Course ID   020756

Calculus of Several Variables

MATH 20132

Unit coordinator: Mark Coleman

Credit rating 10
ECTS credits 5

Semester 2

School of Mathematics
Undergraduate

Level 2

FHEQ level ' Middle part of Bachelors'

Marketing course unit overview

Functions of several variables were briefly considered in first year calculus courses when the notion of partial derivative was introduced. Although there are some similarities with the familiar theory of one real variable, the theory for functions of several variables is far richer. For example, for functions of several variables, the critical points might be maxima, minima or saddle points (which are minima in one direction and maxima in another direction). A key idea is to generalize the definition of the derivative at a point to the the derivative of a map f: Rn Rm at a point a of Rn. This is the Frchet derivative, which is a linear map df(a): Rn Rm (often represented by a matrix whose entries are partial derivatives) which gives the best approximation to the function at the point a. This derivative is used in a number of very elegant and useful results, in particular the Inverse Function Theorem and the Implicit Function Theorem, and is a key notion in the study of the critical points of functions of several variables.

The Frchet derivative is an example of a differential 1-form on Rn and so naturally leads on to an introduction to the basic ideas of differential k-forms. Differential k-forms are fundamental in the integral calculus of functions of several variables and this is briefly considered.

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Aims
The aim of this lecture course is to introduce the basic ideas of calculus of several variables.

Learning outcomes
On the successful completion of this lecture students should:

understand the notion of the limit of a function of several variables at a point and be able to find simple limits;
understand the notion of a continuous function of several variables;
understand the directional derivatives, the partial derivatives and the Frchet derivative of a function of several variables at a point; be able to find these; and understand the relationship between these notions;
be able to find the critical points on a real-valued function of several variables and determine the nature of non-degenerate critical points using the Hessian matrix;
understand and be able the use the Chain Rule, the Inverse Function Theorem and the Implicit Function Theorem;
be able to apply the method of Lagrange multipliers to simple extremum problems with a constraint;
understand the notion of a differential k-form on an open subset of Rn; be able to evaluate such forms at a point; be able to evaluate the wedge product of two forms and the derivative of a form; be able to evaluate line intgrals of 1-forms and surface integrals of 2-forms over a surface parametrized by a rectangle.

Syllabus
1.Continuous functions of several variables.
2.Differentiation of real-valued functions of several variables.
3.Critical points and higher partial derivatives.
4.Differentiation of vector-valued functions of several variables.
5.Differential forms and integration of differential forms.

Assessment methods

Other	15%
Written exam	85%

A coursework test in first week after Easter (to be confirmed - Easter is very late next year so it may be before Easter): weighting 15%;
2 hours end of semester examination: weighting 85%.

Feedback methods
Tutorials will provide an opportunity for students’ work to be discussed and to provide feedback on their understanding.

Requisites

NONE

Available as free choice? N

Recommended reading

- M.J. Field, Differential Calculus and its Applications, Van Nostrand 1976.
- W. Fleming, Functions of Several Variables, Addison-Wesley 1965.
- J. and B. Hubbard, Vector Calculus, Linear Algebra, and Differential Forms, Prentice Hall 1998.
- C.H. Edwards, Jr., Advanced Calculus of Several Variables, Dover Publications 1994.
- R. Courant and F. John, Introduction to Calculus and Analysis, Volume 2, Wiley 1974.
- H.M. Edwards, Advanced Calculus: a Differential Forms Approach, Birkhauser 1994.

Scheduled activity hours

Lectures	22
Tutorials	11

Independent study hours 67 hours

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