The chaotic-representation property

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The martingale $X = (X_t : t \geq 0)$ said to be normal if $(X_t^2 - t : t \geq 0)$ is also a martingale. A theorem of Lévy states that Brownian motion is the only continuous normal martingale; the compensated Poission process is a normal martingale which is discontinuous.

For any normal martingale X, it is possible to define iterated Wiener integrals - integrals with respect to X of square-integrable deterministic functions - using Ito isometry. As is well known, for Brownian motion these integrals (together with the constant functions) are total in the space of square-integrable functions on the underlying probability space. The same holds for the Poisson process, and a normal martingale for which this is true is said to have the chaotic-representation property.

A necessary condition for the CRP is the predictable-representation property; applying this to the quadratic variation process $[X]$ give the structure equation

$$d[X]_t = F_t dX_t + dt,$$

where F is some predictable process. In this talk, we will explain how to demonstrate the CRP when F belongs to a family of affine functions of X. This is joint work with Stéphane Attal, Université Claude Bernard Lyon 1.