Coarse Grid Correction for the Neumann–Neumann Waveform Relaxation Method

Felix Kwok
University of Geneva
felix.kwok@unige.ch

Abstract

In the recent paper [2], a new variant of the waveform relaxation (WR) method based on Neumann–Neumann iterations has been proposed for the solution of linear parabolic PDEs. Just like for the steady case, one step of the method consists of solving the subdomain problems using Dirichlet traces, followed by a correction step involving Neumann interface conditions. However, each subdomain problem is now in both space and time, and the interface data to be exchanged are also functions of time. One advantage of the WR framework is that it allows the use of different spatial and time discretizations for each subdomain. Moreover, it has been shown in [1] that for finite time intervals, the Neumann-Neumann waveform relaxation (NNWR) method converges superlinearly both in one spatial dimension and for 2D decompositions into strips. Unfortunately, convergence deteriorates significantly as the number of subdomain increases, since the method does not allow communication between far-away subdomains.

The goal of this talk is to introduce a coarse grid component to the NNWR method in order to make it scalable with respect to the number of subdomains. In 1D, an exact coarse grid correction can be calculated by considering shape functions that satisfy the homogeneous PDE; this yields an iteration that converges to the exact solution in two steps. In 2D, however, the exact coarse grid correction yields a matrix problem that is too large and dense to be of practical value. Thus, we must seek corrections in a smaller subspace, consisting of coarse hat functions or discontinuous piecewise linear elements. We analyze the convergence of the method for different choices of coarse spaces and show its scaling behaviour when we vary the number of subdomains.

This is joint work with Martin J. Gander, Sébastien Loisel and Kévin Santugini.

References

[1] M. J. Gander, F. Kwok, and B. C. Mandal. Dirichlet–Neumann and Neumann–Neumann waveform relaxation methods for the time-dependent heat equation. *In preparation.*

[2] F. Kwok. Neumann–Neumann waveform relaxation for the time-dependent heat equation. *Submitted to the Proceedings of the 21st International Conference on Domain Decomposition Methods*, 2012.