Robin Bullough Symposium

Abstracts of Contributions

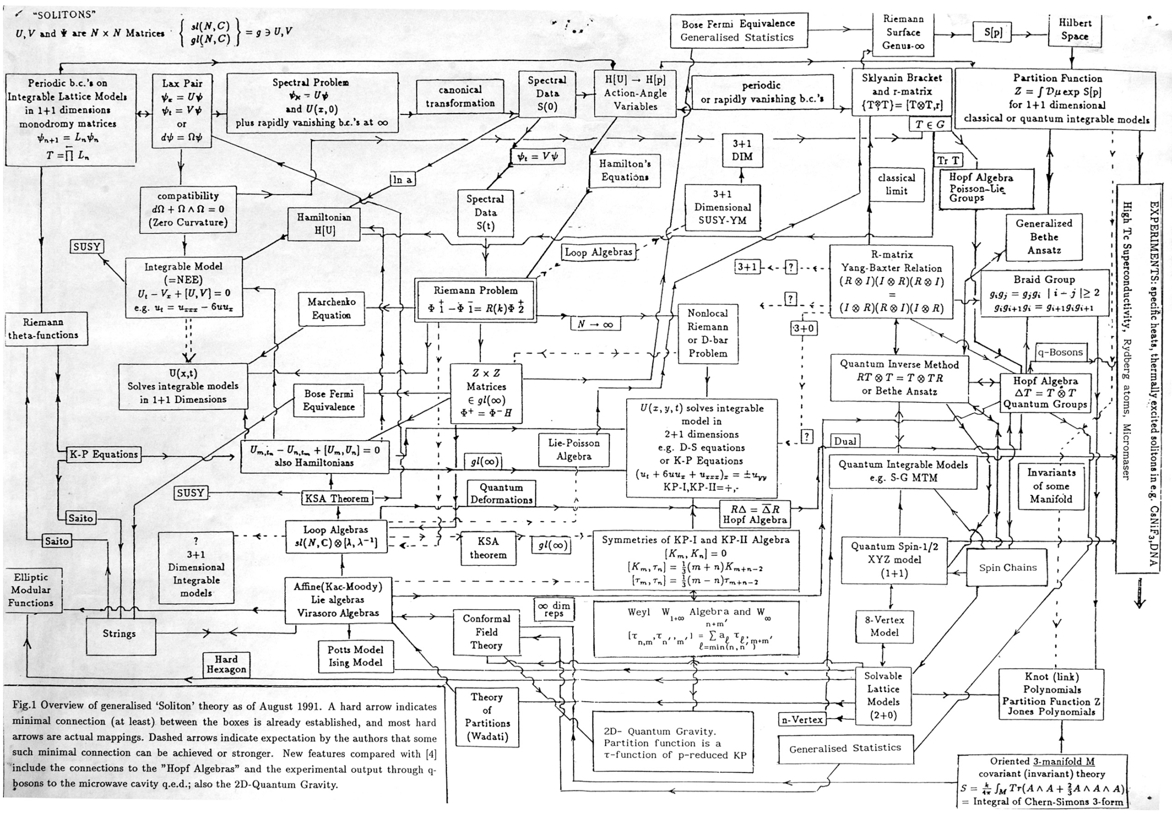

Fig.1 Overview of generalised 'Soliton' theory as of August 1991. A hard arrow indicates minimal connection (at least) between the boxes is already established, and most hard arrows are actual mappings. Dashed arrows indicate expectation by the authors that some such minimal connection can be achieved or stronger. New features compared with [4] include the connections to the "Hopf Algebras" and the experimental output through q-bosons to the microwave cavity q.e.d.; also the 2D-Quantum Gravity.

School of Mathematics, University of Manchester

10th & 11th June 2009

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Per Bullough

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Front cover picture: one of Robin's *grand synthesis of soliton theory*.

