

Functional transcendence via o-minimality

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Schanuel's conjecture efficiently encapsulates the expected transcendence properties of the exponential function. It implies the standard known results (Hermite-Lindemann, Lindemann-Weierstrass, Gelfond-Schneider, Baker,..) as well as the standard conjectures (independence of logarithms,...).

Ax (1971) proved Schanuel's conjecture in the setting of a differential field, the result being known as "Ax-Schanuel". This course will concentrate on Ax-Schanuel and related formulations in more general settings related to the Diophantine settings of the Manin-Mumford and Andre-Oort conjectures, in particular for the elliptic modular function (the j -function).

Schanuel's conjecture is central to the work of Boris Zilber on the model theory of exponentiation which led him to formulate "CIT", a Diophantine conjecture which (in the broader setting considered by Pink) includes those mentioned above as well as far-reaching generalisations.

These functional versions play a key role in the proofs of special cases of these conjectures using o-minimality. Ax gave two proofs of his results: one using differential algebra, and another using differential geometry. We will concentrate on proofs of such results using o-minimality and point-counting.