Schrödinger (1944) and Turing (1952) on the Logic of Life: from the “coding” to the “genesis” of organization and forms.

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A comparison:

- **E. Schrödinger**, *What is life?*, Part I *vs.* II, 1944;
- **A. Turing**, the *Logical* or *Discrete State Machine*, 1936-1950 *vs.* 1952 (on Morphogenesis)
- Hints to recent work

Schrödinger, What is life? (1944), part I

- « All the physical and chemical laws … in the life of organisms are of a **statistical kind**; any other kind of lawfulness and orderliness … is being perpetually disturbed and made inoperative by the unceasing heat motion of the atoms.
- …. incredibly **small groups of atoms**, much too small to display exact statistical laws, do play a dominating role in the **very orderly and lawful** events within a living organism. » (Chap. 6)

A contradiction?

No, understanding **variability**, a blend of *always sligthly disordered order*.

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My key point: **randomness** *implies* **variability** *implies* **diversity** *a component of* **structural stability** (« order and lawfulbess »)

Schrödinger, What is life? (1944), part I

- Remarks on chromosomes and drosophila eye colour

  «We call … “locus”, or, if we think to the hypothetical material structure which serves as support of it, a “gene”. In my view, the fundamental concept is more the ***difference of properties*** than the property it-self »

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The (great) physicist attitude: propose **general principles** (Galileo’s gravitation and inertia… geodetics principles).

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- *Chromosomes*: a **code-script** *for a form* (encoded *information*?)

  « In calling the structure of the chromosomes a *code-script*, we mean that the all-penetrating mind, **once conceived by Laplace**… could tell from their structure how the egg would develop… . »

Schrödinger’s ***right consequences*** of his principles!

Today, the code-script has been fully decoded …

Laplace's Determinism?
Poincaré … Schrödinger and Turing

Physical Determination (Classical)

Laplace's view:

A) determination *implies* predictibility

and

B) determination =/= randomness

[Laplace, Philosophie des Probabilités, 1786]

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*[**J. Monod**, Le **hasard** et la **nécessité**, 1970]*

Consequences of the **Laplacian view**:

Any *predictable determination* is **programmable**

(the "DNA is a program" theory and its "*Central Dogma*")

Next: programmable *is equivalent to* deterministic predictable

(since: **unpredictable** *implies* **algo-random** ...)

Schrödinger, 1944, *part II*: **the alternative view**

Schrödinger**,** the other hinted proposal:

« … let's try to hint to the possible meaning of the principle of **entropy** at the **global scale** of a living organism, ***while forgetting*** *for the time being all what we know on* ***chromosomes*** »

Schrödinger's suggestions:

Consider negative entropy as part of *Gibbs Free Energy* (available work): **G = H - TS**,

where enthalpy H = U + PV (U = internal energy)

(negative) entropy as (part of)

Gibbs Free Energy" ≠ Shannon's information !

(it is defined by different principles)

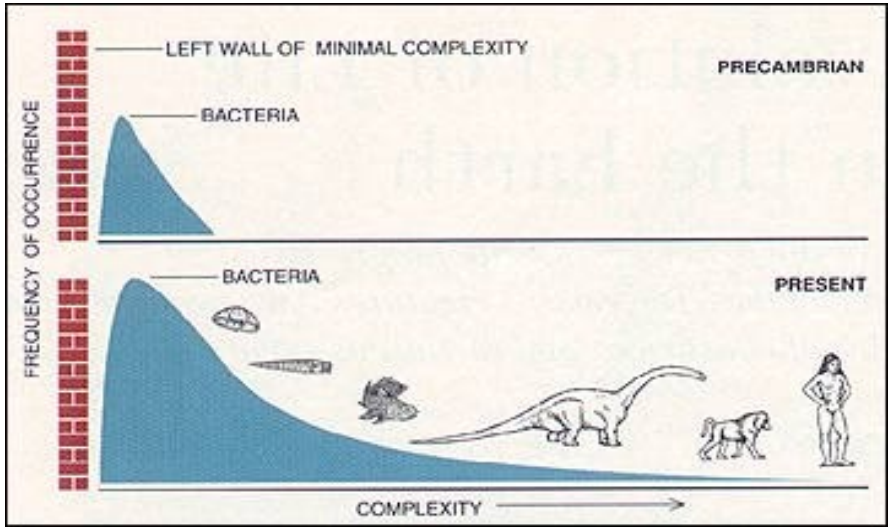
Recent work

Our approach: A new observable, following Schrödinger, part II:

A quantifiable notion of phenotypic **complexity** as ***anti-entropy***

The result:

model Gould's remarks **on increasing phenotypic complexity** along *evolution* (as "diffusion" of bio-mass over complexity).