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Abstracts of Contributions

Form-factors for strongly coupled boson system, plane partitions and random walks

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The exactly solvable model of strongly coupled bosons, the so called "phase model", introduced by R.K. Bullough in the paper (N.M. Bogoliubov, R.K. Bullough, and J. Timonen, "Critical behavior for correlated strongly coupled boson systems in 1+1 dimensions", Phys. Rev. Lett., vol. 25, 3933 (1994)) is considered. The form-factors for the model are calculated explicitly and represented as determinants. The relation of the model with the problems of the contemporary combinatorics is discussed. It is shown that the natural model describing the behavior of the friendly walkers, the ones that can share the same lattice sites, is the "phase model". The expression for the number of all admissible nests of lattice paths made by the fixed number of the friendly walkers for the certain number of steps is obtained. The connection between the form-factors and boxed plane partitions - the three dimensional Young diagrams placed into a box of a finite size is established.

Robin Bullough and the crystallographic phase problem – a tale of scientific inheritance

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I will talk about my father's life outside mathematics and also the inspiration he gave me for a career in research. I will discuss my father's early work as a Ph.D. student in attempting to solve the 'phase problem' in X-ray crystallography. Remarkably, and in more ways than one, this work connects with my own research and I will discuss the importance of the 'phase problem' in biology. I will show how I have used diffraction techniques (both X-ray and electron) to reveal the mode of action of a number of molecules found in different biological systems.

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Isochronous dynamical systems and the arrow of time

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A vector-valued time-dependent function is called isochronous if all its components are periodic in time with the same fixed period T. A dynamical system is called isochronous if its generic solution is isochronous: periodic in all its degrees of freedom with a fixed period T independent of the initial data. It will be shown how essentially any (autonomous) dynamical system can be modified or extended into another (also autonomous) dynamical system which is isochronous with an (arbitrarily !) assigned period T, and which moreover behaves, over time periods very short with respect to T, essentially as the original (unmodified) system—up to a constant time rescaling. This can also be done for a large class of Hamiltonian systems, including the Hamiltonian describing the most general many-body problem (provided it is, overall, translation-invariant). Some implications of this fact for statistical mechanics and thermodynamics will be mentioned, and for the distinction among integrable and nonintegrable dynamical systems (all isochronous systems are integrable, in fact maximally superintegrable). These findings have all been obtained together with F. Leyvraz: some of them are reported in my monograph entitled Isochronous systems (Oxford University Press, 2008), others are more recent.

Solitons without Inverse Scattering

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Who remembers "Hirota's method"? In the early days of solitons, although the Korteweg-de Vries equation had been solved by the "Inverse Scattering Method" [1] most solutions to integrable non-linear equations were found by simpler more direct methods. Outstanding among these was a method due to mainly to Hirota [2, 3, 4] which involved casting the equation into a "bi-linear form" and then applying intelligent guesswork. In this talk I shall take a journey down memory lane looking again at this method.

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Quantum Mechanical and Relativistic Scattering by Short-Range Potentials

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Scattering by short-range potentials in the Schrödinger equation and the Dirac equation is discussed. The potentials are represented by Dirac's delta function and its derivatives. Regularisation is used and uniqueness and non-uniqueness of the procedure is discussed. Recent results for a fourth order equation describing vibrations on a beam will be included.

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Generalising the Weierstrass $\wp$ function to curves of higher genus

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Abstract: The theory of elliptic functions connected to a curve of genus one can be developed in a number of ways. One of these is through the Weierstrass $\wp$ function, which provides the travelling wave solution of the KdV equation and many other integrable PDEs. Generalising this to curves of higher genus, we get periodic solutions of the KdV hierachy, Boussinesq, KP, etc., together with interesting addition theorems. I will concentrate on recent developments for trigonal and tetragonal curves, but also mention some new(?) results for the elliptic case.

