LABORATORY

Solving an Eigenvalue Problem arising from Nonlinear and Non-Deterministic Aeroelasticity

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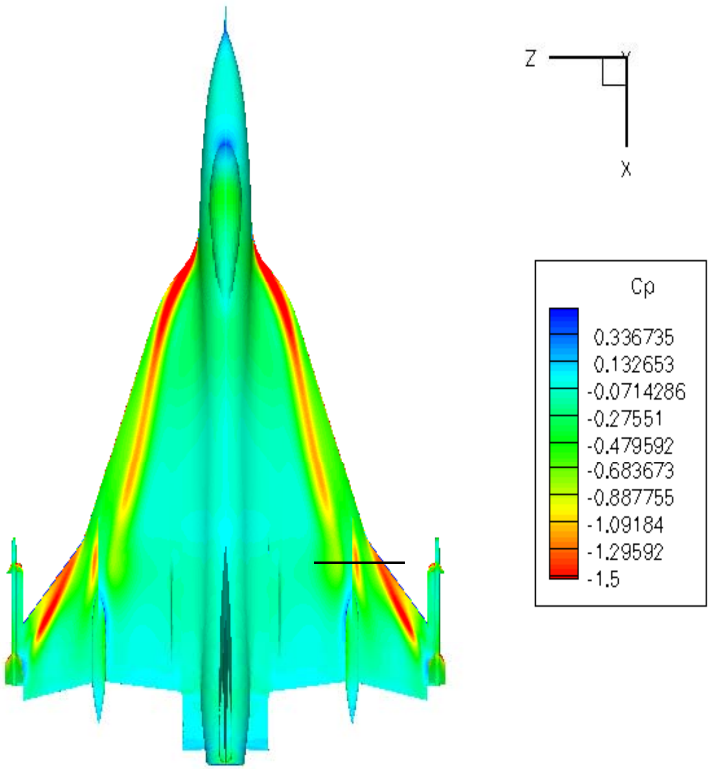
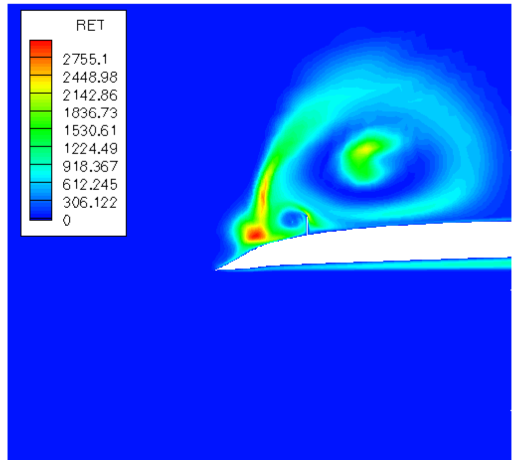
Hamed Khodaparast and John Mottershead

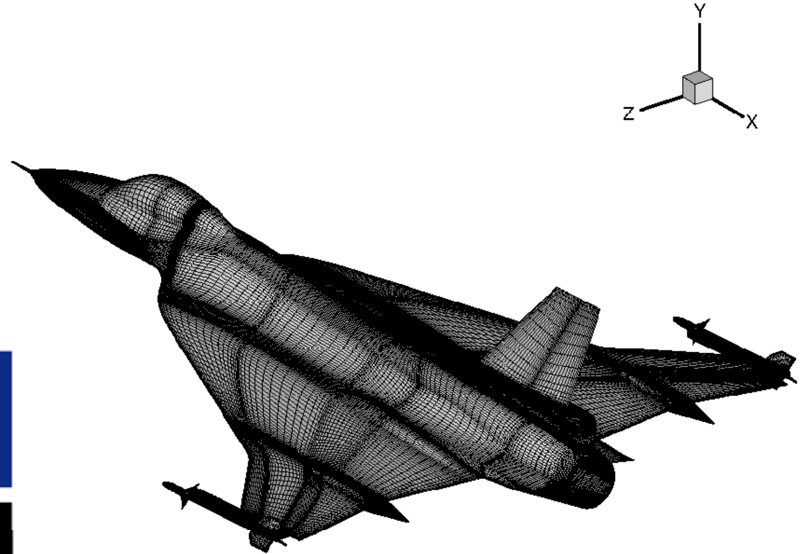
www.cfd4aircraft.com

Marie Curie Excellence Team

Enabling Certification by Analysis



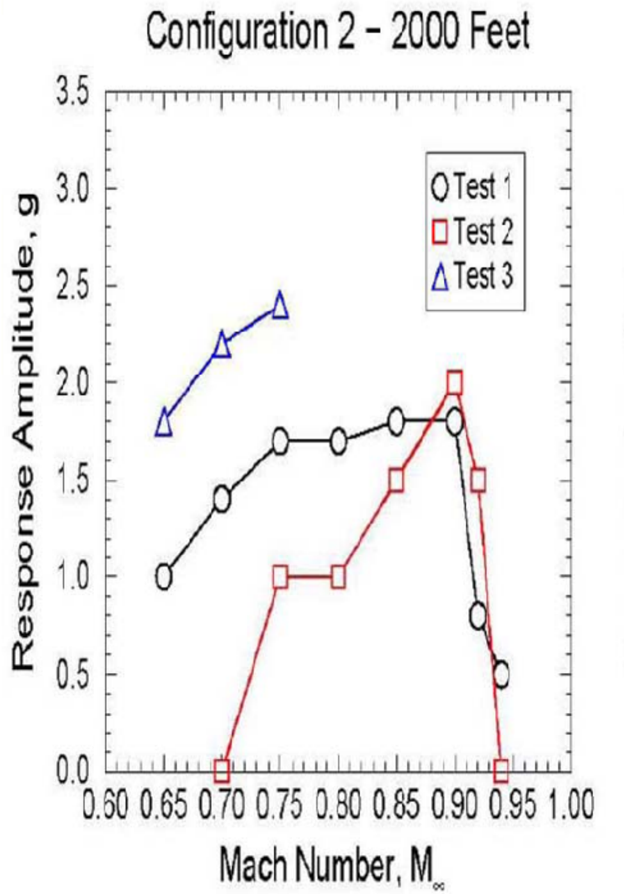




CAWAPI

COBALT-Springfield,OH • EADS-Munich • KTH-Stockholm • LMAC-Fort Worth
NASA Langley-Hampton • NLR-Amsterdam • TUSAS-Ankara • University of Glasgow
University of Wyoming-Laramie • USAFA-Colorado Springs • UTSimCenter-Chattanooga • VU Brussels

Boelens, O.J., Badcock, K.J., Elmilgui, A., Abdol-Hamid, K.S. and Massey, S.J., Comparison of Measured and Block Structured Simulations for the F-16XL, **Journal of Aircraft**, 46(2), 2009, 377-384.



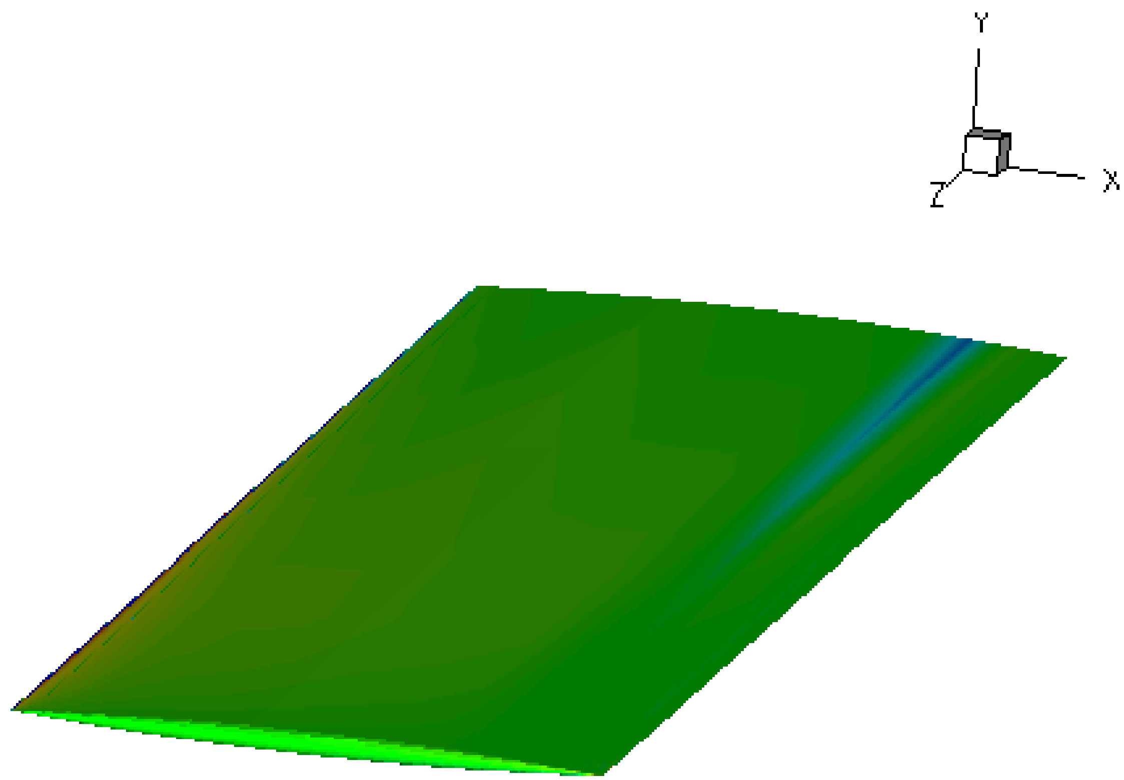
Thomas, SDM, 2006

Simulation Requirements

- Physics Based Simulation
  – Nonlinearity
- Envelope Search for Unanticipated Events
  – Computational cost
- Sensitivity and Variability
  – Probabilistic/Possibilistic
- Integration of Available Measurements
  – Updating/Identification

$$\frac{d\mathbf{w}}{dt} = \mathbf{R}(\mathbf{w}, \mu)$$

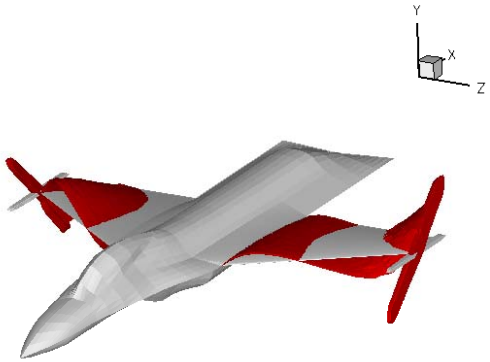
$$\mathbf{w} = \begin{bmatrix} \mathbf{w}_f \\ \mathbf{w}_s \end{bmatrix} \qquad \mathbf{R} = \begin{bmatrix} \mathbf{R}_f \\ \mathbf{R}_s \end{bmatrix}$$