F. Bailly, G.Longo. *Biological Organization and Anti-Entropy*.
In **J. Biological Systems**, Vol. 17, No. 1, pp. 63-96, 2009.

Turing's "part II": the alternative view

Turing (1936, 1950) *vs.* 1952:

a radical change of perspective and tools for knowledge

Alan M. Turing (1952) "The Chemical Basis of Morphogenesis",
Philo. Trans. Royal Soc., B237, 37-72.

Some hints already in the **1950** paper (The imitation game), *next* …

Turing '50 (The imitation Game): *hints to (remarkable) mathematical physics*

Turing, 1950:

- *from* the **Logical Computing Machine** (LCM, 1936), a “man in the act of computing”,
- *to* the **Discrete State Machine** (DSM, 1950), as a *physical* device

Turing '50 (The imitation Game): ***hints to (remarkable) mathematical physics***

Turing: a DSM is ***laplacian*** (i. e. determination implies predictability)

Turing '50 (The imitation Game): *hints to (remarkable) mathematical physics*

Turing: a DSM is ***laplacian*** (i. e. determination implies predictability)

While: "The system of the 'universe as a whole' is such that quite **small errors in the initial conditions** can have an overwhelming effect at a later time.
The displacement of a single electron by a billionth of a centimetre at one moment might make the difference between a man being killed by an avalanche a year later, or escaping.
It is an essential property of the mechanical systems which we have called **'discrete state machines' that this phenomenon does not occur**. Even when we consider the actual physical machines instead of the idealised machines … "

[Later: ***Measurement*** and *prediction*: in practice/ in principle]

Turing '50 (The imitation Game): *hints to (remarkable) mathematical physics*

The Brain ? Beyond Logic and the imitation

"The nervous system is ***certainly not*** a discrete-state machine [DSM]. A ***small error*** in the information about the size of a nervous impulse impinging on a neuron, may make a large difference to the size of the outgoing impulse …." (the issue of measurement)

"In the nervous system chemical phenomena are at least as important as electrical."

Turing ‘52: Morphogenesis

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Turing ‘52: Morphogenesis

A ***model*** of morphogenesis by “action/reaction/difusion”:

- a set of partial differential equations describing a continuous system (tissue – medium -, space, time …)
- (the linear approximation of) a dynamical system highly sensitive to initial conditions (“**the exponential drift”**, p. 43).

“This **model** will be a simplification and an idealization, and consequently a falsification.” *Not an “**imitation**”*

$$\begin{cases} \frac{\partial u}{\partial t} = D_u \Delta u + \frac{ru^2}{v} - \mu u + r, \\ \frac{\partial v}{\partial t} = D_v \Delta v + ru^2 - \alpha v, \end{cases}$$

Chromosomes, genes and the emergence of forms

Turing's "morphogens" are **chemical components** of an action/reaction/diffusion system.

On "genes" (and chromosomes):
« The function of genes is presumed to be **purely catalytic** …
The genes might thus be said to influence the anatomical form of the organism by determining the **rates of those reactions** which they catalyze. » (Turing, 1952, p. 38)

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A very **different role** from carrying ***information*** and ***coding***.

As *reported by Gandy* (Hodges, 1983):
Turing is against the "**argument from design**"

Extended references to D'Arcy Thompson and Waddington, the british "**emergentist" school** (biological forms *emerge*).

Exponential drift

"The investigation is chiefly concerned with the **onset of instability**"

"Such a system, although it may originally be quite homogeneous, may later develop a pattern or structure due to an instability of the homogeneous equilibrium, which is **triggered off by random disturbances"** (Turing, 1952, p. 37)

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"… the presence of irregularities, including **statistical fluctuations** in the numbers of molecules undergoing the various reactions, will, if the system has an appropriate kind of instability, result in this homogeneity disappearing". p. 42.