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Cluster Mutation-Periodic Quivers and Associated Laurent Sequences

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We consider quivers/skew-symmetric matrices under the action of mutation (in the cluster algebra sense). We classify those which are isomorphic to their own mutation via a cycle permuting all the vertices, and give families of quivers which have higher periodicity. The periodicity means that sequences given by recurrence relations arise in a natural way from the associated cluster algebras. We present a number of interesting new families of nonlinear recurrences, necessarily with the Laurent property, of both the real line and the plane, containing integrable maps as special cases. In particular, we show that some of these recurrences can be linearised and, with certain initial conditions,

give integer sequences which contain all solutions of some particular Pell equations. We extend our construction to include recurrences with *parameters*, giving an explanation of some observations made by Gale. Finally, we point out a connection between quivers which arise in our classification and those arising in the context of quiver gauge theories. Based on arxiv.0904.0200

Cavity QED: Theory, experiment, & the cat that got the cream.

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The basic interaction of light with matter does not reveal its quantum nature unless we take care in what we observe and how we observe it. In the fields of cavity QED and Quantum Optics the use of high-Q resonant cavities for microwaves, and light, allows us to see such quantum effects. In this talk there will be an overview of theory and experiment concerned with such cavities interacting with small numbers of atoms. Schrödinger's cat will make an appearance. There will also be some new results on the old issue of the spectrum of light [1]: that is, the light from cavity-atom systems with multiple excitations [2] and dynamic reservoir structures [3].

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Extreme events in solutions of the 3D Navier-Stokes equations and in Primitive Climate models

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Mathematicians tend to look at turbulence through the eyes of the 3D Navier-Stokes (NS) equations. Their technical tools (norms) inevitably involve volume integrals which, by their nature, average out variations in the vorticity and strain associated with intermittency. Thus the subtlety of the vortical spatial structure is lost. Colour graphics vividly demonstrate that tube/sheet 'thin sets' tend to dominate the vortical landscape, yet mathematically it is very hard to explain the origin of these by rigorous methods of analysis. In this talk an attempt is made to (i) make a partial analytical explanation for this pheneomenon; (ii) to see if these ideas can also shed some light on the sudden appearance and disappearance of locally intense fronts in solutions of the viscous hydrostatic & non-hydrostatic primitive equations (HPE/NPE) that form the basis of most numerical climate models.

Non-Linear Optical Extinction Theorem

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Starting from the model Bloch-Maxwell equations for two-level atoms forming an extended system of Fabry-Perot cavity configuration, the fundamental Optical Extinction Theorem due to Ewald-Oseen (1915,1916) is generalized to the non-linear regime. Generalized form of the Lorentz-Lorenz dispersion relation for the refractive index (m) is derived. Within the context of optical multistability phenomenon, the characteristic input-output filed relation is non-linearly dependent on (m). A self-consistent numerical scheme shows that the multistable behaviour is exhibited for dense medium data.

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Landau–Lifshitz–Gilbert (LLG) Equation: Integrability, Chaos, Patterns and All That

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The world of Landau–Lifshitz–Gilbert (LLG) equation is quite large and represents many physically interesting spin and other physical systems. It encompasses very many nonlinear evolution equations, both integrable and nonintegrable. It admits a large class of nonlinear excitations, depending on the nature of the interactions, additional forces, anisotropy and damping. These include spin waves (magnons), elliptic function waves, solitary waves and solitons, vortices, instantons, axisymmetric solutions, dromions, chaotic structures, various spatiotemporal patterns and their switching. Many

special cases of the LLG equation have close connection with differential geometric aspects and group theoretical notions. In recent times, it has drawn renewed attention as the basic model equation in an extended form in the field of spintronics, specifically in connection with spin transfer torque effect in nanoferromagnets. Particularly, when a spin polarized current passes through layers of ferromagnetic films of nanometer level thickness interspersed with conducting nonferromagnetic spacers, it can give rise to magnetic switching normally produced by external applied magnetic fields, leading to novel technological MRAM devices. The phenomenon is essentially represented by an extended LLG equation. Robin Bullough was a great admirer of the underlying physics and mathematics of the LLG equation and I will be reviewing some of the recent developments on LLG equation as a tribute to him.