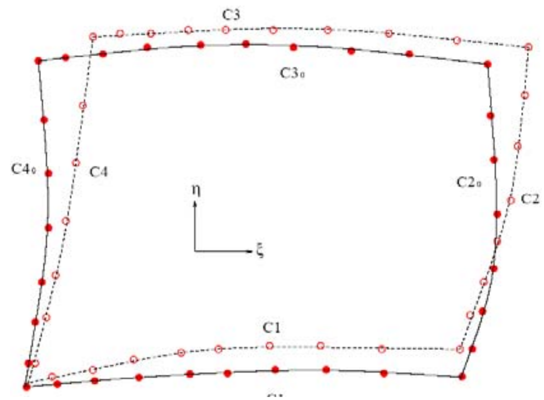

Y
X
Z

Eigenvalue Problem

- Stability studied from an eigenvalue problem:

$$\begin{bmatrix} A_{ff} & A_{fs} \\ A_{sf} & A_{ss} \end{bmatrix} \begin{bmatrix} p_f \\ p_s \end{bmatrix} = \lambda \begin{bmatrix} p_f \\ p_s \end{bmatrix}$$





Eigenvalue Problem

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- Shift and Invert
  - Good estimate of the target eigenvalue needed
  - Work is in solving a sparse linear system as part of the IPM iteration
  - Better shift – faster convergence but worse conditioned linear system
- (Subspace Iteration methods)

Badcock, K.J. and Woodgate, M.A., On the Fast Prediction of Transonic Aeroelastic Stability and Limit Cycles, AIAA J 45(6), 2007.

SCHUR METHOD

- Stability studied from an eigenvalue problem:

$$\begin{bmatrix} A_{ff} & A_{fs} \\ A_{sf} & A_{ss} \end{bmatrix} \begin{bmatrix} p_f \\ p_s \end{bmatrix} = \lambda \begin{bmatrix} p_f \\ p_s \end{bmatrix}$$

- Schur Complement formulation:

$$E = S(\lambda)\, p_s - \lambda p_s = 0$$

$$S(\lambda) = \left(A_{ss} - \lambda I\right) - A_{sf}\,(A_{ff} - \lambda I)^{-1} A_{fs}$$

Badcock, K.J. and Woodgate, M.A., Prediction of Bifurcation Onset of Large Order Aeroelastic Models, AIAA Journal, 48(6), 2010, 1037-1046

$$E = S(\lambda) p_s - \lambda p_s = 0$$
$$S(\lambda) = \left(A_{ss} - \lambda I\right) - A_{sf} (A_{ff} - \lambda I)^{-1} A_{fs}$$

- Solved by Newton's Method

$$\frac{\partial \mathbf{E}}{\partial \mathbf{u}} \Delta \mathbf{u} = -\mathbf{E}$$

Approximate Jacobian to drive convergence

Exact or Approximate Residual

$$\mathbf{u} = [\mathbf{p}_s, \lambda]^T$$

$$E = S(\lambda) p_s - \lambda p_s = 0$$

$$S(\lambda) = \left(A_{ss} - \lambda I\right) - A_{sf} (A_{ff} - \lambda I)^{-1} A_{fs}$$

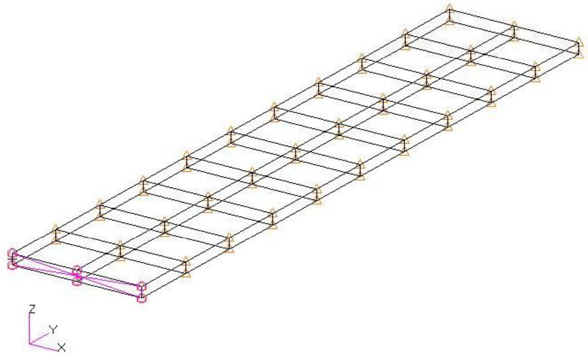
- Solved by Newton's Method

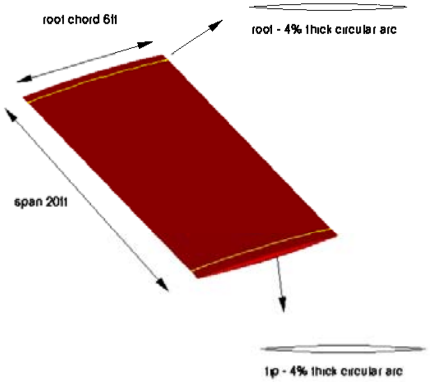
$$\frac{\partial \mathbf{E}}{\partial \mathbf{u}} \Delta \mathbf{u} = -\mathbf{E}$$

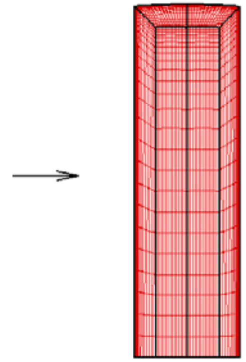
Approximate Jacobian to drive convergence	Exact or Approximate Residual

$$(A_{ff} - \lambda I)^{-1} \approx A_{ff}^{-1} + \lambda A_{ff}^{-1} A_{ff}^{-1} + \ldots$$