"Thus there is an **exponential drift** away from the equilibrium condition. It will be appreciated that a drift away from the equilibrium occurs with almost any small displacement from the equilibrium condition". p. 43 [Gordon et al.: unstable equilibrium]

Catastrophic instability

“ …some qualitative conclusions about the effects of non-linear terms.

… it would result in the amplitude becoming infinite in a finite time. This phenomenon may be called '**catastrophic instability**'.....”

(this may halt the growth; a *critical transition*?) (Turing, ‘52, p. 58-59)

“The set of reactions chosen is such that the instability becomes 'catastrophic' when the second-order terms are taken into account, i.e. the growth of the waves tends to make **the whole system more unstable than ever**”. p. 64

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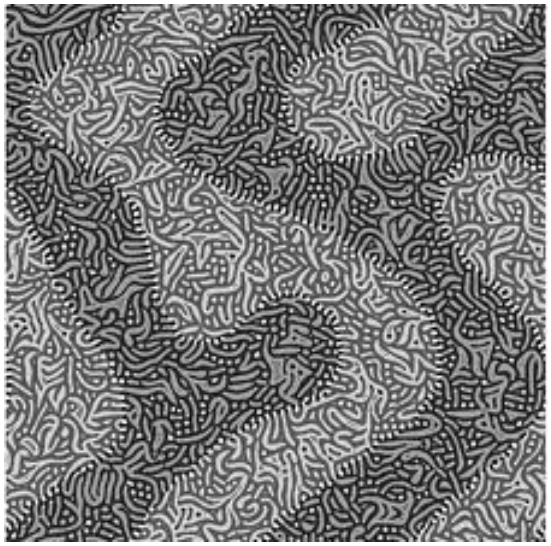
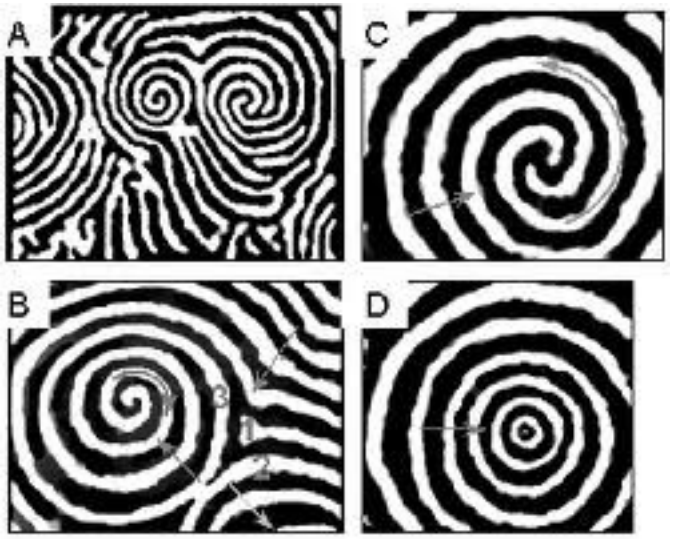
In general: “differential equations for spread of morfogen in a ring produce standing wave forming a whorl”.

Instability, **noise** … “determine” their forms.

“Just” a **material** (*hardware*) **dynamics** of forms: ...

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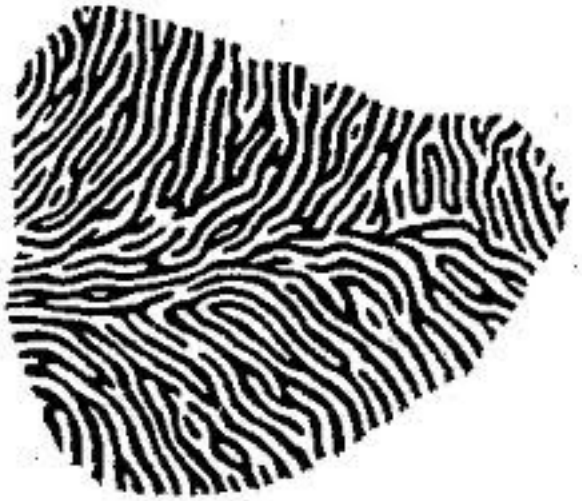

