar{a}), 1\bar{a}1\bar{a}1 \cdot p(\bar{a})$		
$\ldots$			
$\to^{fs}$	$\prod_{w \in 2^k} 0w(a) \cdot p(a), \prod_{w \in 2^k} 1w(\bar{a}) \cdot p(\bar{a})$	$2^k, 2^k$	$2^k!$

is there a potential that controls the above?

upper bound on the number of such (entropy)

lower bound on energy of a deep state

3

construction of a potential

V_1 : total stack size potential

$$V_1(p_1, \ldots, p_n) = V_1(p_1) + \ldots + V_1(p_n)$$
$$V_1(\Gamma \cdot p) = V_1(\Gamma i) = V_1(\Gamma)$$
$$V_1(\Gamma(\vec{\Gamma}, a, q)) = V_1(\Gamma) + \epsilon_{\vec{a}}$$

$$\vec{\epsilon} \cdot \tilde{\Gamma}(p)$$

inner product of the vector of communication cost and history

V_1 : energy deltas

$\Delta V_1 = (n-1)V_1(\Gamma)$ n-ary fork with memory Γ

$\Delta V_1 = m\epsilon_{\vec{a}}$ synch on $\vec{a}$

ratio constraint ✓

$$k_f^- = 1$$
$$k_f^+ = e^{-(n-1)V_1(\Gamma)}$$

$$k_{\vec{a}}^- = 1$$
$$k_{\vec{a}}^+ = e^{-m\epsilon_{\vec{a}}}$$

V_0 : total synch potential

Given a path γ from $\varnothing \cdot p_0$ to p:

$$V_0(p) \quad = \quad \sum_{\vec{a} \in A^\star} \sum_{x \to_{\vec{a}}^{s} y \in \gamma} (-1)^{v(s)} \epsilon_{\vec{a}}$$

$\boxed{\text{ratio constraint}}$

$$k_f^- = k_f^+ \qquad\qquad k_{\vec{a}}^- / k_{\vec{a}}^+ = \exp(\epsilon_{\vec{a}})$$

V_0 vs V_1

V_1 is truly concurrent
= sensitive to sequential expansion

V_0 < or equal to V_1

potentially more divergent

No matter how costly a synch, V_0 diverges

What about V_1 ?

4

epilogue

bounds

upper bound on entropy

Lemma *For large ns,* $\log|T(n)| \leq \beta_+ \alpha^2 O(n \log n)$

lower bound on energy

Lemma *Suppose* $\beta_- > 1$, $\epsilon_m > 0$, $p \in \Sigma_n(p_0)$*:*

$$\frac{\epsilon_m}{\log 4 + \log(\beta_+ + 1)} \cdot n \log n \leq V_1(p)$$

sufficient condition for equilibrium

Proposition 1 *Suppose* $1 < \beta_-$, *and* $\beta_+\alpha^2 \log(4(\beta_+ + 1)) < \epsilon_m$, *then:*

$$Z(p_0) \quad := \quad \sum_{p \in \Omega(p_0)} e^{-V_1(p)} \quad < \quad +\infty$$

Nicolas Oury

Giorgio Bacci (Udine)

Ohad Kammar

David Mark

discussion

simulated annealing with "local" temperatures

$$k_f^- = 1$$
$$k_f^+ = e^{-(n-1)V_1(\Gamma)}$$

energy as syntax

self-organised energy-based dynamics

$$\operatorname{argmax} \vec{\epsilon}. \sum_{p\in\partial X} \pi(\vec{\epsilon}, p) = \int \mathbf{1}_{\partial X}\, d\pi$$

discussion (2)

the bounds are sharp but ...

control growth rate/use specialized potentials?

what with irreversible actions?

other potentials?

work with general steady states?

work with non-universal covers

what kind of problem?

implementation?