GOLAND WING



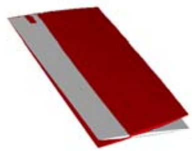




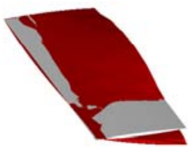
236k points

1×10^6 DoF

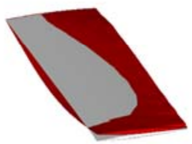
1.72 Hz



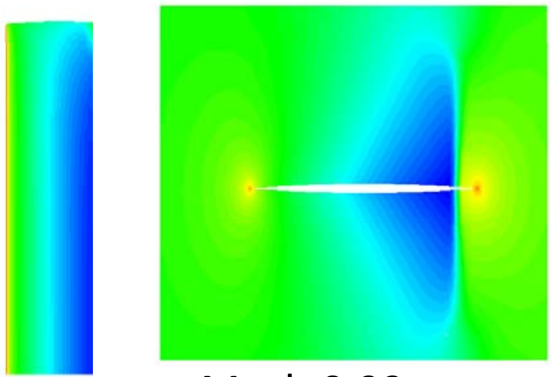
3.05 Hz



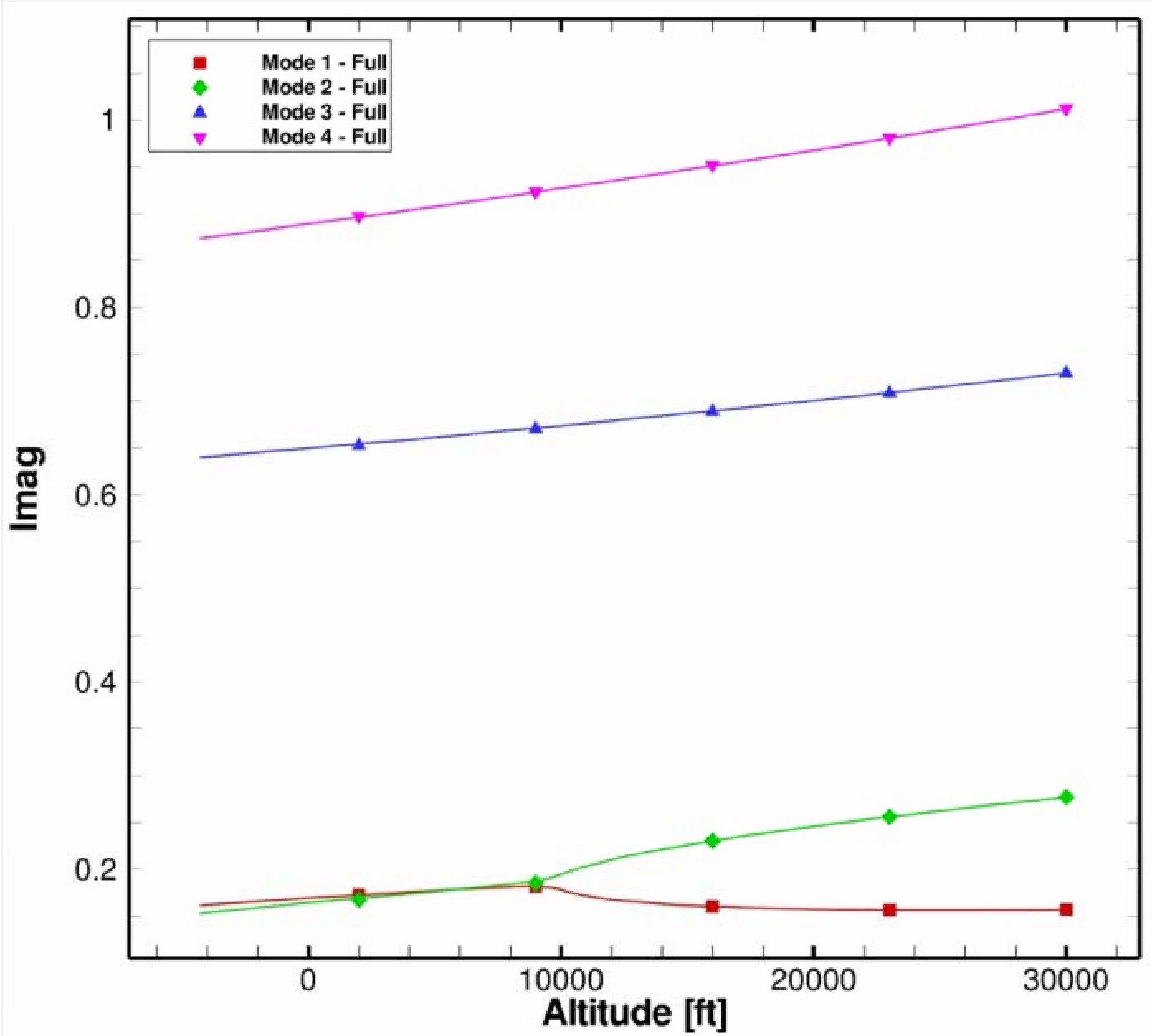
9.18 Hz



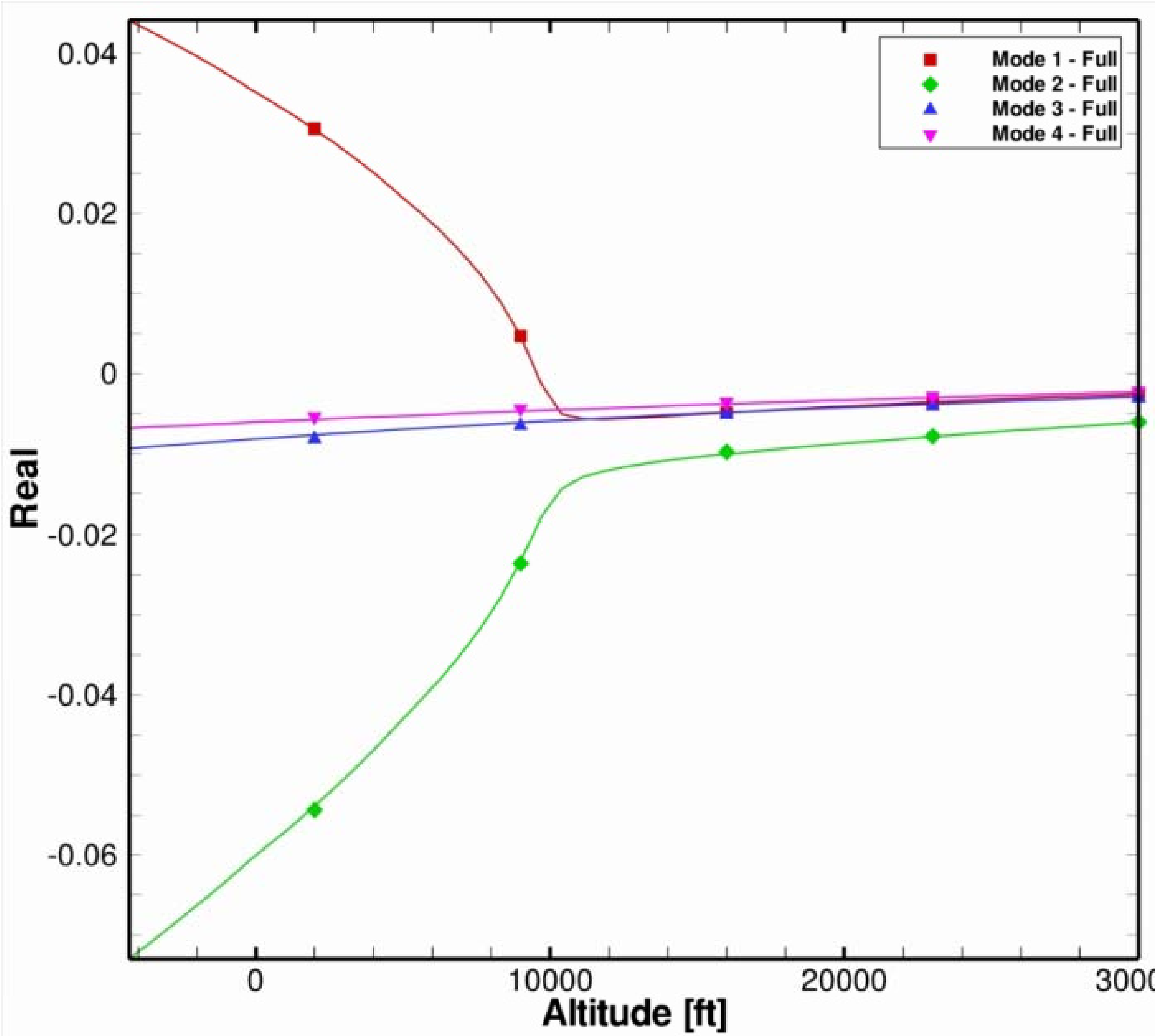
11.10 Hz



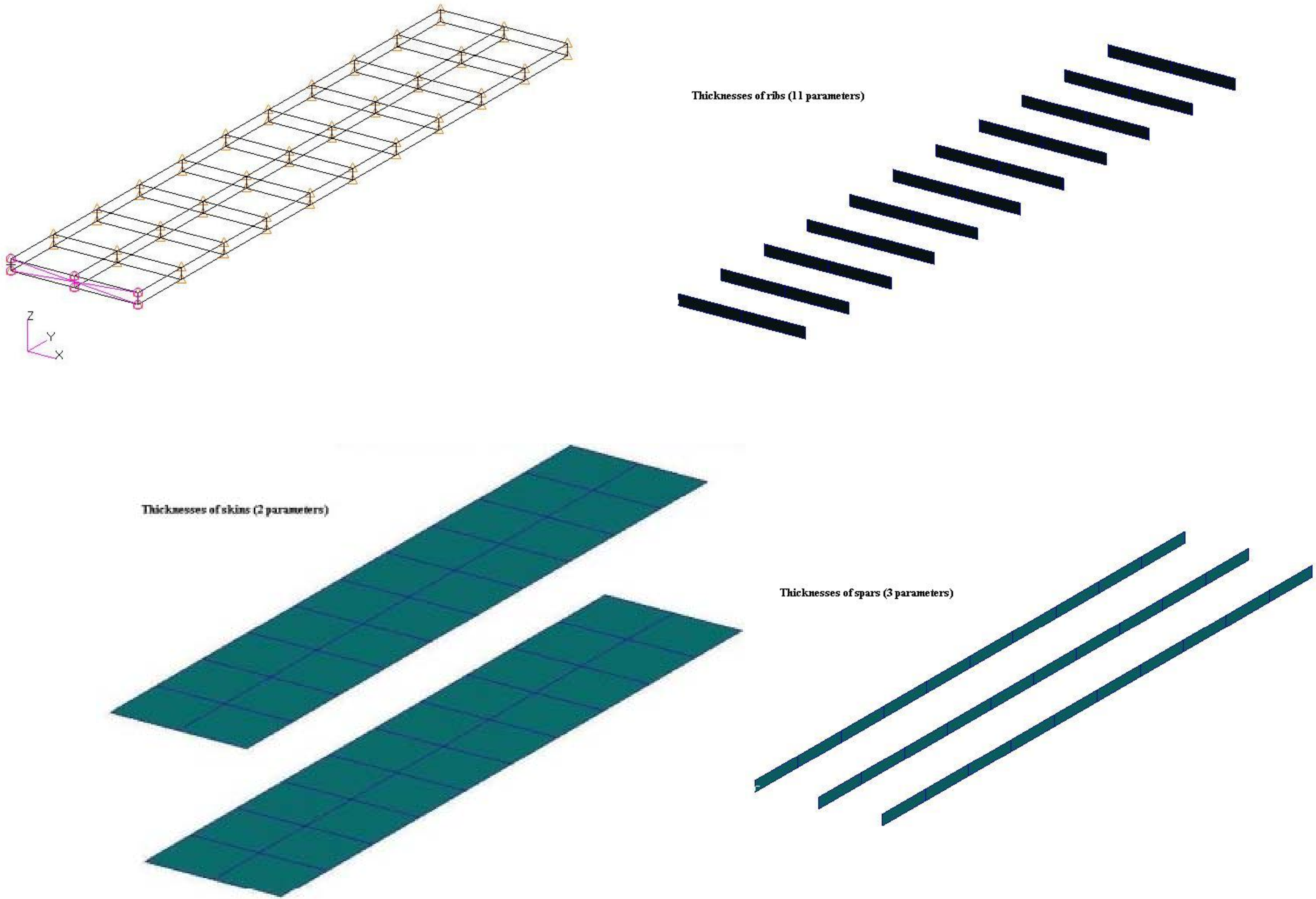
Mach 0.92



Mode 1 - Full
Mode 2 - Full
Mode 3 - Full
Mode 4 - Full
Imag
1
0.8
0.6
0.4
0.2
0
10000
20000
30000
Altitude [ft]



Mode 1 - Full
Mode 2 - Full
Mode 3 - Full
Mode 4 - Full
0.04
0.02
0
-0.02
-0.04
-0.06
Real
0
10000
20000
Altitude [ft]



Z
Y
X
Thicknesses of ribs (11 parameters)
Thicknesses of skins (2 parameters)
Thicknesses of spars (3 parameters)

Structural Model Variability

- Vector of structural parameters θ
- θ is uncertain in real engineering structure
  - Lack of knowledge
  - Variability
- Probabilistic
  - θ is defined by a PDF
  - What are the eigenvalue PDF’s?
- Possibilistic
  - θ is defined by an interval
  - What is the worst case eigenvalue?