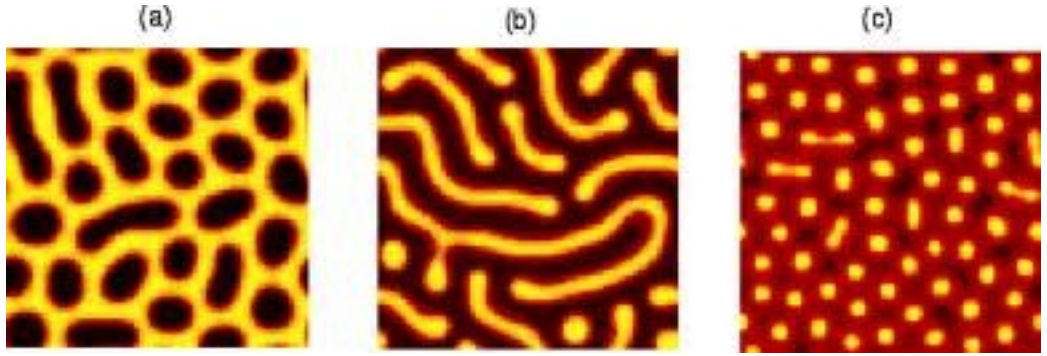
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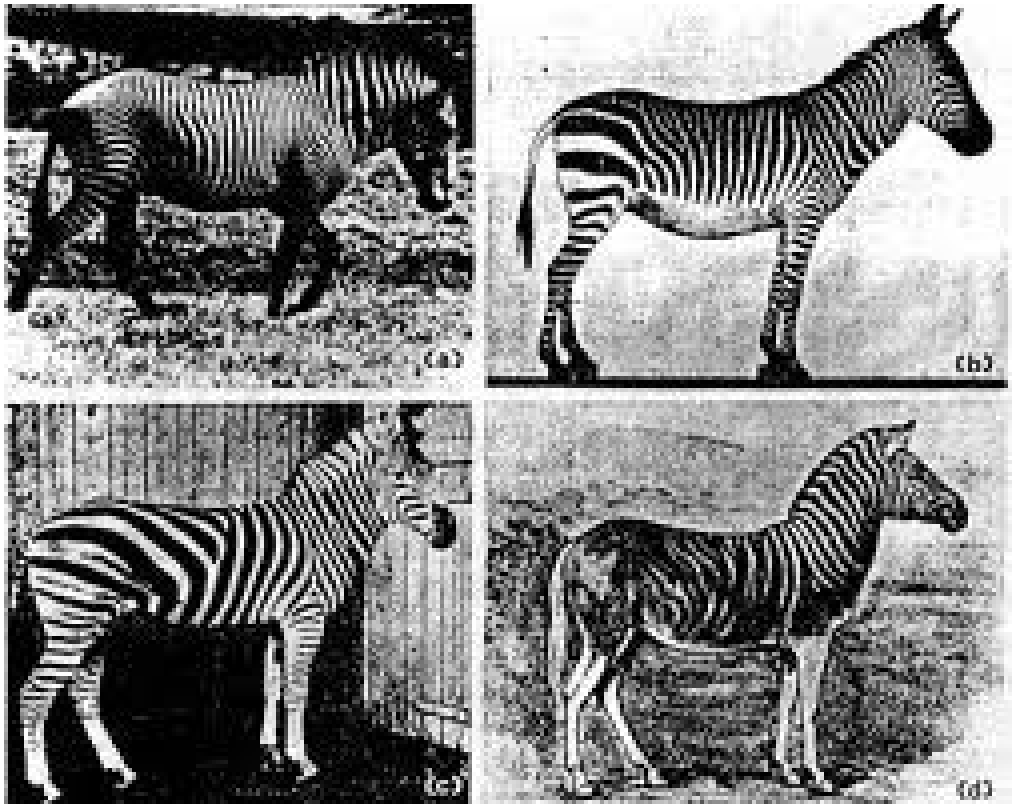
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(d)

Turing's Morphogenesis: key aspects

1 – The role of **Instable Equilibria**:

Instabilities in action-reaction-diffusion processes lead to differentiation of spatial patterns by *symmetry breakings*

2 – The role of **randomness**:

Initial random concentration of chemical morphogens are "amplified" by the dynamics:

E. g. two cells, with *nearly* the same amount of a morphogen inside, end up, by proliferation, with *very different* concentrations (approximation, measurement)

"This **breakdown of symmetry** or homogeneity may be illustrated by the case of a pair of cells originally having the same, or very nearly the same, contents … [yield] an **exponential drift** away" ('52, p. 42-3).

