Sparse space-time Petrov-Galerkin discretizations for parabolic evolution equations

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In view of applications such as optimal control problems with parabolic PDE constraints and massively parallel computations of time-dependent problems, space-time compressive discretizations of parabolic evolution equations are of significant interest.

We discuss space-time (sparse) tensor product simultaneous Petrov-Galerkin discretizations of parabolic evolution equations, and propose efficient preconditioners for the iterative solution of the resulting single linear system of equations. Therein, space-time stability of the discretization, i.e., the validity of the discrete inf-sup condition with respect to suitable space-time norms uniformly in the discretization parameters, is essential.

Viewing the Crank-Nicolson time-stepping scheme as a space-time Petrov-Galerkin discretization, we can show that it is conditionally space-time stable for those space-time norms. This motivates a general minimal residual Petrov-Galerkin discretization framework along with space-time stable families of trial and test spaces of (sparse) tensor product type, resulting in space-time compressive discretization algorithms.

Additional interesting properties of the proposed algorithm include: very low regularity requirements on the input data; modularity in the spatial discretization; possibility of high-order nonuniform temporal discretization. Several natural questions, such as the validity of the maximum principle, and incorporation of space-time adaptivity while maintaining stability, are open.

References:

For a condensed description and a basic Matlab implementation of the algorithm see:

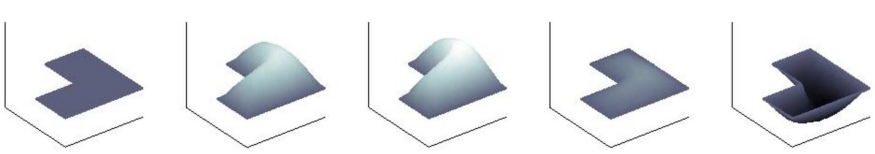
Space-time discretization of the heat equation. A concise Matlab implementation.
eprint arXiv:1212.6037 (includes code), **2012** → WWW: http://arxiv.org/abs/1212.6037

The essentials of the theoretical background and some numerical results are given in:

Stability of sparse space-time finite element discretizations of linear parabolic evolution equations.
IMA J Numer Anal (**2013**) 33(1): 242–260 → DOI: 10.1093/imanum/drs014

Further details and extensions are discussed in:

Stability of space-time Petrov-Galerkin discretizations for parabolic evolution equations.
PhD thesis, ETH Zurich, ETH Diss No 20842, **2012** → DOI: 10.3929/ethz-a-007563932
and
R.A. and C. Tobler: *Multilevel preconditioning and low rank tensor iteration for space-time simultaneous discretizations of parabolic PDEs.*
SAM report 2012-16. ETH Zürich, **2012** → WWW: http://www.sam.math.ethz.ch/sam_reports



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