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- Possibilistic
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  - What is the worst case eigenvalue?

Monte-Carlo Simulation
Many individual e-value calculations

Optimisation Problem
e-value calculations for residual and Jacobian

$$E = S(\lambda)\, p_s - \lambda p_s = 0$$

$$S(\lambda) = \left(A_{ss} - \lambda I\right) - A_{sf}\,(A_{ff} - \lambda I)^{-1} A_{fs}$$

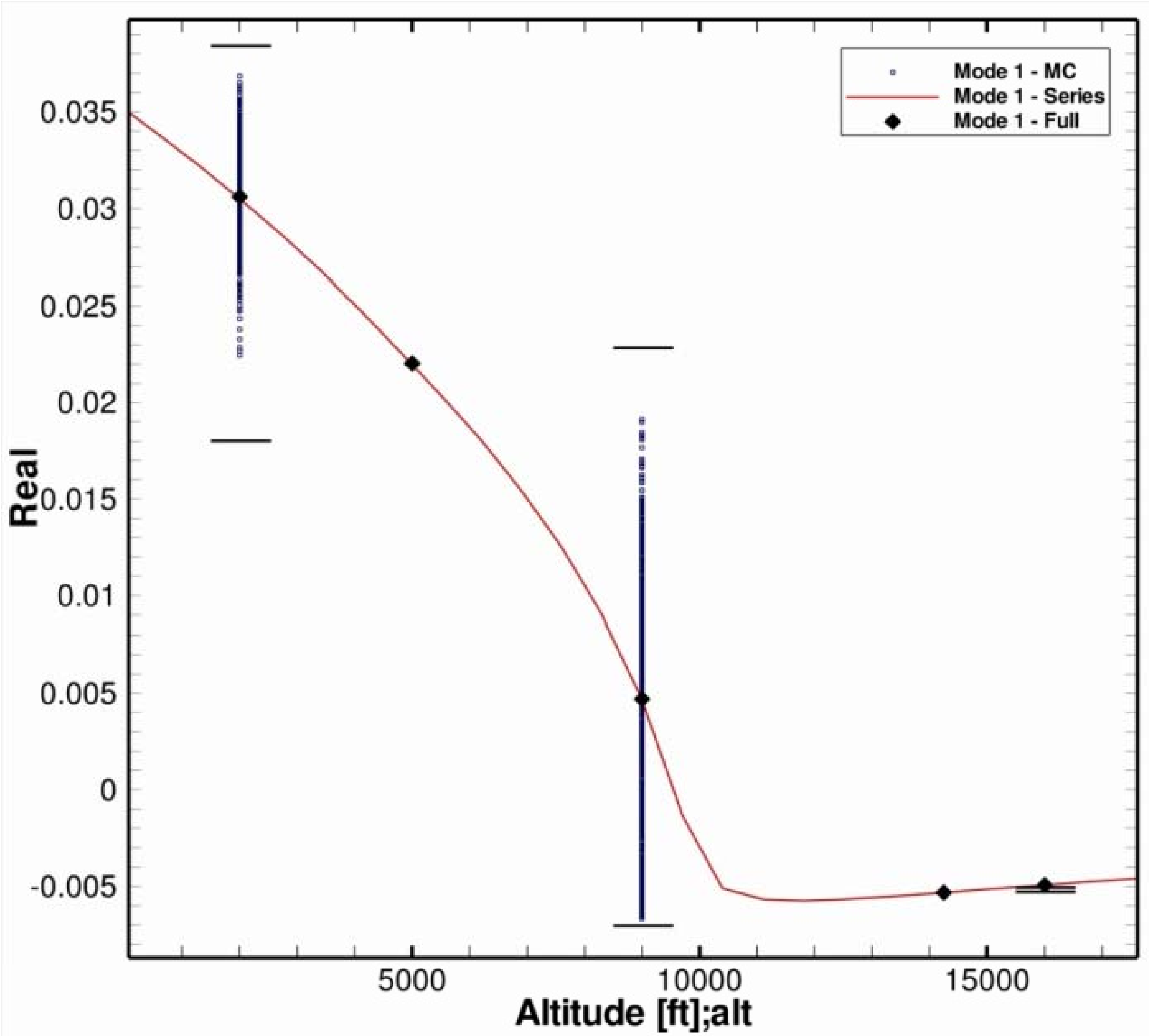
- Solved by Newton´s Method

$$\frac{\partial \mathbf{E}}{\partial \mathbf{u}} \Delta \mathbf{u} = -\mathbf{E}$$

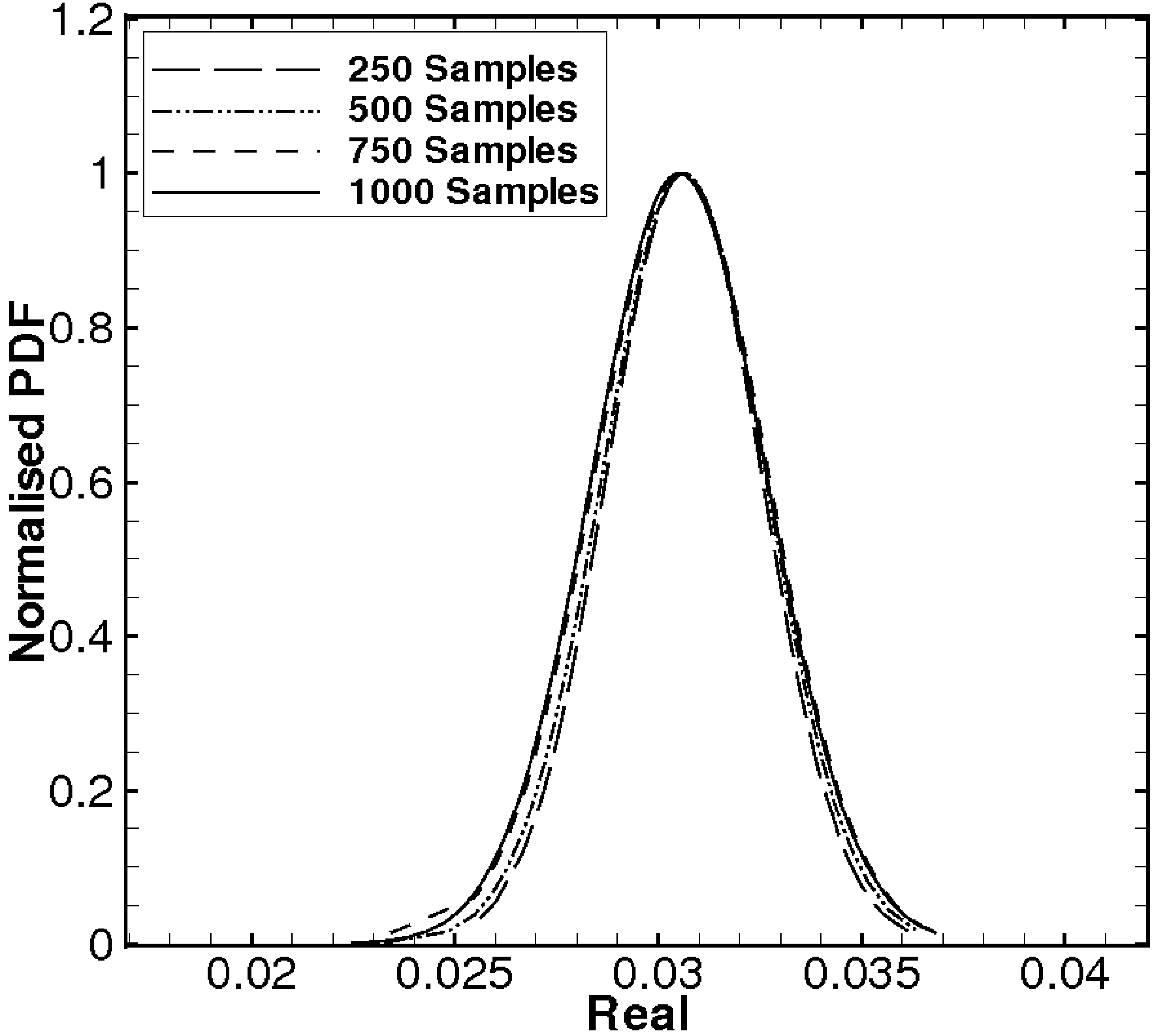
Approximate Jacobian at mean value parameters to drive convergence