(Today's tentative extensions to *cell differentiations:* **Gordon**, 2011)

Turing ‘52: Morphogenesis

In Turing’s analysis, **continuity** of *models* crucially steps in:
- **approximation** (an open interval of *measurement* or of the initial/border conditions)
- various forms of *instability*, *criticality*, *symmetry breakings* …

Key issue: Discret (space-time) dynamics *are not* an approximation of *non-linear* continuous dynamics.

“It might be possible, however, to treat a *few particular cases* in detail with the aid of a digital computer.
The essential disadvantage of the method is that one only gets results for particular cases” (Turing, 1952, p. 72)

Sensitivity of the dynamics implies divergent trajectories, *yet* …

Today's Shadowing Theorem: the "reverse" approximation

Computational problem: the **round-off**

Shadowing Theorem for hyperbolic dynamical systems (D, f)

For any x_0 and δ there is an ε such that, for any ε-approximated (or rounded-off $\leq \varepsilon$) trajectory, there is one in the continuum which goes δ-close to it, at each step.

Informally:

Given a "sufficiently regular" non-linear iterated function system, *any discrete (space-time) trajectory* can be actually approximated by a continuous one **(but, in general *not the converse!)***

Or … *there are so many continuous trajectories*, that, given a discrete trajectory, you can find a continuous one which goes close to it, see:

Pilyugin, S.Yu. (1999). *Shadowing in Dynamical Systems*.

Lecture Notes in Math. 1706, Springer-Verlag, Berlin.

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Key mathematical *dualism*: **software / hardware; signs / meaning**

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Back to Turing's Catastrophic instability

1. Basic ideas for
(**Physical) Criticality**

Back to Turing's Catastrophic instability

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Next: symmetry breakings in ***critical transitions*** : …

Mathematics and Physics
For Critical Transitions, first

Critical Transitions:

- **Change of symmetry /change of object**
- **Global correlations**

Causality and complexity

(Classical) Physics:

Causal relations are local; global only in the sense of a field (by propagation of *local* interactions; i. e. by transitivity) or of the global determination (by equations over *global* variables; cf. Quantum Physics: *entanglement*).

Biology:

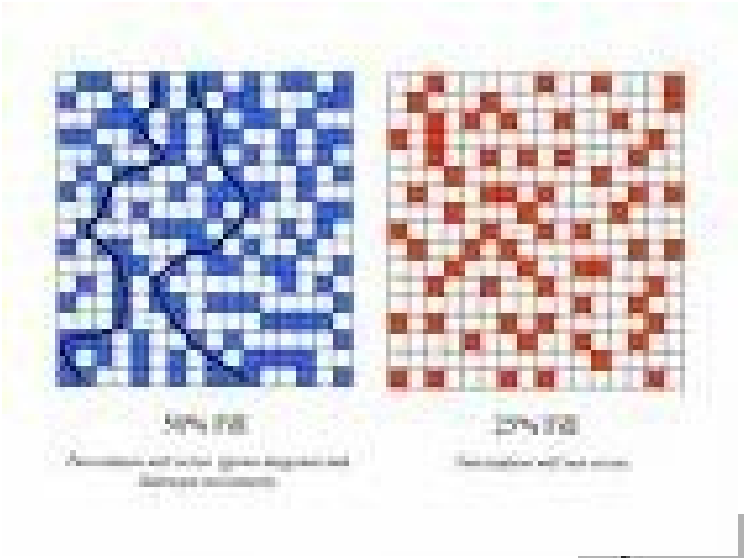
local causality may *differ* radically from **global** correlations, yet it *cannot be isolated* from the latter: integration and regulation, typically, *causally affect* local interactions