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Integrable models in nonlinear optics and soliton solutions

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Systems of PDEs which model nonlinear wave propagation in optics play a particularly interesting role if they are both relevant to experiments and integrable. The second property (integrability) gives these models a very special status because of our capability of solving important problems such as, among others, construction of analytic soliton solutions, soliton interactions, stability, asymptotic states, conservation laws, parametric control.

Notable examples of wave equations in this class are multi component Schrödinger type systems and resonant interaction models; in particular, in this talk I will consider a systems of multi component wave equations in 1+1 dimensions, introduced first by Calogero and Degasperis [1], [2]. Soliton solutions of these equations are constructed by specializing the Dressing Darboux Transformation to deal with different boundary conditions which describe all–bright as well as mixed bright–dark and all–dark pulses

[3], [4]. Contact with nonlinear optics is made by considering the simplest model of boomeronic equation, namely the well–known three wave resonant interaction system [5]–[7].

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Slow-light solitons

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In the framework of the nonlinear Λ-model we investigate propagation of solitons in atomic vapors and Bose-Einstein condensates. We show how the complicated nonlinear interplay between fast solitons and slow-light solitons in the Λ-type media points to the possibility to create optical gates and, thus, to control the optical transparency of the Λ-type media. We provide an exact analytic description of decelerating, stopping and re-accelerating of slow-light solitons in atomic media in the nonadiabatic regime. Dynamical control over slow-light solitons is realized via a controlling field generated by an auxiliary laser. For a rather general time dependence of the field; we find the dynamics of the slow-light soliton inside the medium. We provide an analytical description for

the nonlinear dependence of the velocity of the signal on the controlling field. If the background field is turned off at some moment of time, the signal stops. We find the location and shape of the spatially localized memory bit imprinted into the medium. We discuss physically interesting features of our solution, which are in a good agreement with recent experiments.

We have applied the transformation of the slow light equations to Liouville theory that we also have developed, to study the influence of relaxation on the soliton dynamics. We solved the problem of the soliton dynamics in the presence of relaxation and found that the spontaneous emission from the upper atomic level is strongly suppressed. Our solution proves that the spatial shape of the soliton is well preserved even if the relaxation time is much shorter than the soliton time length. This fact is of great importance for applications of the slow-light soliton concept in optical information processing. We also demonstrate that the relaxation plays a role of resistance to the soliton motion and slows the soliton down even if the controlling field is constant.

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Exactly and Quasi-Exactly Solvable "Discrete" Quantum Mechanics

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Exactly and quasi-exactly solvable "discrete" Quantum Mechanics (QM) were constructed by Odake and myself [1]. In "discrete" QM, the momentum operator $p = -i\hbar\partial$ appears in exponentiated forms $e^{\pm\gamma p}$, which generate shifts of a wavefunction $\psi(x)$: $e^{\pm\gamma p}\psi(x) = \psi(x \mp i\gamma)$ ($\gamma \in \mathbb{R}$ or $\sqrt{-1}\mathbb{R}$), giving rise to two different versions of 'discrete' QM. The Schrödinger equation becomes a *difference equation*. For real γ, *i.e.* the pure imaginary shifts, the coordinate x takes a continuous range, whereas for the real shifts, the coordinate x is on a lattice with the uniform interval $|\gamma|$. All the other axioms of quantum mechanics are well preserved. In the *exactly solvable* QM, on top of the complete set of eigenvalues and eigenfunctions, the Heisenberg operator solutions are exactly obtained; the latter indicating the underlying dynamical symmetry algebras, including the q-oscillator algebra [2]. The eigenfunctions consist of various hypergeometric orthogonal polynomials, e.g. the Askey-Wilson polynomials. They describe the classical equilibrium points of exactly solvable multi-particle dynamics [3] and also provide exact solutions of birth and death processes, a typical example of stationary Markov chains [4]. In *quasi-exactly solvable* QM, constructed with one or two compensation terms [5], a finite number of eigenvalues and eigenfunctions are obtained exactly.