Exact or Approximate Residual at current structural realisation

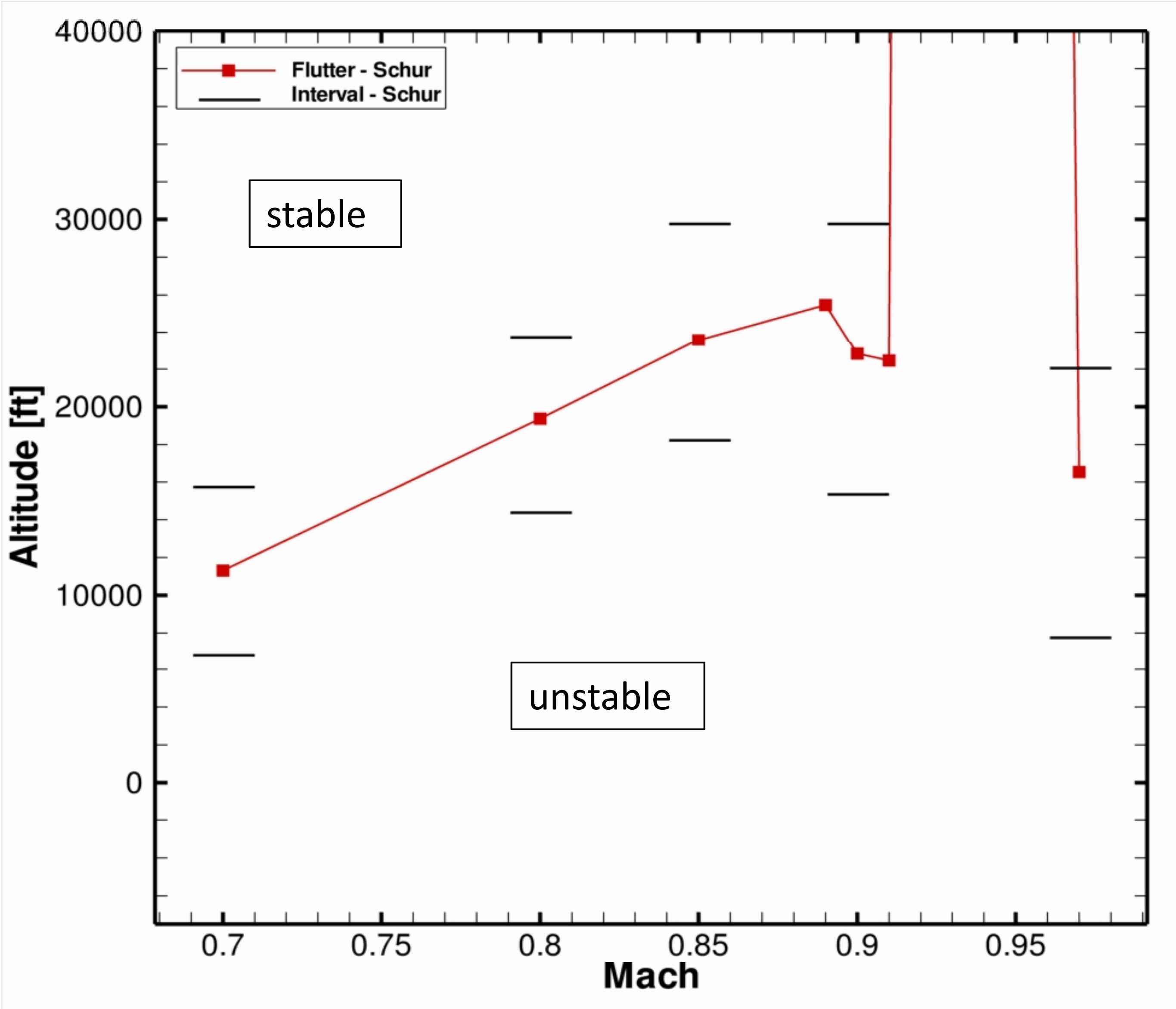
$$(A_{ff} - \lambda I)^{-1} \approx A_{ff}^{-1} + \lambda A_{ff}^{-1} A_{ff}^{-1} + \ldots$$



Badcock et al, CFD Based Aeroelastic Stabilty Predictions Under the Influence of Structural Uncertainty. Journal of Aircraft, 47(4), 2010, 1229-1239

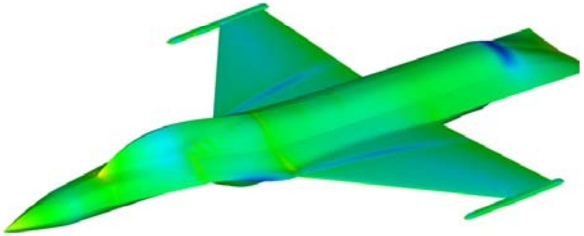


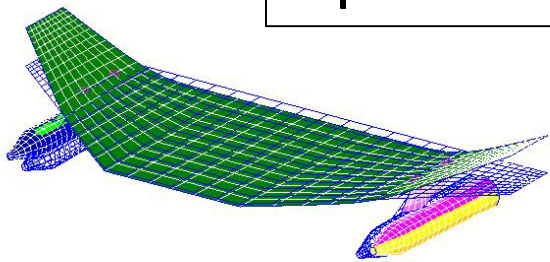
250 Samples
500 Samples
750 Samples
1000 Samples
Normalised PDF
Real
0
0.2
0.4
0.6
0.8
1
1.2
0.02
0.025
0.03
0.035
0.04

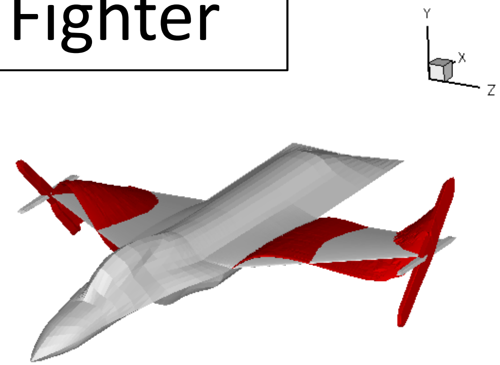


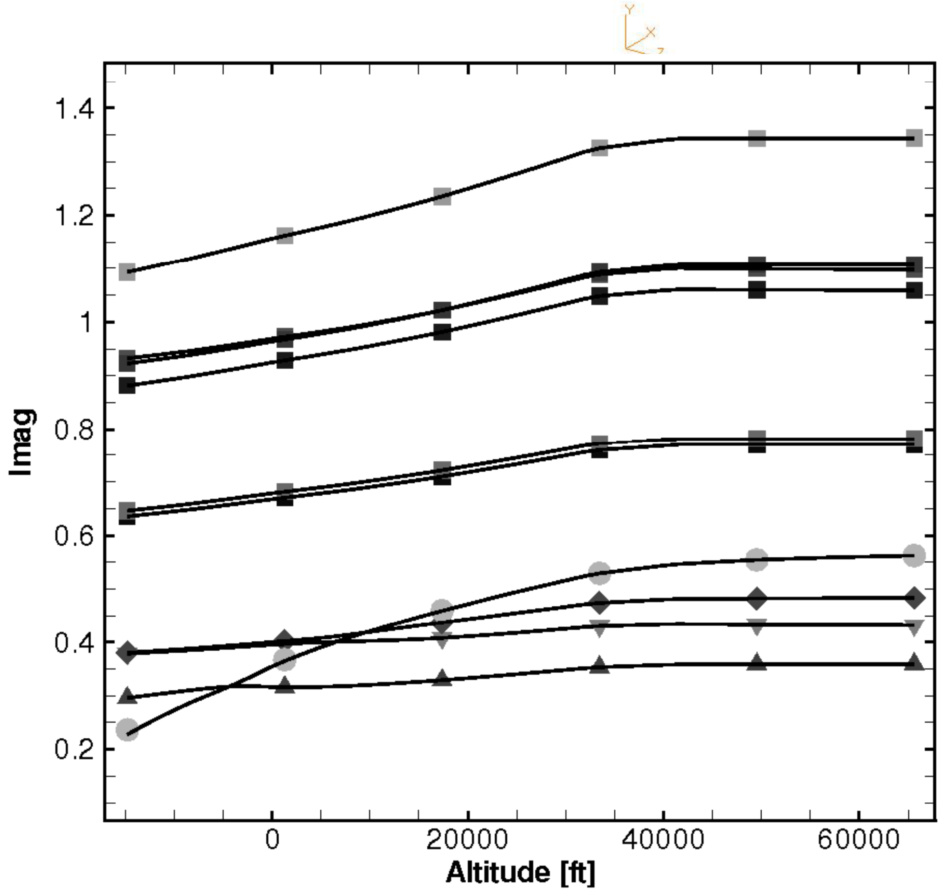
Flutter - Schur
Interval - Schur
stable
unstable
40000
30000
20000
10000
0
Altitude [ft]
0.7
0.75
0.8
0.85
0.9
0.95
Mach