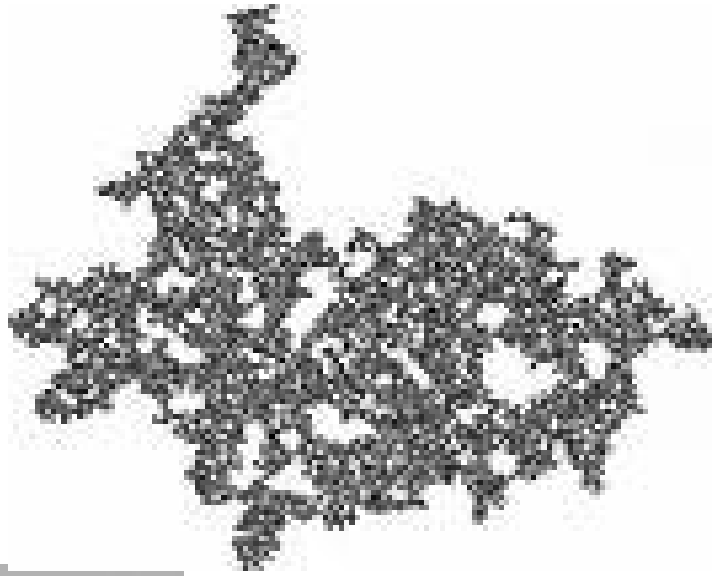
(e.g. local bio-chemical exchanges regulated by cascades of hormones or neural signals of a different *theoretical* nature).

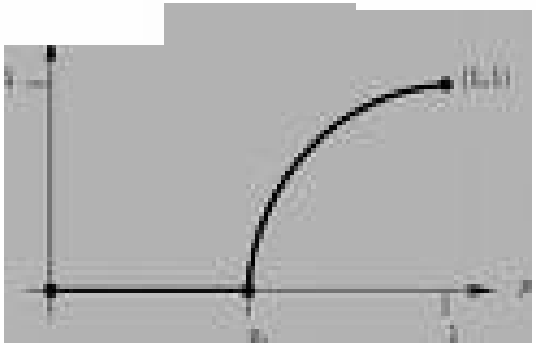
Possible connection, since the ‘80s:

The Physics of **Criticality**: the formation of global “Structures of Coherence” in critical transitions (*singularities*).

Percolation:



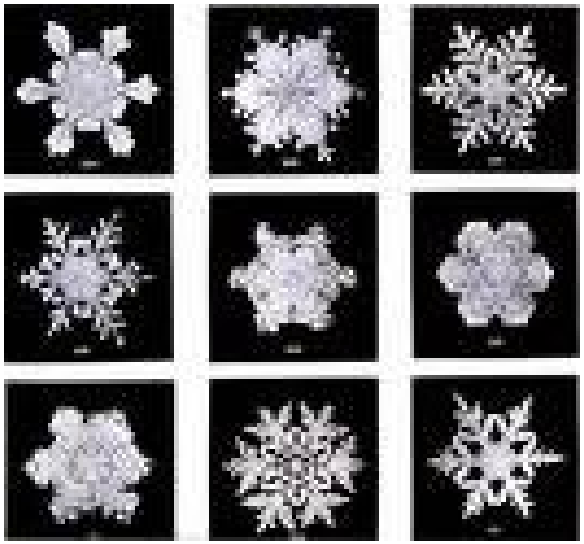




Signature: divergence of derivatives.

“Fractal” autosimilarity: New Symmetries

Snowflakes



- *From* themodynamic potential (potential molecular links),
- *to* a ***sudden*** and ***local,*** **decrease** of entropy

  (in view of a slow-down of the brownian motion)
- **Different symmetries** (sensitivity)

In short: a critical state, in *Physics*

- An **isolated** point: a **singularity** in a process (or a bifurcation or a Thom's *catastrophe*, if irreversible: cf. *Turing*!).

- **Long range correlations:** *minor fluctuations*, possibly below the level of observability, may lead towards radically different evolutions (cf. *Turing*).

