Diophantine Properties of Torsion Points and Special Points

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Let A be an abelian variety defined over a number field. One version of the Manin-Mumford Conjecture states that a subvariety of A does not contain a Zariski dense set of torsion points unless it is a component of an algebraic subgroup. Many new proofs have appeared after Raynaud originally proved this conjecture in 1983. For example, Pila and Zannier used the Pila-Wilkie Theorem on the rational points of definable sets together with a transcendence result on certain analytic functions due to Ax and finally Galois-theoretic properties of torsion points.

The André-Oort Conjecture bears a striking similarity to the Manin-Mumford Conjecture. Roughly speaking, torsion points are replaced by points on a moduli space corresponding to abelian varieties with complex multiplication. Pila gave a first unconditional proof of the André-Oort Conjecture for a product of modular curves.

The purpose of this course is to present the arithmetic ingredients involved in a general strategy to attack these problems. The emphasis will lie on explaining the Galois-theoretic properties of torsion
points on elliptic curves and special points on modular curves. This will enable us to complete a proof-sketch of the Manin-Mumford Conjecture for a product of elliptic curves and the André-Oort Conjecture for a product of modular curves. If time permits I will also discuss higher dimensional unlikely intersections and connections with heights.