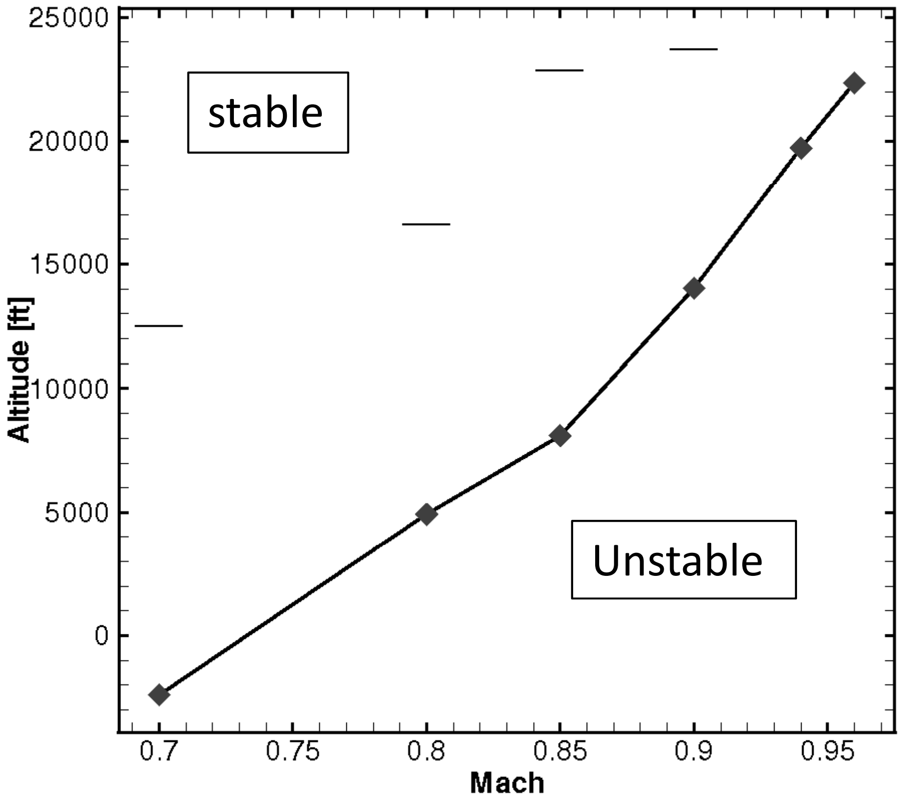
Open Source Fighter

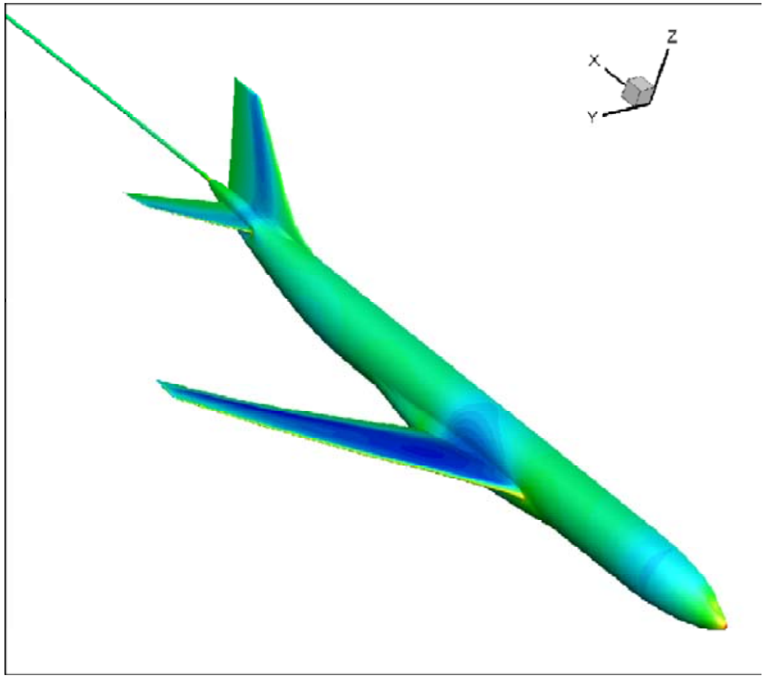


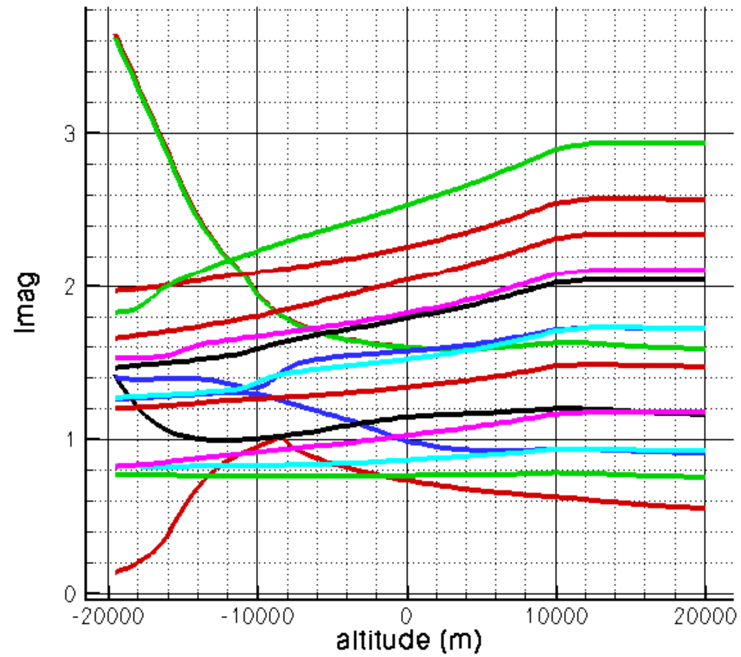


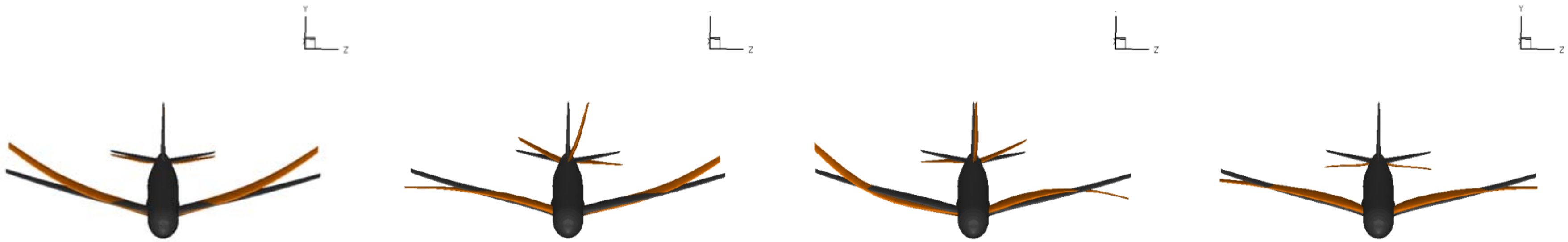












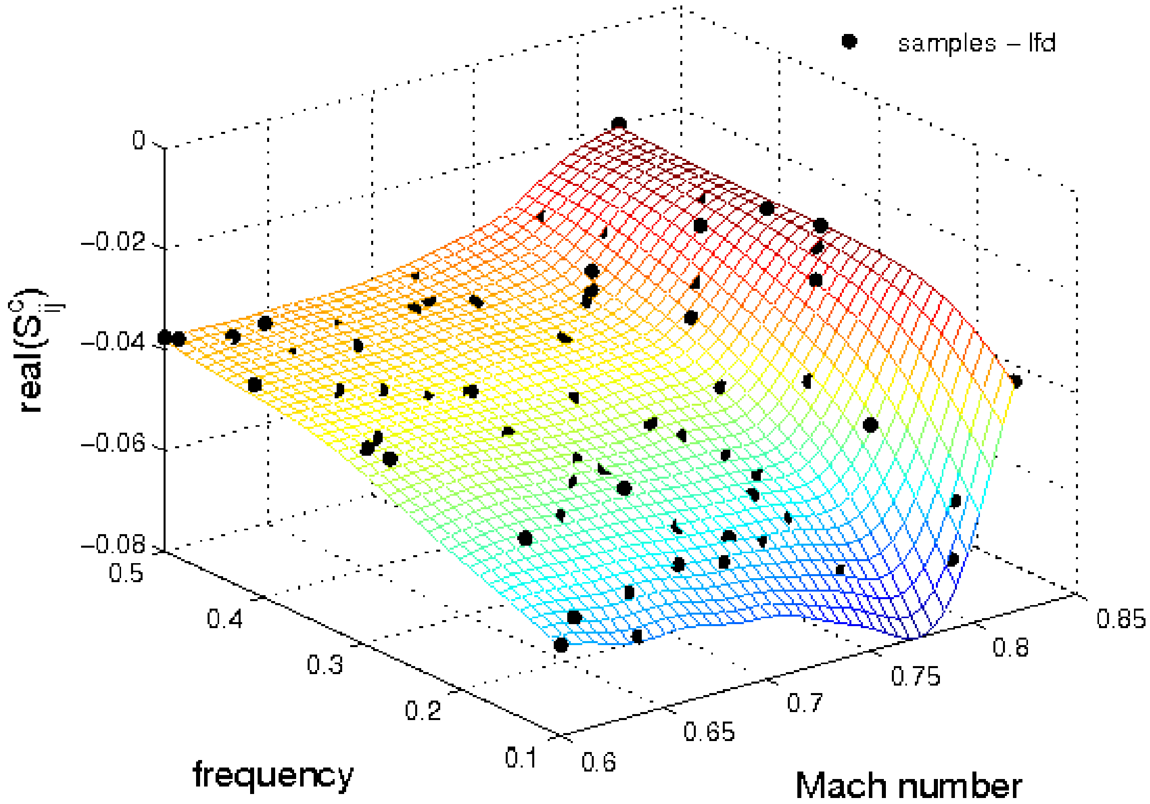
Airbus XRF

Improvements

- Key Issue
  - Approximation of S(λ)
  - Better approaches than series approximation