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From soliton statistical mechanics to crumpling of stiff membranes

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Some recollections will be presented from the early years of soliton statistical mechanics. It will be explained how formulation of the partition function of the sine Gordon and sinh Gordon systems in terms of the Ôaction angle variablesÕ appeared to solve the ÔbreatherproblemÕ.

The rest of the talk will be devoted to a completely different problem, to new results [1,2] on folding and crumpling of stiff membranes, or thin sheets of material with nonzero bending stiffness. Self-similarity of the ridge patterns, fractal dimension of the crumpled configurations, and effects of plasticity on these properties, will be discussed. Effects of self-adhesion will be touched upon.

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From Berezinians to formal characteristic functions of maps of algebras

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Consider two algebras (commutative, associative, with unit) A and B, and a linear map φ from A to B. For example, A can be the algebra of functions on some space X and B be the field of real numbers. Which interesting classes of linear maps of algebras do exist? Obviously, there is a class consisting of the algebra homomorphisms. Geometrically algebra homomorphisms correspond to points of a space or to maps between two spaces. Are there any other good classes? Using a technique motivated by mathematical physics we introduce the notion of a characteristic function for a linear map φ, which is a formal power series in an auxiliary variable:

$$R(\varphi, a, z) = e^{\varphi \log(1+az)}.$$

Its functional properties reflect algebraic properties of the linear map φ. For example, the algebra homomorphisms correspond to the linear binomials $1 + \varphi(a)z$. If the characteristic function is a polynomial of degree n, then the map of algebras is a so-called "n-homomorphism". Examples of n-homomorphisms date back to Frobenius's works on matrix representations. Recently this theory was revived by Buchstaber and Rees motivated by studies of multi-valued groups. Using our approach we can very easily recover their results, in particular, their non-trivial generalization of the famous Gelfand–Kolmogoroff theorem. Moreover, it is possible to go further by considering rational characteristic functions. Consideration of rational characteristic functions is motivated by supergeometry, in particular, the theory of Berezinians (superdeterminants). On this way we obtain classes of maps satisfying very interesting identities. We can guess that such objects may be useful in integrable systems.

References

[1] H. Khudaverdian and Th. Voronov. On Berezinians, exterior powers and recurrent sequences. Lett. Math. Phys. **74** (2005), 201–228, `arXiv:math.DG/0309188`

[2] H. Khudaverdian and Th. Voronov. On generalized symmetric powers and a generalization of Kolmogorov–Gelfand–Buchstaber–Rees theory. *Russian Math. Surveys* **62** (3) (2007), 209–210, `arXiv:math.RA/0612072`

[3] H. Khudaverdian and Th. Voronov. Operators on superspaces and generalizations of the Gelfand–Kolmogorov theorem. In book: XXVI Workshop on Geometric Methods in Physics. Białowieża, Poland, 1–7 July 2007., AIP CP 956, Melville, New York, 2007, p. 149–155, `arXiv:0709.4402` [math-ph]

Robin Bullough Symposium Programme

Day/time	WED 10		THU 11	Day/time
09:00-09:15	OPENING		Sasaki	09:00-09:40
09:15-09:55	Gibbon		Eilbeck	09:40-10:20
09:55-10:35	Lakshmanan		Coffee Break	10:20-10:35
10:35-11:00	Coffee Break		Fordy	10:35-11:15
11:00-11:40	Christiansen		Hassan	11:15-11:55
11:40-12:20	Garraway		Bogoliubov	11:55-12:35
12:20-14:15	LUNCH with the Bullough family		LUNCH	12:35-14:30
14:15-14:55	Per & Patrick Bullough		Rybin	14:30-15:10
14:55-15:35	Caudrey		Lombardo	15:10-15:50
15:35-16:00	Coffee Break		CLOSING	15:50-16:00
16:00-16:40	Calogero		Coffee Break	16:00-16:30
16:40-17:20	Timonen			

For your Notes

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