- ***Different*** *from* "being far from equilibrium" (which does not imply *possibly different evolutions* of the system (bifurcations))

Symmetries in critical transitions, in *Physics*

The *Mathematics* of criticality:

> A (critical) **phase transition** is a "***point-wise*" symmetry change** (breakings/formation of symmetries): essential for the mathematics of renormalization.

The **renormalization semigroup** gives the passage between different correlation lengths (a "*flow in a set of models*" [Lesne, '08]):

> at the infinite limit, *it yields a fixed point* and the system is ***closed under scale invariance*** (this **symmetry** is broken away from the critical point)

Key point:
Once the point-wise transition is made, ***symmetries stabilize***

The Kosterlitz-Thouless transition: a marginally critical interval (limit critical / not critical), correlations at all scales, but no symmetry changes.

Nicolis G., Prigogine I., **Self-organization in non-equilibrium systems: from dissipative structures to order through fluctuations**, Wiley, N. York, 1977.

(cascades of bifurcation, as transitions towards chaos)

Bak P., C. Tang C. and Wiesenfeld K. "Self-organized criticality". **Physical Review** A 38: 364--374, 1988.

(the sand pile: a critical angle)

Lesne A. **Regularization, renormalization, and renormalization groups: relationships and epistemological aspects**, World Sci., London, 2008.

(renormalization as symmetry properties; a flow in the space of models)

This is (very interesting) ***Physics*** …

Towards Biology:

Kauffman S.A., **The origins of Order**, Oxford U. P., 1993.

*("order for free"; **metabolic networks**)*

Thesis *(on symmetry changes)***:**
Life as *extended critical transition* (a physical singularity)

- ***Life*** is (not only) a dynamics, a process, but an **extended** (permanent, ongoing … in time, space ..) **critical transition**

- A critical **interval**, not just a (mathematical) point

- Key understanding: **continual symmetry changes**

- Mathematical challenge: in Physics, both genericity of objects and *determination* (specificity) of trajectories ***are given by symmetries***

Thesis (the role of randomness): a **random** event is (always) correlated to a **symmetry breaking.**

More on the *Mathematics* of criticality

Physical Criticality: *consequences of the* ***point-wise*** **symmetry changes**:

- **Sensitivity:** *infinitesimal* variations yield *finite* changes (or *finite* variations lead to *infinite* changes, mathematically)
- instantaneous process as **divergence** of some functional descriptions (non-analyticity w. r. to the intended parameters);
- **maximal complexity** (fixed point for renormalization), which also yields instability.

Extended Criticality, in Biology:

1. **Each point** is a **critical transition**, thus a symmetry change where **1, 2** and **3** as above also hold;
2. Critical points: **a dense subset of an interval** (w.r. to all pertinent parameters: time, pressure, temperature …)

The surprise of many physicists

- Are biological systems poised at criticality? [Mora et al., 2010]
- Brain processes as Critical Transitions [Werner, BioSyst., 2009]
- Critical Oscillations in Hear “Hair Cells” [Camelet, PNAS, 2000]
- Criticality in mitochondrial networks [Aon, PNAS, 2004]
- Fluctuations in Blood cells [Perazzo, 2008]
- Gene expression … exhibit criticality [Nykter, Kauffman et al., PNAS, 2008]
- **Each mitosis** (bifurcation, symmetry change …)

Yes, *they are maintained in an extended interval of criticality by* ***regulation*** *and* ***integration*** *effects!*

From Physics to Biology in terms of *extended critical transitions*

Physics (critical states, phase transitions…), *summary*:

- a **coherence structure** (order) may appear;
- **local** to **global** correlation length
- **isolated critical points** (0 measure) of the parameter(s);
- some **diverging** observables (related to **sensitivity**).

*Living matter as **extended critical transition*** (add to 1 - 4):

- **(+++)** Far from **equilibrium** and **symmetry changes**
- **Local to global**: internal correlation length *extended over time*: "same" order of the system itself
- **Fluctuations** within *extended* limits (several control parameters)
- Stabilizing integration and regulation: nesting and coupling of **levels of organization** (e.g. supported by tensegrity*).*
- **Infinite physical complexity** (in view of mathem. divergences)
- Maintained into criticality by **autopoiesis** and **homeorhesis**

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