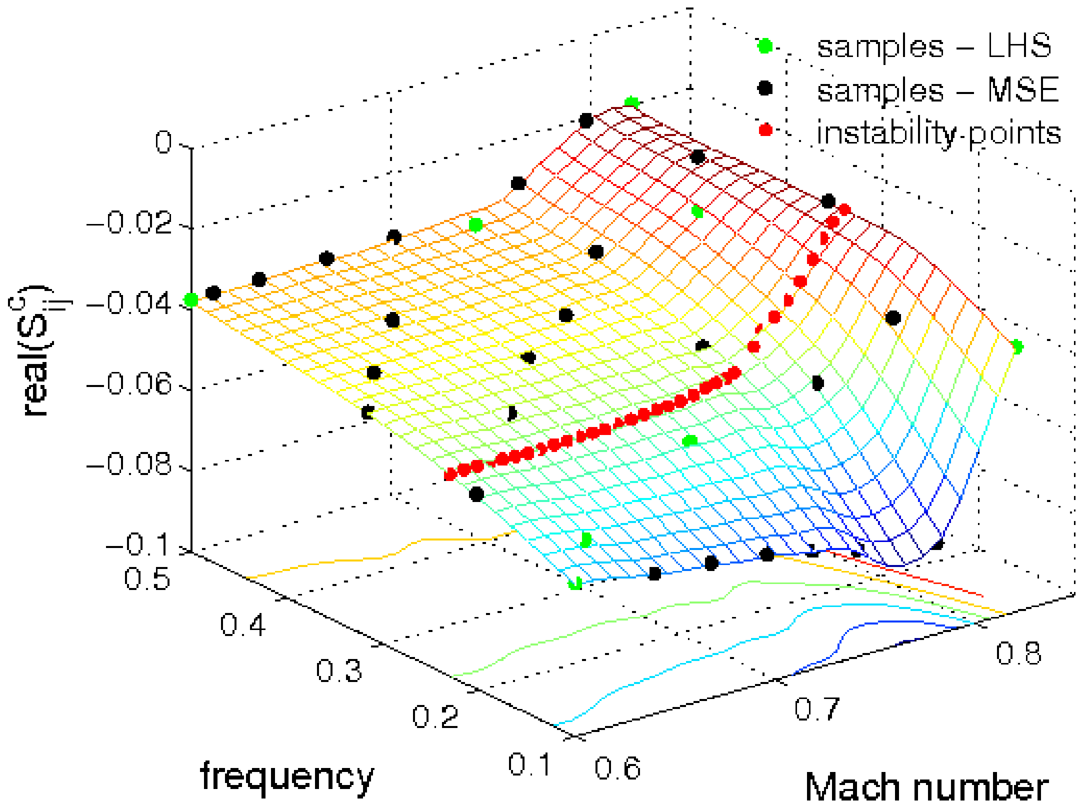
- Kriging Approximation
- Sampling Methods
- Update with higher order information

Timme, S. and Badcock, K.J., Searching for Transonic Aeroelastic Instability Using an Aerodynamic Model Hierarchy, to appear in Journal of Aircraft



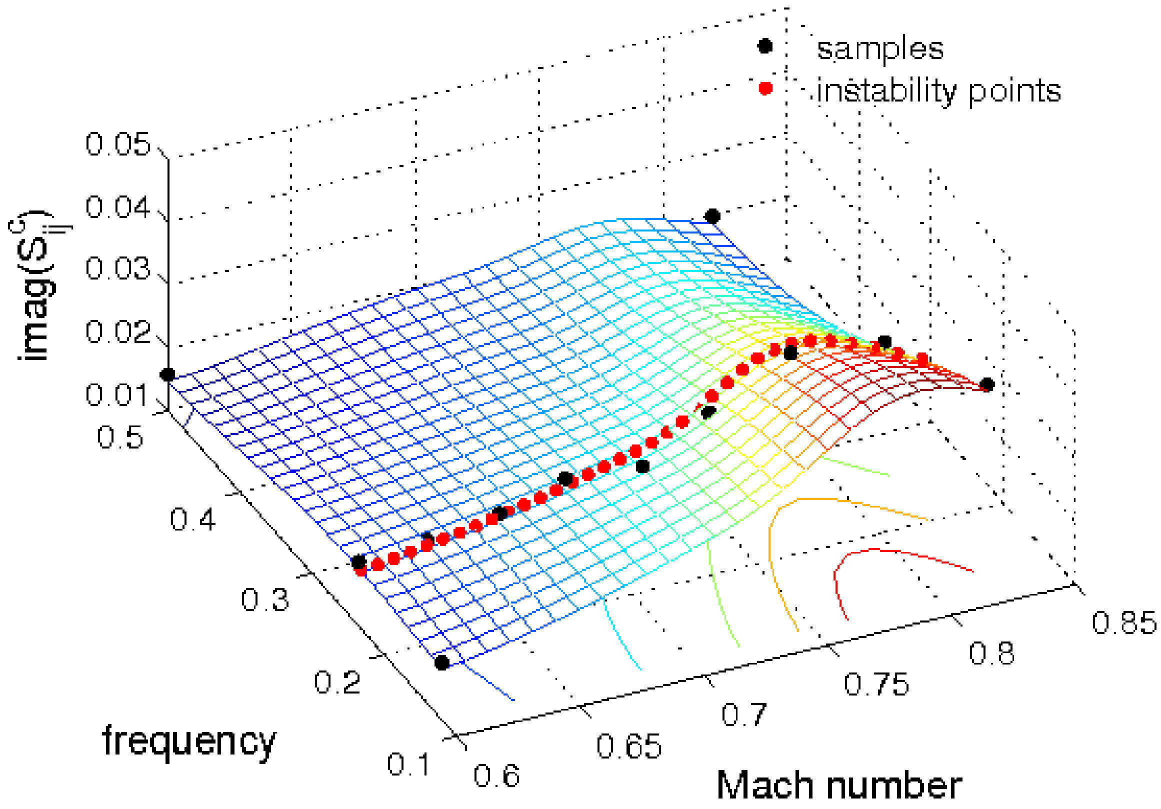
Random Sampling

Better basic sampling approaches...



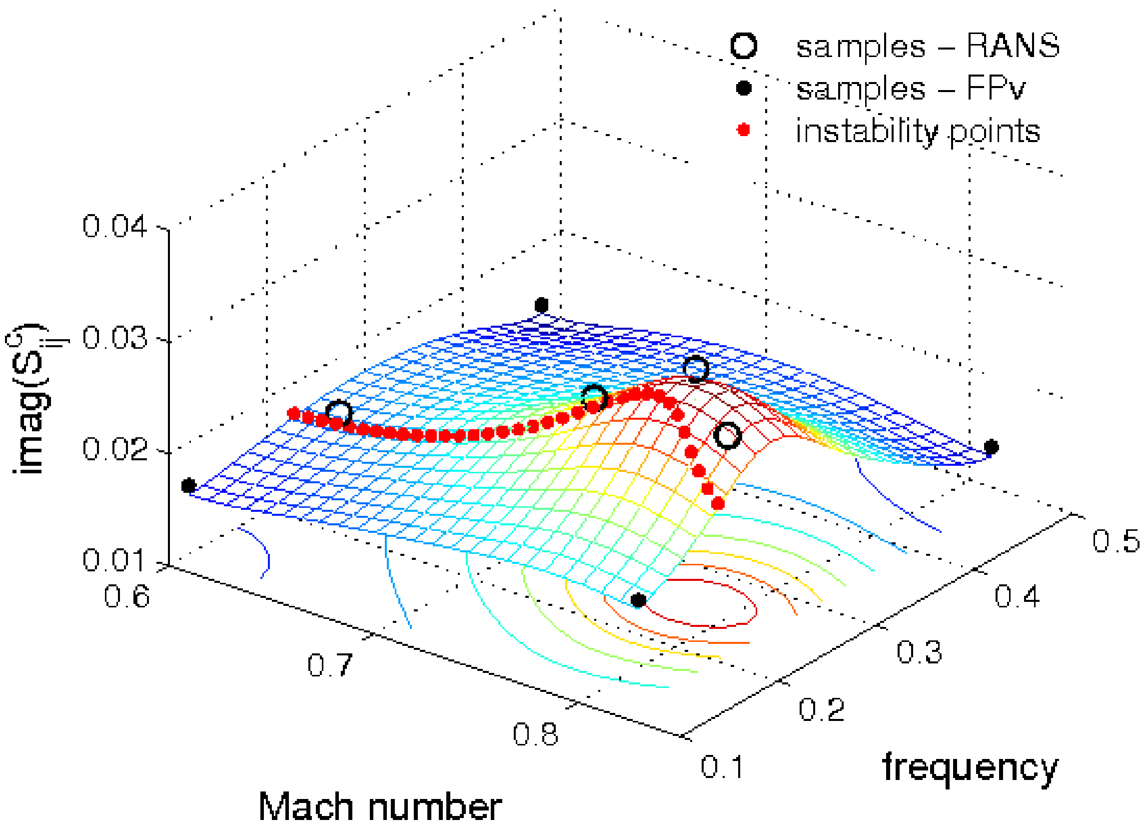
Mean Squared Error Sampling

For any given approximation to S, solving the e-value problem is cheap...



Instability Guided Mean Squared Error Sampling

Can also exploit different levels of modelling.....



Model Hierarchy

Goland Wing Cost Summary

- Euler steady state in wall block 30mins
- RANS steady state
  - Coarse grid: wall clock 1 hr
  - Fine grid: wall clock 6 hrs
- RANS time accurate
  - Coarse grid: 5 cycles in wall block 20hrs
- Flutter boundary in M space
  - 6 fine grid RANS samples for co-Kriging Euler
    - approx 2 days

Simulation Requirements

- Physics Based Simulation
  - Nonlinearity
- Envelope Search for Unanticipated Events
  - Computational cost
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- Integration of Available Measurements
  - Updating/Identification

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- ✓ Physics Based Simulation
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$$AP = iwP \qquad A^T Q = -iwQ$$
$$A\overline{P} = -iw\overline{P} \qquad A^T \overline{Q} = -iw\overline{Q}$$

Change coordinates

$$z = < P, \overline{w} >$$
$$y = \overline{w} - < P, \overline{w} > Q - < \overline{P}, \overline{w} > \overline{Q}$$

Taylor Series Expansion of R

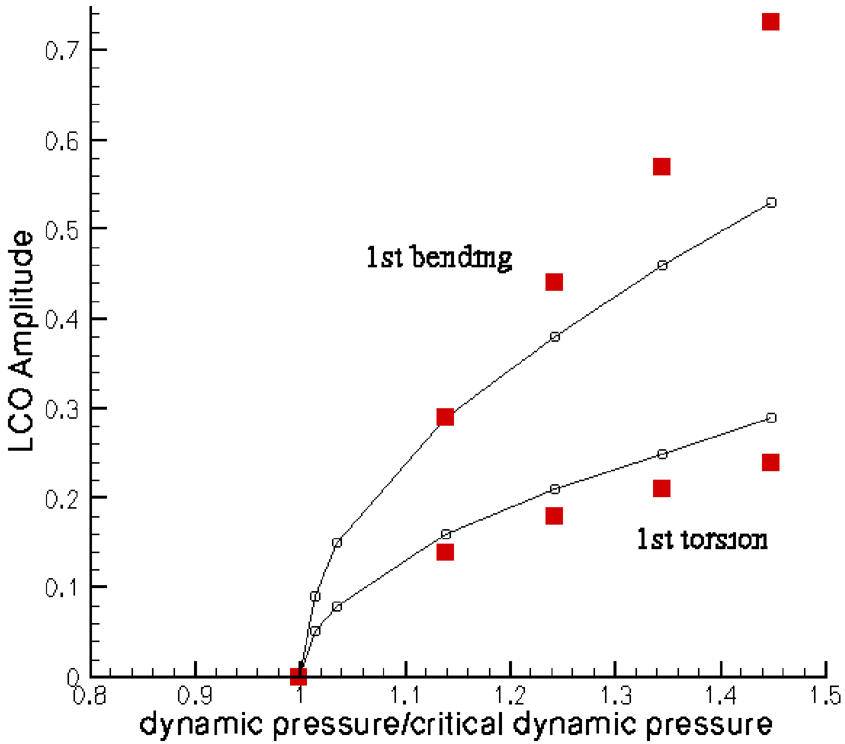
Expand system

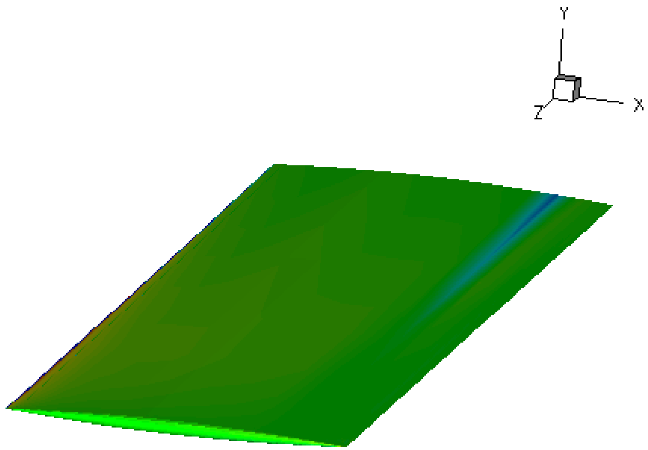
$$\frac{d\overline{w}}{dt} = A\overline{w} + \frac{\partial R}{\partial \mu}\overline{\mu} + \frac{\partial A}{\partial \mu}\overline{\mu w} + \ldots..$$

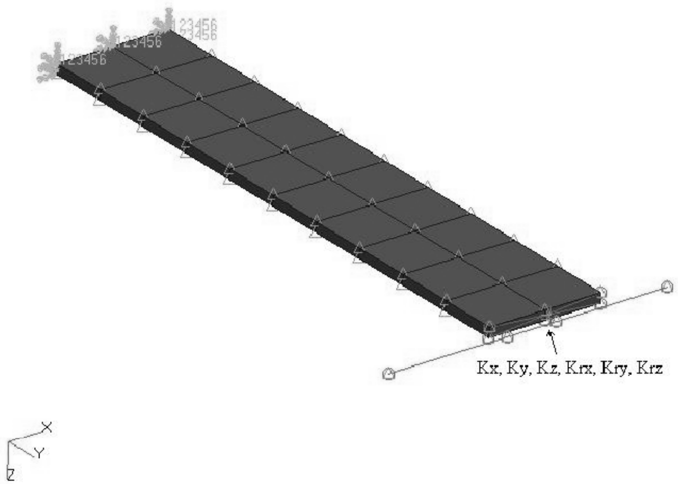
Inner Product with P

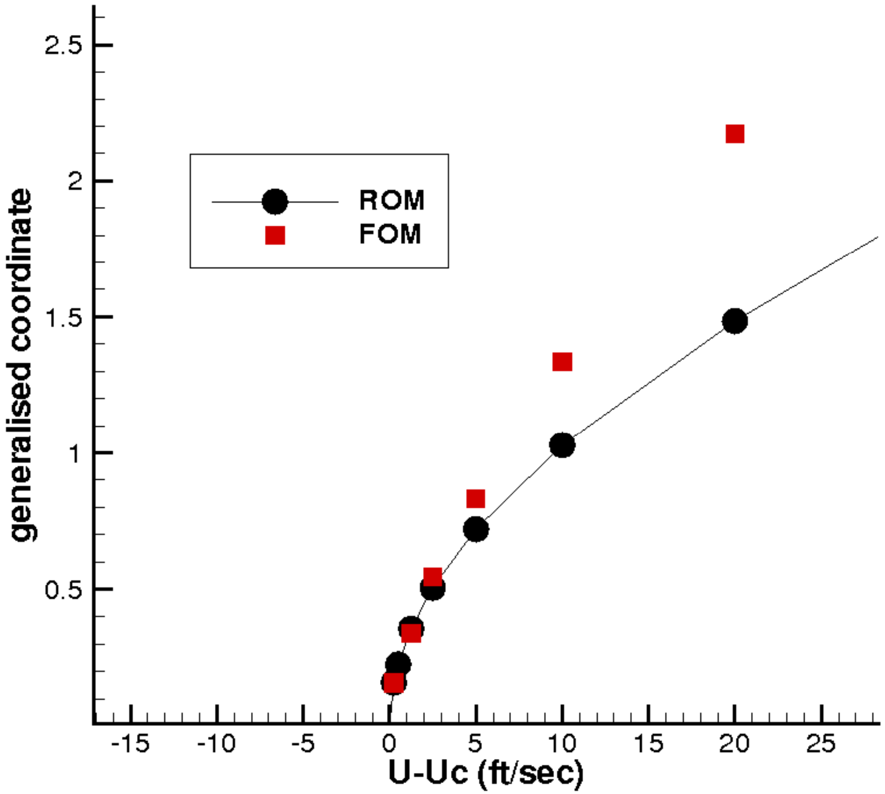
$$\dot{z} = i\omega z + < P, R_\mu \overline{\mu} > + < P, A_\mu \overline{\mu w} >$$

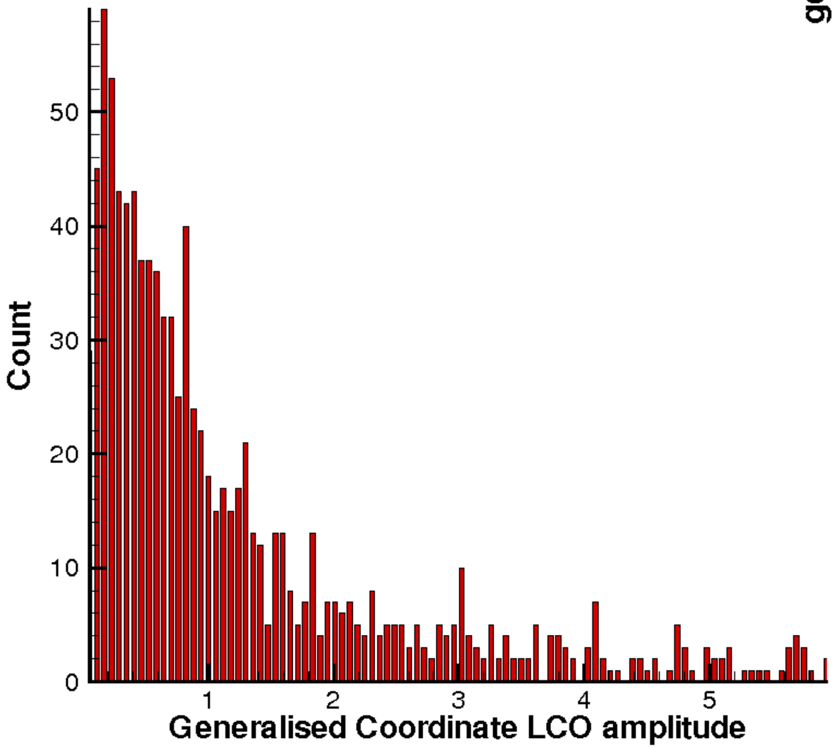
Badcock, K.J. and Woodgate, M.A., On the Fast Prediction of Transonic Aeroelastic Stability and Limit Cycles, AIAA J 45(6), 2007.


LCO Amplitude
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.8
0.9
1
1.1
1.2
1.3
1.4
1.5
dynamic pressure/critical dynamic pressure
1st bending
1st torsion


Y
X
Z


123456
123456
123456
Kx, Ky, Kz, Krx, Kry, Krz
X
Y
Z


ROM
FOM
generalised coordinate
U-Uc (ft/sec)
2.5
2
1.5
1
0.5
-15
-10
-5
0
5
10
15
20
25


Count
Generalised Coordinate LCO amplitude
50
40
30
20
10
0
1
2
3
4
5

Current Focus

- How to define tests/exploit data
  - Identify corrections to Schur matrix to match measurements of response
- LCO Amplitude Uncertainty
- Flexible aircraft flight control
  - New project with Palacios at Imperial College